# Synthesis of Phase and Radiation Pattern for Microstrip Reflectarray using Discretization of Elementary Geometrical Functions 

\#Piyaporn Krachodnok and Rangsan Wongsan<br>School of Telecommunication Engineering, Suranaree University of Technology Nakhon Ratchasima, 30000, Thailand, Email: priam@sut.ac.th

## 1. Introduction

In the wireless communication applications such as the large-scale indoor base station of wireless local area network (WLAN) system, it is desirable for antenna beam to cover a broad area. Therefore, the widely circular beam antenna is an alternative for WLAN applications as shown in Fig.1. The related literatures have been reported by several authors. Smulders et al.[1] presented the design of a 60 GHz shaped reflector antenna for WLAN access points by using backscatter reflector, which fabricated from the modified parabolic surface. Also, Wongsan and Thaivirot [2] presented the synthesis of radiation patterns of the variety of shaped backscatters to provide the wide beam for indoor WLAN applications. From these papers, the backscatters have been fabricated from the circular metal sheet that their surfaces are shaped to be geometric curvature. In case of WLAN systems, such antennas are improper because their structures suffer from mechanical drawbacks such as bulkiness and the need for an expansive custom mold for each coverage specification.


Figure 1: Reflectarray for WLAN Large-Scale Indoor Base Station.


Figure 2: Synthesis Model of Microstrip Backscatter Reflectarray.

Recently, a novel type of antenna that combines some of the best feature of microstrip arrays and parabolic reflector, namely the microstrip reflectarray [3], has essentially no limitation in its dimensions and has much less distortion in its planar shape. The reflectarray antenna consists of a flat reflecting surface and an illuminating feed as shown in Fig.2. On the reflecting surface, there are many isolated elements (e.g. printed patches, dipoles, or rings), which array on flat PCB without any power division transmission lines.

A reflectarray configuration is attractive because it allows a single mechanical design to be used repeatedly for a wide variety of different coverage specifications without the need for expensive fabrication of a new mold. The only changes are required that the printed reflecting element dimensions be changed for each design in order to generate the different beam. Thus, many of the high recurring costs associated with shaped-reflector antennas can be eliminated with flat printed reflectarrays [4]. The flat geometry of a reflectarray also lends itself to easier placement and deployment on the WLAN large-scale indoor base station and also in terms of manufacture. In addition, a flat printed reflectarray fulfils the antenna requirement for low profile and light weight.

This paper proposes a microstrip reflectarray antenna using discretization of elementary geometrical functions to form a wide beam antenna. To achieve such broad-beamwidth, phase of each array element in the reflectarray antenna is specific designed to emulate the curvature of the backscatter by using patches of different size [5].

At first, the general approach will be presented to determine the required phase delay of each patch on the reflectarray (Section 2). In Section 3, we apply this approach into the variety of elementary geometrical functions to calculate the radiation patterns of reflectarrays. Finally, the conclusions are given in Section 4.

## 2. Determination of the Required Aperture Phase Delay

Fig. 2 illustrates the incidence of wave on the surface of a synthesis model of printed microstrip reflectarray. In general, the feed may be positioned at distance from the reflectarray. The path lengths from the feed to all reflectarray elements are all different, which lead to different phase delays. In this paper, the desired phase delay is determined on the construction of the curvature of a shaped backscatter surface with the help of Snell's law. The relations of the microstrip reflectarray surface to the shape of backscatter are two coordinate systems in this figure: one describes the coordinates $\left(x_{r}, y_{r}, z_{r}\right)$ of the reflectarray and the other one describes the coordinate $\left(x_{b}, y_{b}, z_{b}\right)$ of the backscatter. The selected shaped reflector is determined by using functions of elementary geometric as shown in Table 1 that aperture cross sections of all backscatters are confined to be circular, same diameter ( $D$ ), and radius of point source is vary small. Since the reflectarray is designed, the following coordinate is used to find the point on the backscatter surface at a given patch element on the reflectarray.

$$
\begin{align*}
& x_{b}=x_{r}+\left|f\left(z^{\prime}\right)-z_{r}\right| \tan \theta_{i} \cos \phi_{i},  \tag{1a}\\
& y_{b}=y_{r}+\left|f\left(z^{\prime}\right)-z_{r}\right| \tan \theta_{i} \sin \phi_{i}, \tag{1b}
\end{align*}
$$

where the phase center of the feed is located at ( $0,0,0$ ) and the incidence angle can be described in terms of geometrical dimensions

$$
\begin{equation*}
\theta_{i}=\tan ^{-1}\left[\frac{\sqrt{x_{r}^{2}+y_{r}^{2}}}{f}\right] ; \quad \phi_{i}=\tan ^{-1}\left[\frac{y_{r}}{x_{r}}\right] . \tag{2}
\end{equation*}
$$

