

# Antenna Design and Channel Characterization for Mobile MIMO Systems

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## 1. Introduction

One significant difficulty associated with the implementation of multiple-input multiple-output (MIMO) technology is that communication node mobility leads to a time-varying channel which can limit the achievable system throughput [1]. This paper quantifies the degradation in channel capacity resulting from channel temporal variation using novel information theoretic metrics, and demonstrates that proper antenna element radiation properties can reduce the impact of this variability. Furthermore, for rapidly fading channels when channel covariance information is used to construct the transmit signaling, recent studies have suggested that capacity is maximized when the transmit antenna spacing is small [2]. This paper shows that this observed phenomenon results from increased radiated power arising from mutual coupling effects. When the radiated power is constrained, the analysis shows that a conventional array design (approximately  $\lambda/2$  element spacing) is optimal.

## 2. Channel Temporal Variation

While optimal MIMO communication requires nodes to have perfect channel state information (CSI) [1], in many cases channel fluctuations result in imperfect CSI. To study the impact of this effect, measured data were collected with a prototype wideband 8×8 MIMO channel sounder designed using a switched array architecture. The transmitter was placed in a central hallway, and the receiver was moved at a constant speed along a prescribed path in 8 different rooms adjacent to the hallway. The antenna arrays were 8-element uniform circular arrays (UCA) composed of monopole antennas with  $\lambda/2$  inter-element spacing. Measurements were taken at both 2.55 GHz and 5.2 GHz with 80 MHz of excitation bandwidth. The frequency bins in this study were spaced 10 MHz apart to achieve statistically independent samples. Each path was about 4.8 m in length and channel snapshots were obtained each 0.8 cm, giving a resolution of  $0.07\lambda$  and  $0.14\lambda$  at 2.55 and 5.2 GHz, respectively. The spatial variation results in this paper can be scaled by velocity to obtain temporal variation.

To characterize the impact of the channel temporal variation on the communication performance of a MIMO system, we analyze the degradation in capacity due to CSI becoming outdated. Consider the case of *transmit CSI degradation* where the receiver has perfect CSI but the transmitter only has the delayed channel estimate  $\hat{\mathbf{H}}$ . We may define capacity for delayed transmit CSI as

$$C_T = \log_2 \left| \frac{\mathbf{H}\mathbf{Q}(\hat{\mathbf{H}})\mathbf{H}^\dagger}{\sigma_\eta^2} + \mathbf{I} \right|, \quad (1)$$

where  $\mathbf{H}$  is the true channel,  $\sigma_\eta^2$  is the receiver noise variance,  $\mathbf{Q}(\hat{\mathbf{H}})$  is the optimal transmit covariance given by the water-filling solution (assuming  $\mathbf{H} = \hat{\mathbf{H}}$ ),  $\mathbf{I}$  is the identity matrix,  $\text{Tr}\{\mathbf{Q}\} \leq P_T$ , and  $P_T$  is total transmit power. In the results that follow,  $P_T$  and  $\sigma_\eta^2$  are always chosen such that the average single-input single-output (SISO) signal-to-noise ratio (SNR) is 10 dB. As the estimate  $\hat{\mathbf{H}}$  becomes more outdated,  $C_T$  will tend to decrease.

Next, consider the case of *receive CSI degradation*, where both the transmitter and receiver have outdated CSI. If the transmitter and receiver attempt to form parallel Gaussian channels using the singular

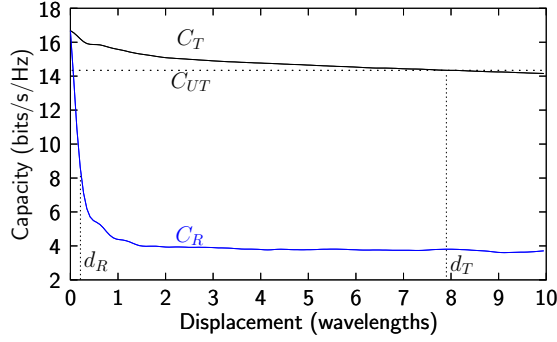


Figure 1: Capacity degradation metrics applied to a single location at 2.55 GHz compared to the capacity  $C_{UT}$  for an uninformed transmitter.

