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AN EFFICIENT ML SCHEME FOR ESTIMATING SPATIAL/FREQUENCY SPECTRUM OF SOURCES BY PASSIVE SENSOR ARRAY

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Abstract-- The MLSUM algorithm incorporated with a specifically-designed passive sensor array is presented to determine the maximum likelihood estimate of spatial/frequency spectrum of closely-spaced sources in spatially/temporally white noise environment. The approximated global-optimum solution is obtained by simply maximizing each harmonic individual log-likelihood function. The computer simulation results are included.

I. INTRODUCTION

The spatial/frequency spectrum of multiple sources can be automatically determined by appropriately processing the observed data at a passive sensor array. This problem can be applied to radar/sonar, geophysics, oceanography and seismology. Several techniques have been proposed to solve this 2-dimensional (2-D) estimation problem [1-5]. In [1], a 2-D linear prediction method was developed. Lim and Malik [2] proposed 2-D maximum entropy method. In [3], Porat and Friedlander developed a parameter estimation procedure based on ARMA modeling. In [4,5], eigenstructure techniques were presented to obtain the suboptimal solution of this estimation problem. Several ML estimators which provided optimal but partial solution were also proposed in [6-8]. In [6], Ziskind and Wax proposed an alternating projection technique. Huang and Barkat [7,8] presented a global-optimum searching algorithm and an Maximum Log-Likelihood Sum (MLSUM) scheme to simultaneously determine the number of sources and their locations. In this paper, we propose a modified MLSUM scheme incorporated with a specifically-designed passive array to determine the spatial/frequency spectrum of closely-spaced wideband sources.

II. PROBLEM FORMULATION AND DATA MODELING

Consider D far-field wideband sources radiating from different unknown directions of angle θ_k, $k=1, 2, \cdots D$, upon N subarrays of L elements of a passive sensor array. We also assume that each source emitting signal $s_k(t)$ with different combinations of N harmonics, that is, in their complex exponential form

$$s_k(t) = \sum_{n=0}^{N-1} c_k(\omega_n) f_k(\omega_n;t)\, exp(j\omega_r t) \qquad k = 1, 2, \cdots, D \tag{1}$$

where ω_r denotes the operating radio frequency. $c_k(\omega_n)$ and $f_k(\omega_n;t)$ are, respectively, the magnitude and unit narrow-band waveform process (low-pass equivalent) corresponding to ω_n-centered harmonic (a frequency bin) of the impinging wavefront emitted from the kth source as received a reference point. The sampled data observed at the subarray which corresponds to the nth harmonic is, by its complex envelope, a $(L\times1)$ vector

$$\mathbf{x}(\omega_n;m) = \sum_{k=1}^{D} \mathbf{a}(\omega_n,\theta_k) c_k(\omega_n) f_k(\omega_n;m) + \mathbf{w}(\omega_n;m) \qquad \begin{array}{l} n = 0, 1, 2, \cdots, N-1 \\ m = 1, 2, \cdots, M \end{array} \tag{2}$$

where N denotes the number of subarrays (or harmonics), and M denotes the number of snapshots. Assume that the harmonic waveform processes are with the same statistical characteristics, and the components in the noise vector $\mathbf{w}(\omega_n;m)$ are ergodic, white Gaussian process of zero mean and finite variance. The noise samples are uncorrelated from sensor to sensor and from the impinging signals. $\mathbf{a}(\omega_n,\theta_k)$ is the nth harmonic steering vector towards the direction θ_k. In matrix notation, equation (2) can be expressed as

$$\mathbf{x}(\omega_n,\theta,\mathbf{c}(\omega_n);m) = \mathbf{A}(\omega_n;\theta)\mathbf{c}(\omega_n)\otimes\mathbf{f}(\omega_n;m) + \mathbf{w}(\omega_n;m) \quad (3)$$