The total path length from the feed to the reflectarray aperture is the sum of the distance from the feed to a point on the backscatter surface and the distance from that point to the corresponding point on the reflectarray with the rays satisfying Snell's law on the backscatter surface. In the analysis of backscatter, it is desirable to find a unit vector that is normal to the local tangent at the surface reflection point.

$$
\begin{equation*}
\hat{n}=\frac{\nabla\left[z-f\left(z^{\prime}\right)\right]}{\mid \nabla\left[z-f\left(z^{\prime}\right)\right]} . \tag{3}
\end{equation*}
$$

With the help of Snell's law of reflection, the reflected angle for the backscatter can be expressed in (4).

$$
\begin{equation*}
\theta_{r}=2 \cos ^{-1}\left[-\frac{\vec{\rho}_{i}}{\left|\vec{\rho}_{i}\right|} \cdot \hat{n}\right]-\theta_{i} \tag{4}
\end{equation*}
$$

The differential path length ( $\Delta L_{n n}$ ) and the phase delay ( $\Delta \Phi_{m n}$ ) for the $m n$-th reflectarray element are given as:

$$
\begin{align*}
& \Delta L_{m n}=\left|\bar{\rho}_{i}\right|+\frac{z_{b}-f}{\sin \theta_{r}}-\left|\vec{r}_{m, i}\right|,  \tag{5}\\
& \Delta \Phi_{m n} \text { in degree }=\left[(1-N) k_{0} \Delta L_{m n}\right] \frac{360}{2 \pi}, \tag{6}
\end{align*}
$$

where $N$ is integer. The above indicates that the compensating phase can be repeated every 360 deg , and the portion that is an integer multiple of 360 deg can be deleted.

The required phase delays of all elements are calculated and will be used to design the dimension of the reflectarray elements as shown in Fig.3. These phase delays are duplicated the same radiating aperture as shape of backscatter. To compensate for these phase delays, the elements must have corresponding phase advancements designed. Its phase change versus element change (patch size, etc.) must be calibrated correctly. In this radiation pattern calculation, the reflectarray is fed with a standard X-band horn, which is placed at front of reflectarray $(f=25 \mathrm{~cm})$. The centre-tocentre elements spacing is fixed at a distance $s=0.6 \lambda_{0}$ in both $x$ and $y$ directions.

Table 1: Formulations of elementary geometrical functions illustrated.

| Backscatter Shapes | Formulations |
| :---: | :--- |
| Circular | $f\left(z^{\prime}\right)=A \sqrt{1-\left(\frac{2}{D} z^{\prime}\right)^{2}}$ |
| Gaussian | $f\left(z^{\prime}\right)=A e^{-\left(\frac{2}{D} \bar{D}^{2}\right)^{2}}$ |
| Quadratic | $f\left(z^{\prime}\right)=A\left(1-\left(\frac{2}{D} z^{\prime}\right)^{2}\right)$ |
| Parabolic | $f\left(z^{\prime}\right)=z^{\prime 2} / 4 f$ |
| Cosine | $f\left(z^{\prime}\right)=A \cos \left(\frac{\pi}{D} z^{\prime}\right)$ |
| Squared cosine | $f\left(z^{\prime}\right)=A \cos 2^{2}\left(\frac{\pi}{D} z^{\prime}\right)$ |
| Triangular | $\left.f\left(z^{\prime}\right)=A\left(\left.1-\frac{2}{D} \right\rvert\, z^{\prime}\right)\right)$ |



Figure 3: Phase Delay for Reflectarray Elements.

## 3. Reflectarray Pattern Calculation

With the compensating phases of all elements known, the far-field radiation patterns can be calculated by the conventional array theory [6], where the radiations of all elements are summed together. Consider a planar array consisting of $M \times N$ elements that are nonuniformly illuminated by a low-gain feed. The reradiated field from the patches in an arbitrary direction $\hat{u}$, will be of the form:

$$
\begin{equation*}
E(\hat{u})=\sum_{m=1}^{M} \sum_{n=1}^{N} F\left(\vec{r}_{m n} \cdot \hat{a}_{z}\right) \cdot A\left(\vec{r}_{i} \cdot \hat{u}_{r}\right) \cdot A\left(\hat{u} \cdot \hat{u}_{r}\right) \cdot \exp \left[-j k_{0}\left(\left|\vec{r}_{m n}\right|+\vec{r}_{i} \cdot \hat{u}\right)-j \Delta \Phi_{m n}\right], \tag{7}
\end{equation*}
$$

where $F$ is the feed pattern function, $A$ is the reflectarray element pattern function, $\vec{r}_{i}$ is the vector from centre of reflectarray to $m n$-th element, $\hat{u}_{r}$ is the reflected field pointing direction, and $\Delta \Phi_{m n}$ is the required compensating phase of the $m n$-th element calculated by (6).