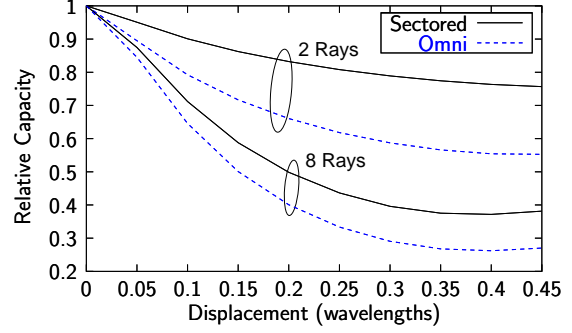


Figure 2: Normalized value of  $C_R$  for a UCA with sectored directional and omni-directional antennas.

value decomposition (SVD) of the delayed channel estimate ( $\hat{\mathbf{H}} = \hat{\mathbf{U}}\hat{\mathbf{S}}\hat{\mathbf{V}}^\dagger$ ), the capacity can be expressed as

$$C_R(\mathbf{H}, \hat{\mathbf{H}}) = \sum_i \log_2(1 + p_i \hat{S}_{ii}^2 / q_i), \quad (2)$$

where  $q_i = \{\mathbf{M}\mathbf{P}\mathbf{M}^\dagger\}_{ii} + \sigma_\eta^2$ ,  $\mathbf{M} = \hat{\mathbf{U}}^\dagger \hat{\mathbf{H}} \hat{\mathbf{V}} - \Phi \hat{\mathbf{S}}$ ,  $p_i$  are found according to water-filling (assuming  $\mathbf{H} = \hat{\mathbf{H}}$  and  $q_i = \sigma_\eta^2$ ),  $\mathbf{P} = \text{diag}(\mathbf{p})$ , and  $\Phi$  is a diagonal matrix with  $|\Phi_{ii}| = 1$ . In this work, we assume  $\arg(\Phi_{ii}) = \arg(\{\hat{\mathbf{U}}^\dagger \hat{\mathbf{H}} \hat{\mathbf{V}}\}_{ii})$ , allowing us to mask the effect of average phase variations of the individual eigenchannels and focus on the effects of the changing spatial structure. Figure 1 depicts the two capacity degradation metrics for one location at 2.55 GHz. These results demonstrate that while outdated CSI at the transmitter does not significantly degrade the capacity, outdated CSI at the receiver results in a dramatic capacity decrease with very small node displacement.

Figure 2 compares the value of  $C_R$  normalized to the value at zero displacement for an 8-element UCA with radius  $4\lambda$  with two possible types of antenna elements: (1) directional elements with  $\cos^2(\cdot)$  radiation patterns and half-power beamwidth of  $\lambda/4$ , with each beam pointing out of the UCA, and (2) omni-directional elements. Channels were computed assuming a simple uniformly distributed ray model with either 2 or 8 rays and normalized to obtain an average SNR of 10 dB. Capacity was averaged over 100 random realizations for each case. Additional analysis reveals that the average capacity for the two array types is the same. The results indicate that when the channel is highly structured (fewer rays), the directional antennas maintain high capacity while reducing the channel temporal variability.

### 3. Array Design for Rapidly Fading Channels

We now focus on the design of arrays for time-varying MIMO channels. Under rapidly fading conditions, it is common to use channel covariance information rather than CSI. It has recently been shown that in this case, increased transmit correlation obtained by compact arrays will improve performance [2]. To explore this phenomenon in more detail, we adopt the block-fading channel model  $\mathbf{X} = \sqrt{(\rho/P)}\mathbf{S}\mathbf{H} + \mathbf{W}$ , where  $\mathbf{S}$  is the  $T \times M$  matrix of complex baseband transmit signals,  $\mathbf{H}$  is the  $M \times N$  channel transfer matrix,  $\mathbf{X}$  is the  $T \times N$  matrix of receive samples,  $T$  is the block length, and  $M$  and  $N$  are the number of transmit and receive antennas, respectively. The quantities  $P$  and  $\rho$  represent the average power generated per unit time by the transmit signal matrix  $\mathbf{S}$  and the average SNR, respectively. The  $T \times N$  matrix  $\mathbf{W}$  of noise samples consists of i.i.d. zero-mean, unit-variance complex Gaussian elements.

