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DIRECTION-OF-ARRIVAL ESTIMATION OF CORRELATED SOURCES BY ADAPTIVE BEAMFORMING

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Introduction : Use of the adaptive beamforming in bearing estimation of closely spaced radiating sources has become very popular due to their superresolution properties [1-4]. It has been shown that this method is capable of resolving two independent sources which are separated by an angle which is of the order of one standard beamwidth (sbw) and sometimes less. The performance of this method is severely degraded, however, when coherent, or highly correlated signals are present [1]. It is the goal of the this work obtain analytical estimates of the potential capability for adaptive beamforming method in the presence of correlation among the sources. Our resolution analysis is based on the assumption that the covariance matrix of the received signal to be determined exactly by averaging over an infinite period of time, thus enabling us to obtain an estimate of the limit power of resolution.

Signal model : We consider a passive array, having N sensors, receiving stationary random narrow-band signals emanating from correlated point sources. The received signals are know to be embedded in spatially white noise with unknow variance in each sensor. Noise is assumed to be statistically independent with the signals. We shall be limited by the assumption, that array is linear, its sensors are omnidirectional and received signals have plane wavefronts. Following [2], adaptive beamforming algorithm involve the evaluation functional

$$P(\varphi) = (S^{+}R^{-1}S)^{-1} \qquad (1)$$

R is the covariance matrix, S is direction-of-look vector. Here, we are interested in the two-source case. Covariance matrix of the received signal can be expressed as

$$R = \sigma_0^2 I + \sigma_1^2 S_1 S_1^{+} + \sigma_2^2 S_2 S_2^{+} + \sigma_1 \sigma_2 (\rho S_1 S_2^{+} + S_2 S_1^{+} \rho^{*}) \qquad (2)$$

σ_0^2 is the noise power, σ_p^2 is variance of p-th signal, S_p is direction vector of p-th signal. The coefficient of correlation between signals is defined as follows $\rho = |\rho| exp\ (j\varphi)$, where $|\rho|$ and φ represent the modulus and argument of the correlation factor. Let us place the origin of the coordinates at the geometric center of the antenna. Following [2], we will determine the inverse covariance matrix

$$R^{-1}=\frac{1}{\sigma_0^2}\left\{ I-\frac{1}{\alpha}[\nu_1(1+N\nu_2(1-|\rho|^2))S_1S_1^+-(N\nu_1\nu_2(1-|\rho|^2)|f(\Delta u)|-\sqrt{\nu_1\nu_2}|\rho|exp(j\varphi))S_1S_2^+-(N\nu_1\nu_2(1-|\rho|^2)|f(\Delta u)|-\sqrt{\nu_1\nu_2}|\rho|exp(-j\varphi))S_2S_1^+ + \nu_2(1+N\nu_1(1-|\rho|^2))S_2S_2^+]\right\}, \quad (3)$$

where $\nu_{1,2}=\sigma^2_{1,2}/\sigma^2_0$, $|f(\Delta u)|=sin(N\Delta u/2)/(Nsin\Delta u/2)$, $\Delta u=(2\pi\delta/\lambda)(sin\varphi_1-sin\varphi_2)$, $\alpha=1+N(\nu_1+\nu_2)+N^2\nu_1\nu_2(1-|\rho|^2)(1-|f(\Delta u)|^2)+2N|\rho|\sqrt{\nu_1\nu_2}|f(\Delta u)|\ cos\varphi$, λ - wavelength, δ - interelement distance.

Resolution of source bearing: When the sources are resolved, the beam energy evaluated at either target bearing must be larger than the beam energy evaluated between the target bearings. The criterion used for resolution of the sources is that the ratio of on-target to between-target beam energies exceed a threshold value of 1 (Rayleigh resolution limit). For sources of identical intensity ($\nu_1=\nu_2=\nu_s$) situated near each other the threshold of resolution is defined by the condition

$$P(\varphi_{1,2}) \geq P((\varphi_1+\varphi_2)/2) \quad (4)$$

Then, making use of (1),(3), from (4) we obtain

$$1+2N\nu_s+N^2\nu_s^2(1-|\rho|^2)(1-|f(\Delta u)|^2)+2N|\rho|\nu_s\ cos\varphi\ (|f(\Delta u)|-|f(\Delta u/2)|^2)+2N^2\nu_s^2(1-|\rho|^2)|f(\Delta u)||f(\Delta u/2)|^2-2N\nu_s(1+N\nu_s(1-|\rho|^2))|f(\Delta u/2)|^2 \geq 1+N\nu_s(1-|f(\Delta u)|^2) \quad (5)$$