The calculated results indicate the different radiation patterns for various geometrical functions, which are shown in Fig.4. The prescribed field requirements have been satisfied by an appropriate choice of the radiating patches selected from the complex design curves obtained in the analysis stage. The steepness of the pattern edges and the angular positions of these edges confirm that the antenna efficiently illuminates the target area to be covered $\left( \pm 65^{\circ}\right)$. Radiation patterns are different due to phase of reflectarray elements which are duplicated the same radiating aperture as backscatter. Because of phase change versus element change, each reflectarray type (as backscatter shapes) provides different characteristics such as -3 dB beamwidth (HPBW) and relative power, which are reported in Table 2.

From Table 2, it is observed that the HPBW of seven backscatter reflectarray types are different. For average consideration, it is apparent that the squared cosine has the widest beamwidth and followed, in order, by gaussian, cosine, quadratic, parabolic, and circular, which are $166^{\circ}, 164^{\circ}$, $156^{\circ}, 150^{\circ}, 140^{\circ}$, and $133^{\circ}$, respectively, while HPBW of triangular is about $27^{\circ}$ on each main beam. Since the relative power is strongly coincided with the HPBW i.e., the narrower the beamwidth the higher the relative power and vice versa. However, reflection surface for reflectarray elements, which are synthesized in this paper, are placed position near centre of shaped backscatter.

Thus, the circularly geometrical function yields the highest relative power and followed, in order, by parabolic, gaussian, triangle, quadratic, cosine, and squared cosine, respectively.

Table 2: Characteristics of Various Reflectarray Types

| Backscatter Shapes | HPBW (degree) | Maximum relative power (dBi) |
| :---: | :---: | :---: |
|  | 133 | 30.29 |
| Gaussian | 164 | 28.49 |
| Quadratic | 150 | 25.58 |
| Parabolic | 140 | 28.87 |
| Cosine | 156 | 24.27 |
| Squared cosine | 166 | 22.01 |
| Triangular | 27 | 28.06 |



Figure 4: Radiation Pattern of Microstrip Reflectarray.

## 4. Conclusions

The synthesis of phase and radiation pattern of various reflectarray types in case that the -3 dB beamwidth and relative power can be controlled in broadside direction is presented in this paper. The desired phase of reflectarray elements which are duplicated the same radiating aperture as backscatters by using functions of elementary geometries. The radiation characteristics such as radiation pattern, half-power beamwidth, and relative powers are investigated and compared each other. From all the aforementioned of radiation characteristics, it can be summarized that these reflectarray can be chosen according to the characteristic requirements in practicable applications. For example, if the widest HPBW for large coverage area is required, then, the geometric function of squared cosine should be the best choice. However, if we need very high gain reflectarray antenna, the circularly geometrical function should be applied.

## References

[1] Peter F.M. Smulder, S. Khusial, and H.A.J. Herben, "A shaped reflector antenna for 60-GHz indoor wireless LAN access points," IEEE Trans. On Verhicular Technology, Vol. 50, No. 2, pp. 584-591, 2001.
[2] R. Wongsan, V. Thaivirot, "Synshesis of radiation pattern of variety of shaped backscatters using physical optic," ECTI-CON Conference Proceedings, Thailand,Vol. 1, pp. 155-158, 2006.
[3] R.E. Munson, H.A. Haddad, and J.W. Hanlen, "Microstrip reflectarray for satellite communications and RCS enhancement or reduction," U.S. patent 4684 952, 1987.
[4] D.M. Pozar, S.D. Targonski, and R. Pokuls, "A shaped-beam microstrip patch reflectarray," IEEE Trans. On Antenna and Propag., Vol.47, Issue 7, pp. 1167-1173, 1999.
[5] P. Krachodnok and R. Wongsan, "Design of microstrip reflectarray antenna using backscattering technique," ITC-CSCC 2006 Conference Proceedings, Thailand, Vol. 3, pp. 513-516, 2006.
[6] J. Huang, "Analysis of a microstrip reflectarray antenna for microspacecraft applications," The Telecommunications and Data Acquisition Progress Report 42-120, pp. 153-173, 1995.