where $\otimes$ denotes the *kronecker* product. $\mathbf{A}(\omega_n;\theta)$ denotes the $(L\times D)$ matrix composed of D steering vectors corresponding to the nth harmonic. θ denotes the direction vector, $[\theta_1\ \theta_2\ \cdots\ \theta_D]^T$. $\mathbf{c}(\omega_n)$ and $\mathbf{f}(\omega_n;m)$ are the nth harmonic spatial spectrum of sources and the corresponding waveform process vector, respectively. Let $H_{\theta_k}(\omega_n)$, $k = 1, 2, \cdots, L-1$, represent the hypothesis corresponding to the nth harmonic spatial spectrum of k sources. The steering vector matrix and the spatial spectrum vector corresponding to the hypothesis $H_{\theta_k}(\omega_n)$ are, respectively, $\mathbf{A}(\omega_n;\theta_k)$ and $\mathbf{c}_k(\omega_n)$. The log-likelihood function in terms of the nth harmonic spatial spectrum given this hypothesis is obtained to be [6-8]

$$g(\theta_k,\mathbf{c}_k(\omega_n)) = \sum_{m=1}^{M} |\mathbf{P}_{\mathbf{A}(\omega_n;\theta_k)}\, \mathbf{x}(\omega_n;\theta,\mathbf{c}(\omega_n;m)|^2, \qquad k = 1, 2, \cdots, L-1 \quad (4)$$

where $\mathbf{P}_{\mathbf{A}(\omega_n;\theta_k)}$, the projection operator onto the space spanned by the columns of the matrix $\mathbf{A}(\omega_n;\theta_k)$ which composes of k nth-harmonic steering vectors, is given by

$$\mathbf{P}_{\mathbf{A}(\omega_n;\theta_k)} = \mathbf{A}(\omega_n;\theta_k)[\mathbf{A}^H(\omega_n;\theta_k)\mathbf{A}(\omega_n;\theta_k)]^{-1}\mathbf{A}^H(\omega_n;\theta_k) \quad (5)$$

where H denotes *Hermitian*. By some algebric manipulation, (4) can also be rewritten as

$$g(\theta_k,\mathbf{c}_k(\omega_n)) = tr[\,\mathbf{P}_{\mathbf{A}(\omega_n;\theta_k)}\,\hat{\mathbf{R}}(\theta,\mathbf{c}(\omega_n)] \quad (6)$$

where $tr[\ ]$ denotes the trace, and $\hat{\mathbf{R}}(\theta,\mathbf{c}(\omega_n))$, the MLE of data covariance, is given by

$$\hat{\mathbf{R}}(\theta,\mathbf{c}(\omega_n)) = \frac{1}{M}\sum_{m=1}^{M}\mathbf{x}(\omega_n,\theta,\mathbf{c}(\omega_n);m)\,\mathbf{x}^H(\omega_n,\theta,\mathbf{c}(\omega_n);m). \quad (7)$$

To maximize the log-likelihood function corresponding to nth harmonic given in (6), we find a number of nth harmonic closely-spaced sources $\hat{D}(\omega_n)$, locations $\hat{\theta}_{\hat{D}(\omega_n)}$ and their point masses (spatial spectrum) $\mathbf{c}_{\hat{D}(\omega_n)}$ such that $g(\theta_k,\mathbf{c}_k(\omega_n))$ is maximum.

III. SPATIAL/FREQUENCY SPECTRUM ESTIMATION SCHEME

The subarrays are constructed uniformly from a linear passive sensor array which consists of a number of isotropic wideband elements uniformly d apart. The intersensor spacing d_n of the nth subarray is a multiple of d. Assume the angle coverage of closely-spaced sources is uniformly quantized. The step size is chosen to be the reciprocal of $L\bar{d}_n$. $\bar{d}_n$ denotes the spacing in terms of the nth harmonic wavelength. The response at lth sensor, $a_l(q)$, of the nth harmonic array manifold toward qth quantization level becomes