The complexity of expression (5) prevent a clear understanding of the effects of input signal-to-noise ratio (SNR), coherence, and bearing separation on resolving capability. To gain such an understanding, we will assume that the sources are localized in the main lobe and that the angular distance between them is small. Then, expanding the functions $|f(\Delta u)|$, $|f(\Delta u/2)|$ in an exponential series over Δu, we retain the first four terms of the expansion. As a result, for minimal Δu_{min} , from (5) we obtain

$$\Delta u_{min}=8.71\ ((1+|\rho|cos\varphi)/(N^5\nu_s\ (1-|\rho|^2)))^{1/4} \quad (6)$$

We can see that the increase in $|\rho|$ leads to a decrease in Δu_{min} ; however, even in the case of strongly correlated sources resolution is possible if the input SNR is sifficiently large. For example, for $|\rho|=0.99$ the minimum SNR is greater by 20 dB than for uncorrelated sources for $\varphi=0$. The relationship between Δu_{min} and φ indicates that the resolving power is determined by the location of the array. In particular, the array located at the maximum of the interference pattern ($\varphi=0$) exhibits a resolving power is lower than the array at the minimum of the interference pattern ($\varphi=\pi$). The maximum gain which can be achieved by changing the position of the adaptive array is determined by the quantity $(1+|\rho|)/(1-|\rho|)$. For coherent signal the resolution independent of the input SNR becomes impossible, with the exception of the case in which $\varphi=\pi$. The effect of resolving the coherent sources for $\varphi=\pi$ when using adaptive beamforming was apparently first detected in [4] by computer investigations. However, it should be noted that estimates of the angular position of the sources are significantly shifted.

Let us consider the case in which the angular distance Δu is on the order of the sbw. Then $|f(\Delta u)| \ll 1$, $|f(\Delta u/2)| \ll 1$. Condition (4) is satisfied for any $N\nu_u$. Proceeding from practical considerations, rewrite condition (4) in the form

$$P(\varphi_{1,2}) \geq \alpha\, P((\varphi_1+\varphi_2)/2),$$

where $\alpha>1$. The criterion used for detection of the sources, for example, is that $\alpha=2$ [2]. It then is not dificult to obtain the following condition

$$N\nu_u\,(1-|\rho|^2)^{1/2} \geq 1$$

We can see that the resolution of the coherent signals is impossible independly of the location of the array. For purposes of comparison let us note that for resolution (detection) of sources located at distances on the order of the sbw of classical or Bartlett beamforming we are confronted with the requirement that input SNR is equal to 1, regardless of the coherence.

Spatial smoothing algorithm : The promissing solution of the coherent signal problem was proposed by Evans et al [3] and developed by Shan et al [5]. Their spatial smoothing algorithm uses spatial averaging tecnhiques to "decorrelate" the signals. In this approach, the N-element linear antenna array is grouped into K overlapping subarrays each with M elements ($N=K+M-1$). Covariance matrices of the subarrays R_k are computed and averaged to obtain the spatially smoothed covariance matrix $\bar{\mathrm{R}}$

$$\bar{\mathrm{R}} = \frac{1}{K}\sum_{k=1}^{K} \mathrm{R}_k$$

It has been show the rank of the noise free spatially smoothed covariance matrix is the same as the number of sources. It was demonstrated in [6] that such procedure leads to a situation in which the modulus of the correlation factor diminishes from $|\rho|$ to

$$|\rho_0| = |\rho| \sin((K+1)\Delta u/2)/((K+1)\sin\Delta u/2) \qquad (7)$$

For simplicity, we will examine the case which $|\rho|=1$ and $\varphi=0$. Then (6) is changed to the form

