SCATTERING OF HERMITE-GAUSSIAN BEAMS FROM A RANDOM ROUGH SURFACE—3D SCALAR ANALYSIS

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INTRODUCTION The wave scattering from rough surfaces is not only one of interesting academic problems but also a practical one closely related to remote sensing such as various kind of measuring or diagnosing techniques by the utilization of micro waves, millimeter waves, lasers, or ultrasonic waves(1). From various points of view, there have been extensive approaches to this kind of problem(2),(3). However, most of them have taken the case of plane-wave incidence into account. There have been a few studies considering the inhomogeneity of the amplitude and phase of an incident wave such as a spherical wave, a cylindrical wave ,or a beam wave(4),(5). Especially, the scattering of Gaussian beam from rough surfaces is becoming more important problem related to measurement and diagnostics with the recent advance of laser or ultrasonic wave technology.

The purpose of this paper is to present a three-dimensional version of our previous paper(5) , that is, the three-dimensional scalar analysis of the scattering of Hermite-Gaussian beams from an irregular surface. Both the incident beam and the rough surface are three-dimensional. In treating the rough surface, we are obliged to use some approximation, i.e., a certain kind of mathematical model of the surface. As the first step toward the complete analysis of the three-dimensional scattering of inhomogeneous waves from rough surfaces, we use the Kirchhoff approximation(2), which is one of the simplest models and also useful and convenient in some cases. In order to utilize the results of plane-wave version, the plane-wave spectrum representaion is used in the treatment of the incident beam-wave. Both coherent and incoherent components of the intensity of the scattered field are derived. In order to examine how the beam parameters and the stochastic quantities such as surface roughness and correlation length affect the scattering characteristics, numerically computed results are obtained and an example of them is shown.

FORMULATION The coordinate system of the present problem is selected as shown in Fig.1. An arbitrary point on the irregular surface is given by the coordinate (x, y, ζ), where ζ is a random variable and the mean level of the surface is the plane z=0. The incident beam is assumed to be a three-dimensional scalar Hermite-Gaussian beam of arbitrary order. Let us take the coordinate system O'(x',y',z') where the incident beam is initially represented as shown in Fig. 1. The beam axis coincides with the x' axis and let the angles (θ_1, ϕ_1) between the x' axis and both the z and x axes be an incident angle of the beam. The beam waist is located at the origin of the coordinate system O', which is separated from the origin of the coordinate system O by r_0. We choose the configuration of two coordinate systems such that the x'-z' plane involves the z axis. The field distribution of the beam waist is expressed as

$$\psi_i(x'=0,y',z')=A_{mn}H_m(\sqrt{2}y'/W_0)H_n(\sqrt{2}z'/W_0)\exp[-(y'^2+z'^2)/W_0^2] \quad (1)$$

where we chose a coefficient A_{mn} as

$$A_{mn}=\sqrt{2}/(2^{m+n}m!n!W_0^2) \quad (2)$$

so that the integration of the square of (1) over the entire y'-z' plane becomes unity. W_0 is the beam radius or spot size of the incident beam and $H_n(x)$ is the Hermite polynomial of order n . The time dependence of the field $\exp(-i\omega t)$ is suppressed.

The spectrum function of (1) G_{mn} is given by the Fourier transform of (1). Hence the incident beam field in the coordinate system (x',y',z') can be expressed by the inverse transform. By using the coordinate transformation between two systems O' and O, we can obtain the plane wave spectrum representation of the incident beam field in the coordinate system O as

$$\psi_i(x,y,z)=\iint_{-\infty}^{\infty} G_{mn}(\eta,\mu)\exp[ik(x\sin\alpha_i\cos\beta_i + y\sin\alpha_i\sin\beta_i - z\cos\alpha_i)]d\eta d\mu \quad (3)$$

where

$$G_{mn}(\eta,\mu)=(kW_o)^2(-i)^{m+n}A_{mn}/(4\pi)H_m(kW_o\eta/\sqrt{2})H_n(kW_o\mu/\sqrt{2}) \times\exp[-k^2W_o{}^2(\eta^2+\mu^2)/4+ikr_o\xi] \quad (4)$$

$$\cos\alpha_i=-(k_z/k)=\xi\cos\theta_i-\mu\sin\theta_i,\ \sin\alpha_i=1-\cos^2\alpha_i,$$
$$\cos\beta_i=[(\mu\cos\theta_i+\xi\sin\theta_i)\cos\phi_i-\eta\sin\phi_i]/\sin\alpha_i,$$
$$\sin\beta_i=[(\mu\cos\theta_i+\xi\sin\theta_i)\sin\phi_i+\eta\sin\phi_i]/\sin\alpha_i \quad (5)$$

$\xi=k_x'/k$, $\eta=k_y'/k$ and $\mu=k_z'/k$ denote the normalized propagation constants in the x', y', and z' directions, respectively.