$$a_l(\omega_n;q) = \exp[-j2\pi(l-1)q/L]. \quad (8)$$

We observe that (8) is valid for all harmonic and the array manifold repeats with a spatial period L. Note that since the array manifolds, $\mathbf{a}(p)$'s, are orthogonal in this case, the projection operator $\mathbf{P}_{\mathbf{A}(\theta_k)}$ can be decomposed into a sum of individual projection operator $\mathbf{P}_{\mathbf{a}(\theta_i)}$ [8]. Employing the linearity of trace operator, the LLF of hypothesis H_{θ_k} becomes

$$g(\theta_k,\mathbf{c}_k(\omega_n)) = \sum_{i=1}^{k} tr[\mathbf{P}_{\mathbf{a}(\theta_i)}\,\hat{\mathbf{R}}(\theta,\mathbf{c}(\omega_n))], \qquad k=1, 2, , \cdots, L-1\,. \quad (9)$$

Note from equation (9) that the log-likelihood function is a power measured by the modulus of the projection of the data vector sequence $\mathbf{x}(m)$ onto the space spanned by the columns of the matrix $\mathbf{A}(\theta_k)$. When the array manifolds are orthogonal, this power measure becomes simply the sum of the individual power measure's. The spatial spectrum can be obtained by simply searching the local maxima of a single ILLF, $tr[\mathbf{P}_{\mathbf{a}(\theta)}\,\hat{\mathbf{R}}(\theta,\mathbf{c}(\omega_n))]$. We also note that the L-dimensional data vector $\mathbf{x}(\omega_n;m)$ in (2) can be interpreted as a signal vector in the D-dimensional signal subspace corrupted by the noise vector in the L-dimensional observation space. The noise term $\mathbf{w}(\omega_n;m)$ in (2) can be decomposed into a linear combination of the D signal steering vectors, $\mathbf{a}(\theta_i)$'s, $i=1, 2, \cdots, D$, and the $(L-D)$ noise steering vectors, $\mathbf{a}(\theta_i)$'s, $i=D+1, D+2, \cdots, L$. The asymptotic power $p_i^{(n)}$ defined by the modulus of the projection of the data vector $\mathbf{x}(\omega_n;m)$ onto the ith array manifold $\mathbf{a}(\theta_i)$, $i=1, 2, \cdots, L$, can be found; that is, when the number of snapshots goes to infinity,

$$p_i(\omega_n) = Lc_i^2(\omega_n) + \sigma_w^2, \quad \begin{array}{l} i = 1, 2, \cdots, D \\ n = 0, 1, 2, \cdots, N-1 \end{array} \tag{10}$$

where $c_i^2(\omega_n)$ is the nth harmonic power corresponding to the ith source. Applying the orthogonality property and the fact that the squared length of any manifold is equal to L, the spatial/frequency power spectrum $S(\omega_n,\theta)$ in terms of harmonic and the directional angle can be rewritten as

$$S(\omega_n,\theta) = tr\,[\mathbf{a}(\theta)\mathbf{a}^H(\theta)\, \hat{\mathbf{R}}(\theta,\mathbf{c}(\omega_n))] \tag{11}$$

By applying parallel processing the data in each harmonic, The spatial/frequency power spectrum is obtained. We observe that the number of sources, their DOA's and point masses for all harmonics considered are determined simultaneously. In addition, this algorithm is less computationally involved since (11) deals only with matrix multiplication.

IV. SIMULATION RESULTS AND CONCLUSIONS

Consider several wideband acoustic sources with different power spectra, emitting plane waves into a uniform linear sensor array. The central wavelength of the impinging signals and the array system was 1.5 m. The subarray structure for the considered harmonics is shown in Figure 1. We performed the MLSUM algorithm for each harmonic and obtained the spatial/frequency power spectra for the case of uncorrelated sources as shown in Figures 2 and 3. Then we performed the experiment for correlated sources and obtained the 2-D spectra as shown in Figure 4. We observed that a superior performance is attained even the noise level is high and/or the number of snapshots is small. In this paper, we have used the MLSUM algorithm incorporated with a specifically-designed structure of subarrays of an uniform passive sensor aray to determine the spatial/frequency spectrum of closely-spaced sources. Without the *a priori* knowledge of the number of sources, the ML estimate of the 2-D spectrum was obtained. This algorithm guarantees the global-optimum convergence of the approximated ML estimator. It is worthy to note that the proposed scheme is equally applicable to both cases of correlated sources and uncorrelated sources.