$$\Delta u_{min} = 8.71\,(1/M^2\nu_u\,(1-|\rho_0|))^{1/4} \qquad (8)$$

Using (8), let us expand the expression for $|\rho_0|$ in a series over and let us limit ourselves to the first two terms of the expansion. Having taken the derivative with a respect to it is not difficult to determine optimal size of subarray

$$M_{opt}= \frac{5}{7} (N + 1) = \frac{5}{7} N \qquad (9)$$

Noted that in [7] it is suggested the spatial smoothing algorithm modification using property of persimmetry of covariance matrix of received signal. It was shown in [8] that this method leads to a sutiation in which the signal coherence diminishes to

$$|\rho_1|=|\rho|(sin((K+1)\Delta u/2)/((K+1)sin\Delta u/2))cos(\varphi+(N-1)\Delta u/2).$$

Our computer investigations shows that for this algorithm optimal size of subarray M_{opt} definites from (9) also.

Simulation results : This section provides several computer simulations. In all examples we considered the array is assumed to be linear and uniformly spaced with ten omnidirectional sensors. The in ten element spacing is assumed to be one-half wavelength, SNR in each sensor is 40 dB. The angular distance between the sources amounted to 5° with a width of about 12°

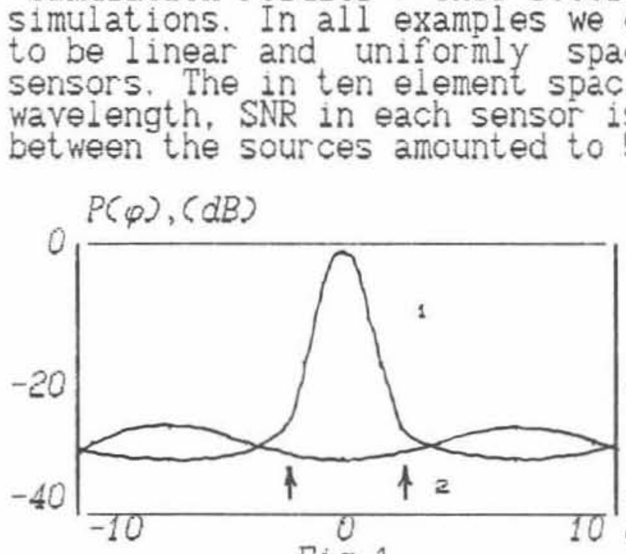

Fig.1

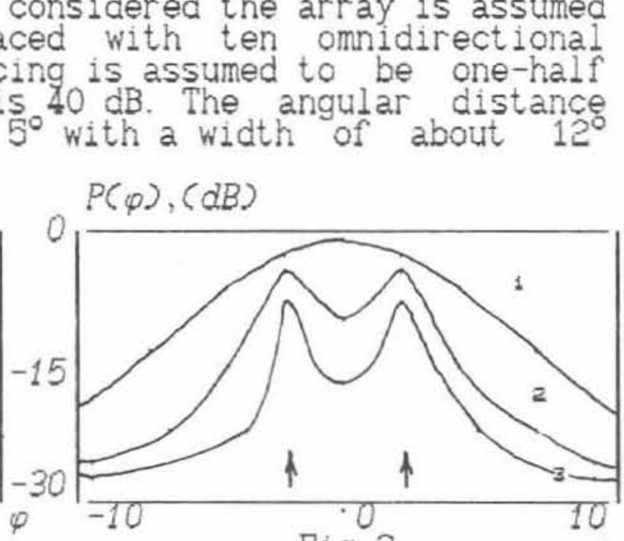

Fig.2

for the main lobe. The first example (fig.1) we considered had two fully correlated planar wavefronts for the various values of the correlation-factor argument (curve 1 corresponds to $\varphi=0$, 2 - $\varphi=\pi$). The second example (fig.2) shows the output power as a function of the angle of rotation in the application of the spatial smoothing method for $\varphi=0$ (1 - M=4, 2 - M=6, 3- M=8). From these curves we can see that the effectiveness of resolution depends significantly on the dimensions of the subaperture and will be the highest when M=8, which corresponds to expression (9).

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