# 3-DIMENSIONAL LOCALIZATION OF NEAR-FIELD MULTIPLE SOURCES USING MUSIC WITH CIRCULAR ARRAY 

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## 1. Introduction

For the realization of the indoor radio communication systems, it is very significant to grasp the circumstances of indoor multipath propagation. For the purpose, it is most effective to estimate the signal parameters of incoming waves at receiving points.

For far-field sources, many signal estimators have been proposed so far[1]. On the contrary, there have been only a few algorithms that localize multiple sources in a nearfield where the incoming waves cannot be regarded as plane waves[2][3]. Besides, only 2 -dimensional(2-D) localization in limited areas have been carried out in the near-field algorithms. In real radio environments, however, the sources distribute in a 3-dimensional (3-D) space and that is why the 3-D localization algorithms are required for such sources.

In this paper, via computer simulation, we show that the MUSIC algorithm[4] can be applied to the 3-D localization of multiple sources in the restricted space by using a circular array as the receiving antenna system.

## 2. Estimation Principle

Consider $L$ near-field sources observed by $N$-element circular array with the radius of $r$ (see Figure 1). All antenna elements are uniformly spaced on the circumference and have an isotropic pattern. The observed data at the $n$th antenna element can be expressed as

$$
\begin{gather*}
v_{n}=\sum_{l=1}^{L} \frac{F_{l}}{d_{l n}} \exp \left(-j k d_{l n}\right)+w_{n}  \tag{1}\\
d_{l n}=\sqrt{\left(x_{l}-X_{n}\right)^{2}+\left(y_{l}-Y_{n}\right)^{2}+\left(z_{l}-Z_{n}\right)^{2}} \tag{2}
\end{gather*}
$$

where $k$ is the wavenumber, $F_{l}$ is the signal amplitude emitted from $l$ th source, $w_{n}$ is the additive noise at the $n$th antenna and $d_{l n}$ is the distance between the location of the $n$th antenna $\left(X_{n}, Y_{n}, Z_{n}\right)=\left(r \cos \frac{2 \pi n}{N}, r \sin \frac{2 \pi n}{N}, Z_{n}\right)$ and that of the $l$ th source $\left(x_{l}, y_{l}, z_{l}\right)$. Equation (1) can be rewritten in vector and matrix forms as

$$
\begin{equation*}
\boldsymbol{V}=A \boldsymbol{F}+\boldsymbol{W} \tag{3}
\end{equation*}
$$

where

$$
\begin{aligned}
& \boldsymbol{V}=\left[v_{1}, v_{2}, \cdots, v_{N}\right]^{T} \\
& A=\left[\boldsymbol{a}\left(x_{1}, y_{1}, z_{1}\right), \boldsymbol{a}\left(x_{2}, y_{2}, z_{2}\right), \cdots, \boldsymbol{a}\left(x_{L}, y_{L}, z_{L}\right)\right] \\
& \boldsymbol{a}\left(x_{l}, y_{l}, z_{l}\right)=\left[\exp \left(-j k d_{l 1}\right) / d_{l 1}, \exp \left(-j k d_{l 2}\right) / d_{l 2}, \cdots, \exp \left(-j k d_{l N}\right) / d_{l N}\right]^{T} \\
& \quad(l=1, \cdots, L) \\
& \boldsymbol{F}=\left[F_{1}, F_{2}, \cdots, F_{L}\right]^{T} \\
& \boldsymbol{W}=\left[w_{1}, w_{2}, \cdots, w_{N}\right]^{T}
\end{aligned}
$$