The channel  $\mathbf{H}$  is assumed to be constant over blocks of length  $T$ , with elements given by the Kronecker model  $\mathbf{H} = \mathbf{R}_T^{1/2} \mathbf{H}_w \mathbf{R}_R^{1/2}$ , where  $\mathbf{R}_T = (1/N) \mathbb{E}\{\mathbf{H}\mathbf{H}^\dagger\}$  and  $\mathbf{R}_R = (1/M) \mathbb{E}\{\mathbf{H}^\dagger \mathbf{H}\}$  are the transmit and receive covariance matrices, and  $\mathbf{H}_w$  consists of i.i.d. zero-mean, unit-variance complex Gaussian elements. Covariance matrices are generated in this work by specifying the probability density function  $p(\phi)$  of departures or arrivals at angle  $\phi$  in the azimuthal plane. For a uniform linear array (ULA)

of infinitesimal dipoles with electrical element spacing of  $\Delta x$ , the covariance matrix has elements

$$R_{ik} = \int_0^{2\pi} d\phi p(\phi) \exp[j2\pi(i-k)\Delta x \cos \phi]. \quad (3)$$

In MIMO analyses, the traditional power constraint is  $P = P_{\text{tr}} = (1/T)\text{ETr}\{\mathbf{S}\mathbf{S}^\dagger\} = M$ , where  $\text{E}\{\cdot\}$  denotes expectation. However, when the antenna elements are electromagnetically coupled, the power radiated during the  $i$ th symbol time becomes  $P_i = \mathbf{s}_i \mathbf{A} \mathbf{s}_i^\dagger$ , where  $\mathbf{A}$  is an  $M \times M$  coupling matrix, and  $\mathbf{s}_i$  is the  $i$ th row of  $\mathbf{S}$  [3]. The average radiated power is therefore

$$P_{\text{rad}} = (1/T)\text{ETr}\{\mathbf{S}\mathbf{A}\mathbf{S}^\dagger\}. \quad (4)$$

For a ULA of vertically-oriented infinitesimal dipoles, we obtain  $A_{ij} = J_0[2\pi\Delta x(i-j)]$ .

For rapidly fading channels ( $T = 1$ ), the results of [2] indicate that only one spatial mode (or beam) should be used and therefore antenna placement should be chosen to maximize the principal channel eigenvalue (achieved with  $\Delta x \rightarrow 0$  in (3)). However, this is troubling from an electromagnetic perspective, since for  $\Delta x = 0$  the antennas should function as a single element. The apparent contradiction arises because the traditional power constraint allows high radiated power for close spacings [3]. Consider the case of  $M = 2$  transmit antennas. The transmit covariance matrix is of the form

$$\mathbf{R}_T = \begin{bmatrix} 1 & \gamma \\ \gamma^* & 1 \end{bmatrix}, \quad (5)$$

with eigenvalues  $\lambda_{1,2} = 1 \pm |\gamma|$  and eigenvectors  $\mathbf{v}_{1,2} = (1/\sqrt{2})[1 \pm \exp(-j\angle\gamma)]^T$ . We consider the case identical to [2], where for  $T = 1$  we use  $\mathbf{S} = \mathbf{s}'\mathbf{v}_1^\dagger$ . For uncoupled antennas, the gain of two antennas over a single antenna is the eigenvalue  $\lambda_1 = 1 + |\gamma|$ . However, to remove the impact of excess radiated power, we simply scale the transmit signals so that the radiated power computed in (4) achieves the desired value. This is accomplished by dividing  $\mathbf{S}$  by the square root of the factor

$$P_{\text{rad}}/P_{\text{tr}} = (1/M)P_{\text{rad}} = (1/M)\text{ETr}\{\mathbf{S}\mathbf{A}\mathbf{S}^\dagger\} = (1/M)\text{E}\{\mathbf{s}'^\dagger \mathbf{s}'\} \mathbf{v}_1^\dagger \mathbf{A} \mathbf{v}_1 = \mathbf{v}_1^\dagger \mathbf{A} \mathbf{v}_1, \quad (6)$$

leading to an effective gain of  $G_{\text{eff}} = \lambda_1/(\mathbf{v}_1^\dagger \mathbf{A} \mathbf{v}_1)$ . For infinitesimal dipoles  $G_{\text{eff}} = [1 + |\gamma|]/[1 + \cos(\angle\gamma)J_0(2\pi\Delta x)]$ . Thus, we are left with finding the antenna spacing that maximizes the effective gain. We will consider three different multipath distributions.