On the other hand, for a plane wave incident on an irregular surface with an arbitrary incident angle (α_i, β_i), the scattered field can be obtained by using the Kirchhoff aproximation [(1),(2)] . Hence the scattered field at the point (R_0 , θ_s, ϕ_s) far from the origin is given by the following spectrum representation:

$$\psi_{mm}{}^s(R_o,\theta_s,\phi_s)=\psi_o\iint_{-\infty}^{\infty} G_{mn}(\eta,\mu)F(\eta,\mu) \times\exp[i(V_xx+V_yy+V_z\zeta)]dxdyd\eta d\mu, \quad kR_o>>1 \quad (6)$$

where

$$\psi_o=ik\exp(ikR_o)/(2\pi R_o),$$
$$F(\eta,\mu)=-[1+\cos\alpha_i\cos\theta_s-\sin\alpha_i\sin\theta_s\cos(\beta_i-\phi_s)]R/(\cos\alpha_i+\cos\theta_s),$$
$$R=\begin{cases}-1 & \text{for Dirichlet boundary condition,}\\ +1 & \text{for Neumann boundary condition,}\end{cases}$$
$$V_x=k(\sin\alpha_i\cos\beta_i-\sin\theta_s\cos\phi_s),\ V_y=k(\sin\alpha_i\sin\beta_i-\sin\theta_s\sin\phi_s),$$
$$V_z=-k(\cos\alpha_i+\cos\theta_s) \quad (7)$$

Equations (6) and (7) can be expressed as functions of (ξ , η, μ). By assuming the height of the surface $\zeta(x, y)$ is normally distributed with the mean value zero and the standard deviation σ , the mean value of the far-zone scattered field is calculated after some tedious manipulations as

$$\langle \psi_{mn}{}^{s} \rangle = (4\pi/k)\psi_{o}{}^{2}G_{mn}(\eta_{o}, \mu_{o})\exp(-2k^{2}\sigma^{2}\cos^{2}\theta_{s})$$

$$\times[\cos\theta_{i}\cos\theta_{s}+\sin\theta_{i}\sin\theta_{s}\cos(\phi_{s}-\phi_{i})]R \tag{8}$$

where ⟨A⟩ stands for taking the mean value of A and

$$V_{z}=-2k\cos\theta_{s}, \quad \eta_{o}=\sin\theta_{s}\sin(\phi_{s}-\phi_{i}),$$
$$\mu_{o}=-\sin\theta_{i}\cos\theta_{s}+\cos\theta_{i}\sin\theta_{s}\cos(\phi_{s}-\phi_{i}),$$
$$\xi_{o}=\cos\theta_{i}\cos\theta_{s}+\sin\theta_{i}\sin\theta_{s}\cos(\phi_{s}-\phi_{i}),$$
$$F(\eta_{o}, \mu_{o})=-R\cos\theta_{s} \tag{9}$$

Equation (8) corresponds to the coherent component of scattered fields which is originated from the specularly reflected plane wave component with the incident angle $\alpha_{i} = \theta_{s}$ and $\beta_{i} = \phi_{s}$. The variance of the scattered field is defined as

$$D\{\psi_{mn}{}^{s}\}=\langle \psi_{mn}{}^{s}\psi_{mn}{}^{s*}\rangle - \langle \psi_{mn}{}^{s}\rangle \langle \psi_{mn}{}^{s*}\rangle$$
$$=\psi_{o}{}^{2}\iiiint_{-\infty}^{\infty} G_{mn}(\eta_{1}, \mu_{1})G_{mn}{}^{*}(\eta_{2}, \mu_{2})F(\eta_{1}, \mu_{1})F^{*}(\eta_{2}, \mu_{2})$$
$$\times\iiiint_{-\infty}^{\infty}\exp[i(V_{x1}x_{1}+V_{y1}y_{1})-i(V_{x2}x_{2}+V_{y2}y_{2})]$$
$$\times \{ \langle\exp\{i(V_{z1}\zeta_{1}-V_{z2}\zeta_{2})\}\rangle - \langle\exp(iV_{z1}\zeta_{1})\rangle$$
$$\times \langle\exp(-iV_{z2}\zeta_{2})\rangle \} dx_{1}dx_{2}dy_{1}dy_{2}d\eta_{1}d\mu_{1}d\eta_{2}d\mu_{2} \tag{10}$$