$a(x, y, z)$ is referred to as the $(N \times 1)$ mode vector characterized by the antenna array. When the additive noise is spatially white and statistically independent of the $L$ signals, the covariance matrix $S$ of the observed data vector $\boldsymbol{V}$ is given by

$$
\begin{equation*}
S=E\left[\boldsymbol{V} \boldsymbol{V}^{\dagger}\right]=A E\left[\boldsymbol{F} \boldsymbol{F}^{\dagger}\right] A^{\dagger}+\sigma_{0}^{2} \boldsymbol{I} \tag{4}
\end{equation*}
$$

where $E[\cdot]$ and $\dagger$ denote the expectation value and the complex conjugate transpose respectively, $\sigma_{0}^{2}$ is the variance of the additive noise, and $I$ is the identity matrix. Assuming that the $L$ signals are not completely correlative with each other, the matrix $E\left[\boldsymbol{F} \boldsymbol{F}^{\dagger}\right]$ has a full rank. An estimate of $S$, denoted by $\tilde{S}$, can be obtained from the $I$ snapshots as

$$
\begin{equation*}
\tilde{S}=\frac{1}{I} \sum_{i=1}^{I} V_{i} V_{i}^{\dagger} \tag{5}
\end{equation*}
$$

where $\boldsymbol{V}_{i}$ is the data vector obtained at the $i$ th snapshot.
We execute the eigen-decomposition of $\tilde{S}$ and then the number of sources $L$ is estimated from that of the prominent eigenvalues of $\tilde{S}$. Let $e_{L+1}, e_{L+2}, \cdots, e_{N}$ be the noise eigenvectors corresponding to the remaining $(N-L)$ smaller eigenvalues. And we define the $N \times(N-L)$ noise eigenvector matrix $E_{N}$ as follow:

$$
\begin{equation*}
E_{N} \triangleq\left[e_{L+1}, e_{L+2}, \cdots, e_{N}\right] \tag{6}
\end{equation*}
$$

To search for the locations of sources, we construct a 3-D source spectrum defined as

$$
\begin{equation*}
P_{M U}(x, y, z)=\frac{\boldsymbol{a}^{\dagger}(x, y, z) \boldsymbol{a}(x, y, z)}{\boldsymbol{a}^{\dagger}(x, y, z) E_{N} E_{N}^{\dagger} \boldsymbol{a}(x, y, z)} \tag{7}
\end{equation*}
$$

We can estimate the locations of multiple sources in the restricted space from the peaks of spectrum of equation (7).

## 3. Computer Simulation

The coordinates of the estimation space are shown in Figure 2. The size of the space is $10 \lambda \times 20 \lambda \times 10 \lambda$, and to search the locations of sources we make the estimation space discrete using $0.2 \lambda$ mesh. In the simulation, 12 -element circular array with its radius $r=3 \lambda$ is used as the receiving antenna system and also the center of it is located on the origin of the coordinate axes. In this space, there are two sources whose parameters are chosen as follows:

$$
\left.\begin{array}{rccc}
\text { 1st } & \text { source : } & \left|F_{1}\right|=0 \mathrm{~dB} & \left(x_{1}, y_{1}, z_{1}\right)=(3,10,2) \\
\text { 2nd } & \text { source : } & \left|F_{2}\right|=-3 \mathrm{~dB} & \left(x_{2}, y_{2}, z_{2}\right)=(3,5,2.2)
\end{array}\right]
$$

The signal-to-noise ratio(SNR) is defined for the first source at the center of the circular array and the computation of estimates is carried out for $\mathrm{SNR}=30 \mathrm{~dB}$. To make the covariance matrix, 20 snapshots are taken.

Figures 3(a), (b), (c) and (d) show the estimation results on $x y$-plane at $z=2 \lambda$, $x y$-plane at $z=2.2 \lambda, y z$-plane at $x=3 \lambda$ and $z x$-plane at $y=10 \lambda$, respectively. It is found that the sharp tips of spectrum almost exactly point out the locations of sources as long as the sources exist in the estimation plane of interest. Even if the sources are not in that plane, we can recognize that the sources exist close to the plane.

## 4. Conclusion

Via computer simulation, we have shown that the MUSIC algorithm is applicable to the 3 -dimensional localization of multiple sources by using circular array. Since the presented method has high resolution in localization, we suppose that employing this method in the general chambers surely contributes to the implementation of indoor radio communication systems.

## References

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Figure 1 Circular array geometry in near field


Figure 2 The coordinates of estimation space in simulation

(c) $y z$-plane at $x=3 \lambda$

Figure 3 Localization by MUSIC in the computer simulation