*Full Angular Spread:* For full angular spread  $p(\phi) = 1/(2\pi)$ , and the covariance elements become  $R_{ik} = J_0[2\pi\Delta x(i-k)]$ , where  $J_0(\cdot)$  is the zeroth order Bessel function. Furthermore,  $\gamma = J_0(2\pi\Delta x)$ , leading to  $G_{\text{eff}} = 1$  regardless of the antenna spacing. Thus, any increase in correlation due to reduced spacing is exactly offset by an increase in radiated power. To avoid difficulties with element coupling, antenna spacing should be as large as possible.

*L-path Model:* Next we consider the case of  $L$  discrete paths, each having a mean power of  $1/L$ . The path directions  $\phi_\ell$  are assumed to be i.i.d. uniform on  $[0, 2\pi]$ . The covariance elements become  $R_{ik} = (1/L) \sum_{\ell=1}^L \exp[j2\pi\Delta x(i-k) \cos \phi_\ell]$ . Figure 3 plots the mean effective gain computed by averaging  $G_{\text{eff}}$  over  $10^4$  channel realizations as a function of spacing. As expected, the effective gain decreases with increasing multipath. Also, for spacings less than about 0.4 wavelengths, coupling begins to counteract the benefits of correlation.

*Von Mises Cluster:* Consider a single departing cluster described with a von Mises angular distribution [4], yielding a covariance with elements  $R_{T,ik} = I_0\left(\sqrt{\kappa^2 - y^2 + j2\pi\kappa y \cos \bar{\phi}}\right)/I_0(\kappa)$  where  $y = 2\pi(i-k)\Delta x$ ,  $\bar{\phi}$  is the mean cluster angle relative to the array, and  $\kappa$  controls the cluster directivity (higher  $\kappa$  indicates narrower cluster). Figure 4 plots  $G_{\text{eff}}$  versus  $\Delta x$  for three values of  $\kappa$  for endfire mean departure angle ( $\bar{\phi} = 0$ ).

The results reveal that increased multipath causes a gain reduction. However, in contrast to the results observed for the discrete path model, very large spacings are now less desirable. This behavior

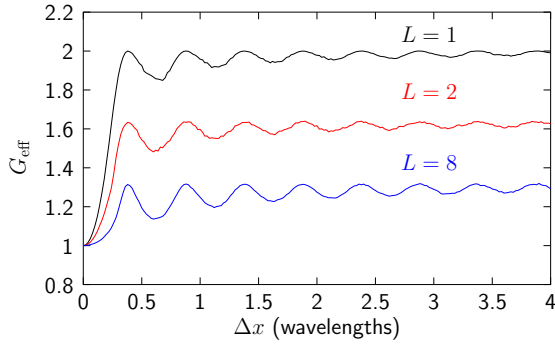


Figure 3: Effective gain as a function of antenna spacing for the  $L$ -path model from Monte Carlo simulations.

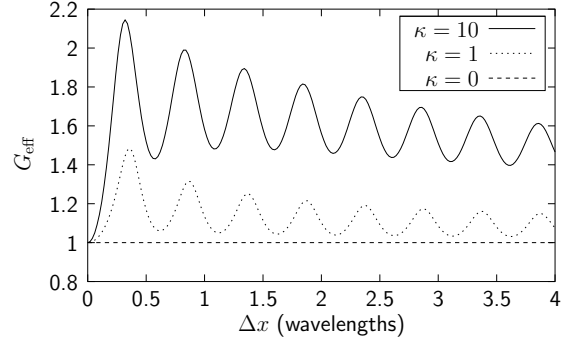


Figure 4: Effective gain as a function of antenna spacing assuming a single departing cluster distributed according to the von Mises distribution for three values of  $\kappa$  with  $\bar{\phi} = 0$ .

likely stems from the fixed mean departure angle, as the effective gain averaged over a uniformly distributed sequence of mean departure angles looks similar to the curves for the discrete path model. The key observation from this result is that if array orientation relative to the multipath can be controlled, close spacings may be advantageous. When the arrival angles are more random, very wide spacings appear to be nearly as optimal as narrow spacings.

## 4. Conclusions

This paper has explored the impact of antenna characteristics on the performance of MIMO systems in time-varying channels. Experimentally obtained channel data was used to develop metrics that allow quantification of temporal variability which were in turn used to show that directional antennas can be used to reduce temporal variation while maintaining high capacity. Results for rapidly fading channels where the system uses covariance information illustrated that arrays should have approximately half-wavelength inter-element spacing for optimal performance.

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