

# Study on Digital Beamforming for Spaceborne SAR Based on Sparse DOA Estimation

Taoli Yang<sup>1</sup>, Yong Wang<sup>1,2</sup>

<sup>1</sup>School of Resources and Environment, University of Electronic Science and Technology of China, Chengdu 611731, China

<sup>2</sup>Department of Geography, Planning, and Environment, East Carolina University, Greenville, NC 27858, USA

**Abstract** - A novel digital beamforming (DBF) algorithm for spaceborne synthetic aperture radar (SAR) system based on sparse direction-of-arrive (DOA) estimation is presented, which can solve the beam mispointing caused by the terrain height. Compared to the traditional subspace-based algorithms, the approach has no strict requirement for the number of snapshots. Finally, the simulated results validate the proposed algorithm.

**Index Terms** — Digital beamforming (DBF), direction-of-arrive (DOA), sparse, synthetic aperture radar (SAR).

## 1. Introduction

Synthetic aperture radar (SAR) system combined with digital beamforming (DBF) technique has becoming a research hotspot [1], [2]. In order to increase the signal to noise ratio (SNR) of the received echo, the scan-on-receive (SCORE) technique can be adopted [3]: multiple subapertures and DBF are employed to form a sharp and high gain beam pattern which follows the echo as it travels along the ground swath. With the hypothesis of a stringent spherical Earth model, the direction-of-arrival (DOA) of the echo is calculated according to the imaging geometry, and then the weights for the subapertures are obtained to form the correct beam. Nevertheless, in the presence of topographic height, there is a mispointing by using the ideal model, which results in the gain loss of the received echo and deteriorates the SNR.

In order to solve the problems, the adaptive DBF approach based on subspace algorithms, such as MUSIC and Capon, are proposed to estimate the DOA to reduce the effect of terrain height [4]-[6]. However, the subspace-based algorithms have strict requirement for the number of snapshots.

## 2. Signal Model

The considered SAR geometry is shown in Fig. 1, where the antenna is divided into  $M$  subapertures in elevation dimension,  $H_s$  is the satellite height,  $d_m$  is the distance from the  $m$ th subaperture to the first subaperture,  $\alpha$  is the antenna plane orientation angle,  $r_1$  and  $r_m$  are the slant range from the first and the  $m$ th subaperture to the target  $P$  located in the swath, respectively.  $h$  represents the target elevation.  $\theta$  is the angle between  $r_1$  and normal direction of the antenna plane, which can be obtained by the following equation

$$\theta = \arccos \left[ \frac{(R_e + H_s)^2 + r_1^2 - (R_e + h)^2}{2(R_e + H_s)r_1} \right] - \alpha, \quad (1)$$

where  $R_e$  is the radius of the Earth.

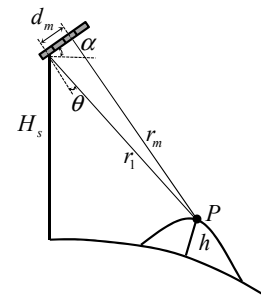


Fig. 1. Geometry of SAR system.

Considering the white noise, the received signal of the  $m$ th subaperture after range compression is [5]

$$s_m(\tau) = s_1(\tau) \exp \left( j2\pi f_c \frac{d_m \sin \theta}{c} \right) + n_m(\tau), \quad (2)$$

where  $s_1(\tau) = \sigma p_r \left( \tau - \frac{2r_1}{c} \right) \cdot \exp \left( -j4\pi f_c \frac{r_1}{c} \right)$ ,  $\tau$  is the range fast time,  $\sigma$  is the complex reflectivity,  $f_c$  is the carrier frequency,  $c$  is the propagation velocity,  $p_r(\tau)$  is the compressed pulse envelop, and  $n_m(\tau)$  is the white noise.

Using vector notation, one can express the signal as

$$s = s_1(\tau) \mathbf{p}(\theta) + \mathbf{n} \quad (3)$$

where

$$\mathbf{s} = [s_1(\tau), \dots, s_M(\tau)]^T \quad (4)$$

$$\mathbf{p}(\theta) = \left[ 1, \dots, \exp \left( j2\pi f_c \frac{d_M \sin \theta}{c} \right) \right]^T \quad (5)$$

$$\mathbf{n} = [n_1(\tau), \dots, n_M(\tau)]^T \quad (6)$$

The superscript  $T$  denotes the transpose operation.

## 3. Processing Algorithm

Considering the sparsity of the received signal, (3) can be reformatted as

$$s = \tilde{\mathbf{P}}(\theta) \tilde{\mathbf{s}}_1 + \mathbf{n} \quad (7)$$

where  $\tilde{\mathbf{P}}(\theta) = [\mathbf{p}(\theta_1), \mathbf{p}(\theta_2), \dots, \mathbf{p}(\theta_Q)]$  is composed of steering vectors corresponding to all the potential signal

directions, and  $Q$  is the number of all the possible signal directions.  $\tilde{s}_1$  is a sparse vector whose  $q$ th element is  $s_1(\tau)$  if the echo signal comes from  $\theta_q$  and zero otherwise. Usually,  $Q > M$ , the DOA estimation is converted into the following sparse representation [7]

$$\min \|s - \tilde{P}(\theta)\tilde{s}_1\|_2^2 + \mu\|\tilde{s}_1\|_1 \quad (8)$$

where,  $\|\cdot\|_2$  and  $\|\cdot\|_1$  represent the  $\ell_2$ -norm and  $\ell_1$ -norm, respectively, and  $\mu$  is the regularization parameter.  $\tilde{s}_1$  can be achieved by solving the above optimization problem, and the DOA of the echo is obtained according to the peak.