where the asteriks denotes the complex conjugate. In order to evaluate (10), we introduce the two-dimensional normal distribution of two variables with mean value zero, variance σ^{2} and a correlation coefficient C as given in references (1) and (2). After somewhat long manipulations, we finally obtain the relatively simple integral form of the variance of the scattered field as follows:

$$D\{\psi_{mn}{}^{s}\}=\psi_{o}{}^{2}4\pi^{3}T^{2}/(k^{2})\iint_{-\infty}^{\infty} | G_{mn}(\eta, \mu)F(\eta, \mu) |^{2}\exp[-(V_{z}\sigma)^{2}]$$
$$\times\sum_{j=1}^{\infty}(V_{z}\sigma)^{2j}/(j!j)\exp[-(V_{x}{}^{2}+V_{y}{}^{2})T^{2}/(4j)]$$
$$\times \xi/(\mu\sin\theta_{1}-\xi\cos\theta_{1})d\eta d\mu \tag{11}$$

where T is the correlation length. Since the spectrum function decreases rapidly as η and μ are increased from zero, we can replace the integration limits $\pm\infty$ by finite values at which G_{mn} becomes sufficiently small. Therefore, equation (11) can be evaluated by using an appropriate numerical integration method.

The mean intensity of the scattered field is expressed as

$$\langle \psi_{mn}{}^{s} \psi_{mn}{}^{s*} \rangle = \langle \psi_{mn}{}^{s} \rangle \langle \psi_{mn}{}^{s*} \rangle + D\{\psi_{mn}{}^{s}\} \tag{12}$$

The first and second terms in the right side of (12) correspond to the coherent and incoherent components of the mean scattered intensity, which are given by (8) and (11), respectively.

NUMERICAL RESULTS For convenience sake of numerical calculation, we normalize the mean intensity and the coherent and incoherent components in (12) by multiplying $2\pi k^2 R_0{}^2$ and let them be $P_{a,m,n}$, $P_{c,m,n}$ and $P_{i,m,n}$, respectively. In order to carry out the double integration with respect to the normalized wave numbers η and μ for the incoherent component, we select the integration limits at which the spectrum function becomes the order 10^{-5}-10^{-6}. These integration limits depend on the spot size, however we verified that the ranges of integration $|\eta|$, $|\mu| \leq 0.21$ satisfies the present condition for $W_0/\lambda \geq 5$. Fig.2 is illustrating the dependence of the back scattered intensity on σ for the normal incidence of the fundamental beam. It is shown that the back scattering is uniformly decreased by both the increase of σ and the decrease of the spot size. The coherent component is dominant in the small range of σ and the incoherent one being in the larger range of σ . The crossover point of of both components decreases as W_0 becomes small.

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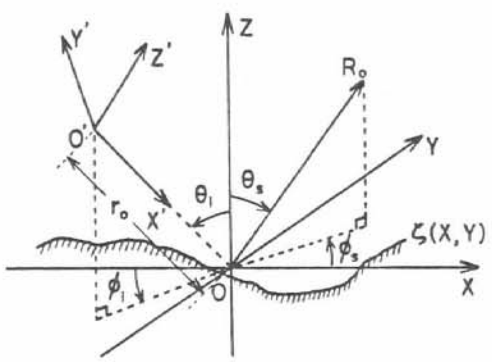

Fig.1 Geometry of the problem.

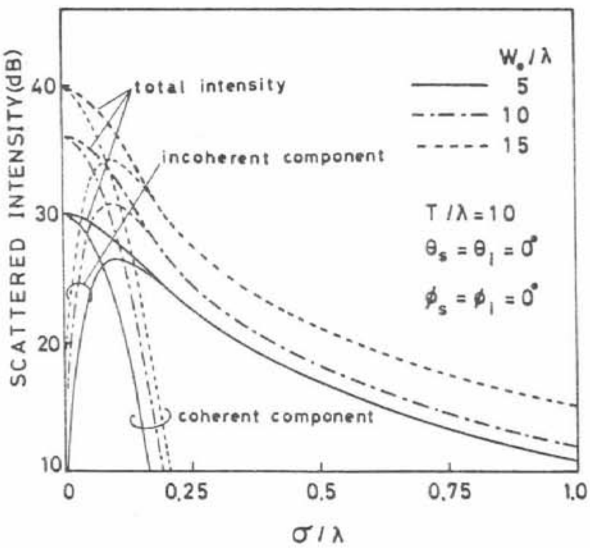

Fig.2 Back-scattered intensity vs. σ .