REFERENCES

[1] R. Kumaresan and D. W. Tufts, "A Two-Dimensional Technique for Frequency-Wavenumber estimation", *Proc. IEEE*, vol. 69, pp. 1515-1517, 1981.

[2] J. S. Lim and N. L. Malik, "A New Algorithm for Two-Dimensional Maximum Entropy Power Spectrum Estimation", *IEEE Trans. ASSP*, vol. 29, pp. 401-413, 1981.

[3] B. Porat and B. Friedlander, "Estimation of Spatial and Spectral Parameters of Multiple Sources", *IEEE Trans. on Information Theory*, vol. IT-29, pp. 412-425, 1983.

[4] R. O. Schmidt, "Multiple Emitter Location and Signal Parameter Estimation", *IEEE Trans. on Antennas and Propagation,* vol. AP-34, pp. 276-280, 1986.

[5] M. Wax, T.-J. Shan and T. Kailath, "Spatio-Temporal Spectral Analysis by Eigenstructure Methods", *IEEE Trans. ASSP*, vol. 32, pp. 817-827, 1984.

[6] I. Ziskind and M. Wax, "Maximum-Likelihood Localization of Multiple Sources by Alternating Projection", *IEEE Trans. ASSP*, vol. 36, pp. 1553-1560, 1988.

[7] Y.-D. Huang and M. Barkat, "Near-Field Multiple Source Localization by Passive Sensor Array", *IEEE Trans. Ant. Propag.*, vol. AP-39, pp. 968-975, 1991.

[8] Y.-D. Huang and M. Barkat, "An Efficient ML Algorithm for Localizing Closely-Spaced Sources by Passive Sensor Array", *Proc. of 34th IEEE Midwest Symposium on Circuits and Systems*, Calif., vol. 2, May 1991.

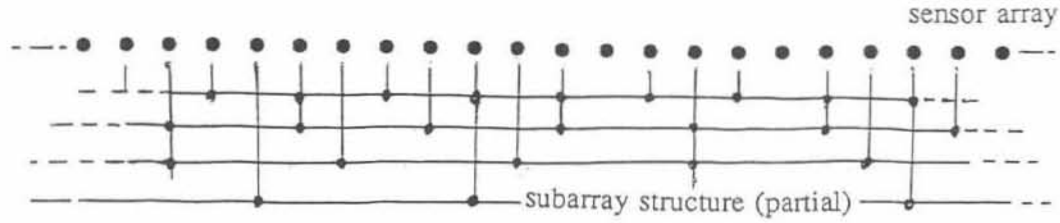

Fig. 1 The L-element subarray structure (partial) of a linear uniform sensor array.