The sparse problem shown in (8) is a convex problem, which can be efficiently solved by many available algorithms. Compared to the traditional adaptive DBF methods, the proposed algorithm does not need multiple snapshots to estimate the covariance matrix, which is of great value since only one snapshot is available in most cases. Of course, multiple snapshots can improve the estimated accuracy.

By using the estimated  $\hat{\theta}$ , the weighted vector of the subapertures at time  $\tau$  is

$$\omega(\tau) = p^*(\hat{\theta}(\tau)), \quad (9)$$

where the superscript \* denotes conjugate operation.

#### 4. Simulations

Simulated data are used to demonstrate the performance of the proposed algorithm. The simulation parameters are listed in Table I, where  $\beta_c$  denotes the center nadir angle, and  $d_r$  is the total antenna size in elevation. One target located in the scene center is simulated. After applying range compression, the DOA is estimated by using one azimuth sample. Fig. 2(a) and (b) show the variation of the beam mispointing and the associated gain loss with target elevation, where the solid blue and dashed red lines represent the results by using the traditional SCORE [3] and the proposed algorithm, respectively. It can be seen that with the increasing of the target elevation, the mispointing and the gain loss of the SCORE method is increasing, while the performance of the proposed method almost remains unchanged. That is to say, the proposed method can deal with the complicated terrain.

It should be noted that the proposed method only adopt one snapshot. For the traditional adaptive method [4]-[5], at least  $2N+1$  snapshots should be used, where  $N$  is the number of the estimated signals. Only one snapshot will cause great estimated biases.

TABLE I  
Simulation Parameters

$H_s$	$\beta_c$	$f_c$	$d_r$	$M$
514km	32°	9.65GHz	15×0.1m	15

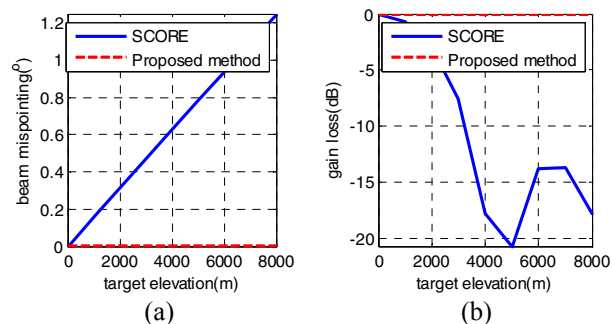


Fig. 2. Beam mispointing (a) and gain loss (b) for different target elevation by using SCORE and the proposed method.

#### 5. Conclusion

A novel DBF algorithm for spaceborne SAR system based on sparse DOA estimation is studied. In the presence of terrain height, the traditional SCORE algorithm would cause beam mispointing and gain loss. By adding the sparse constraint, the proposed algorithm solves the problem. In addition, it relaxes the requirement for quality of the estimated covariance matrix, or the number of snapshots compared to the subspace-based algorithms. The simulated results verify the effectiveness of the proposed algorithm.

#### Acknowledgment

This work was funded by Key Laboratory of Satellite Mapping Technology and Application, National Administration of Surveying, Mapping and Geoinformation under grant KLSMTA-201507, the Fundamental Research Funds for the Central Universities under grants ZYGX2015J115 and ZYGX2013Z006.

#### References

- [1] G. Krieger, N. Gebert and A. Moreira, "Multidimensional waveform encoding: a new digital beamforming technique for synthetic aperture radar remote sensing," *IEEE Trans. Geosci. Remote Sens.*, vol. 46, no. 1, pp. 31-46, 2008.
- [2] A. Moreira, G. Krieger and I. Hajnsek, et. al., "Tandem-L: a highly innovative bistatic SAR mission for global observation of dynamic processes on the earth's surface," *IEEE Geosci. Remote Sens. Mag.*, vol. 3, no. 2, pp. 8-23, 2015.
- [3] A. Currie, and M. A. Brown. "Wide-swath SAR," *IEE Proc Part F Radar Signal Process.*, vol. 139, no. 2, pp. 122-135, 1992.
- [4] F. Bordoni, M. Younis and E. M. Varona, et. al., "Adaptive scan-on-receive based on spatial spectral estimation for high-resolution, wide swath synthetic aperture radar," *IGARSS'09.*, 2009, Cape Town, South Africa: IEEE, pp. 1-64-1-67.
- [5] F. He, X. Ma and Z. Dong, et. al., "Digital beamforming on receive in elevation for multidimensional waveform encoding SAR sensing," *IEEE Geosci. Remote Sens. Lett.*, vol. 11, no. 12, pp. 2173-2177, 2014.
- [6] G. Kriegerr, S. Huber and M. Villano, et. al., "CEBRAS: Cross elevation beam range ambiguity suppression for high-resolution wide-swath and MIMO-SAR imaging," *IGARSS'15.*, 2015, Milan, Italy, pp. 196-199.
- [7] D. Malioutov, M. C. etin, and A. S. Willsky, "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010-3022, Aug. 2005.