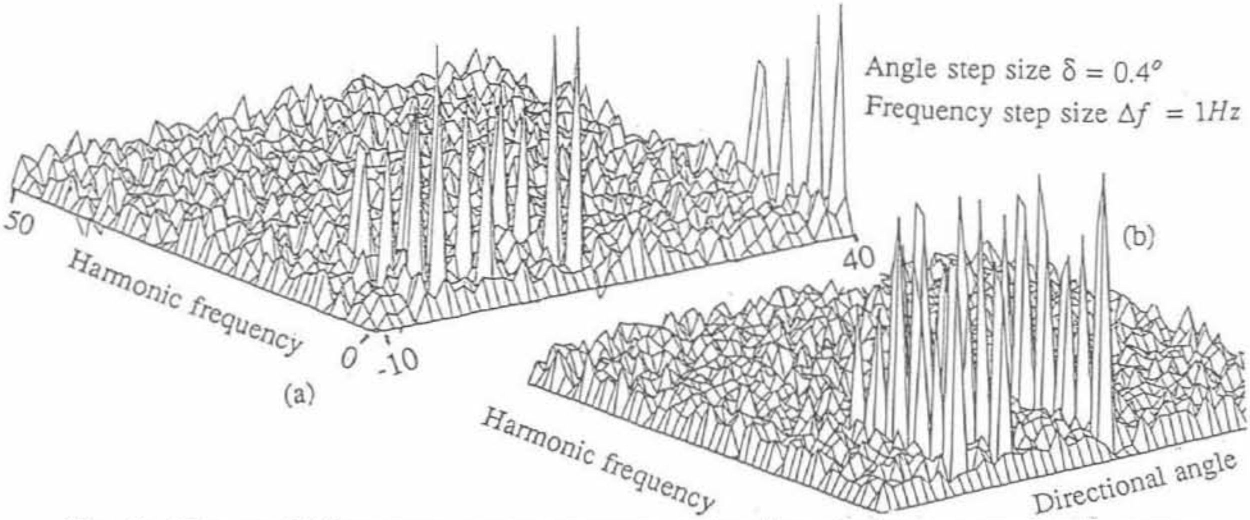

Fig. 2 The spatial/frequency power spectrum estimation of three uncorrelated sources. Noise level = $-5\ dB$. (a) $M = 5, N = 4$. (b) $M = 10, N = 6$. The spatial repeatedness with period $L = 41$ is shown in (a).

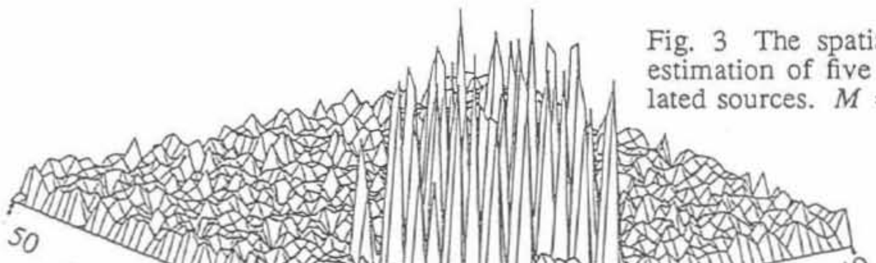

Fig. 3 The spatial/frequency power spectrum estimation of five very closely-spaced uncorrelated sources. $M = 5$. $N = 10$.

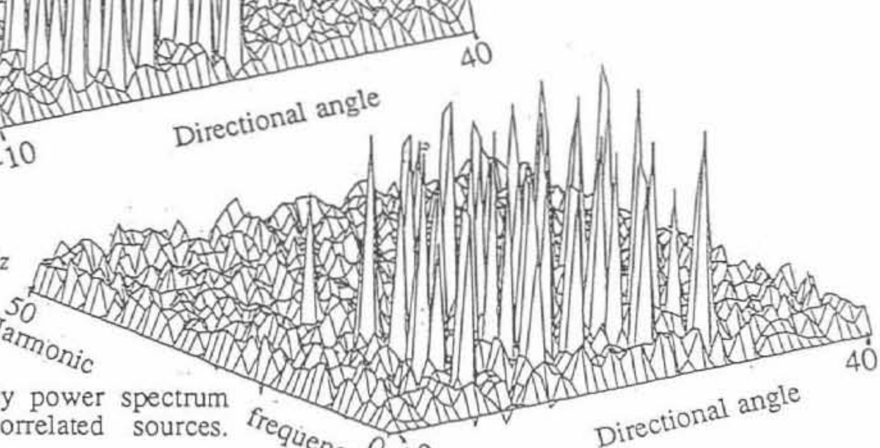

Fig. 4 The spatial/frequency power spectrum estimation of five fully-correlated sources. $M = 5$. $N = 10$