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CONSIMILARITY CLASSIFICATION OF GENERAL RADAR SCATTERING MATRICES

E. Lüneburg[1] and W.-M. Boerner[2]

[1]Institute for Radio Frequency Technology
German Aerospace Research Establishment (DLR)
D-82234 Oberpfaffenhofen, Germany;
Tübitak–Marmara Research Center
Department of Basic Sciences
41470 Gebze-Kocaeli, Turkey

[2]University of Illinois at Chicago
UIC-EECS Communications, Sensing and Navigation Laboratory
840 W. Taylor Str.; SEL 4210
Chicago, IL 60607-7018

1. INTRODUCTION

Radar polarimetry started about 40 years ago with the pioneering works of Sinclair [11], Graves [4], Kennaugh [8] et al. This early work culminated in the famous 1970 Ph.D. thesis of Richard Huynen [7]. In the last decade the importance of radar polarimetry in problems of remote sensing with real and synthetic aperture radar, clutter suppression and target recognition was gradually recognized. The full appreciation of polarimetric methods is hampered, however, by a lack of understanding of basic polarimetric concepts and principles, obscuring the inherent polarimetric structure of radar responses. This is related, above all, to the often missing comprehension that radar polarimetry makes use of two related but different definitions of states of polarization for transmitted and received electromagnetic waves travelling in opposite directions to each other. This affects the proper definition of co- and cross-polarized components and the correct form of polarization basis transformations of the Sinclair radar scatter matrices.

Arbitrary coherent 2 × 2 complex Sinclair are classified according to the number of coneigenvectors and to their possible reduction to diagonal or triangular canonical form under general and unitary consimilarity transformations.

2. BASIC RADAR POLARIMETRY

A plane electromagnetic wave propagating in the direction of the wave vector $\vec{k}$ can be written is the form

$$\vec{E}(\hat{k}, \vec{r}, t) = \vec{E}(\hat{k}) \exp[j\{\omega t - \vec{k} \cdot \vec{r}\}] \tag{1}$$

where $\hat{k}$ is the unit propagation vector, $\vec{r}$ is the observation point. $\vec{E}(\hat{k})$ is an element of the 2-dimensional complex column vector space $\mathcal{P}(\hat{k})$ and is called a Jones vector as introduced by Jones in optical polarimetry. In the Euclidean orthonormal polarization basis $B = \{\hat{e}_1, \hat{e}_2\}$ this vector reads

$$\vec{E} = E_1\hat{e}_1 + E_2\hat{e}_2 \equiv \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} |E_1|\exp\{j\phi_1\} \\ |E_2|\exp\{j\phi_2\} \end{bmatrix}. \tag{2}$$

The derivation of the geometric descriptors of the polarization ellipse of such plane waves from their Jones vector is described extensively in the literature, cf. [1] and [9]. A Jones vector uniquely determines the shape (locus) of the polarization ellipse including its sense of rotation once the explicit time dependence and a cartesian basis $B = \{e_1, e_2\}$ has been fixed. The handedness (helicity) and, hence, the state of polarization of an e.m. plane wave, however, incorporates by definition statements with respect to the direction of propagation of the plane wave and, therefore, cannot be derived from a Jones vector alone. The additional information about the direction of propagation is incorporated into the sign of the axial ratio of the polarization ellipse. The introduction of the concept of handedness removes the ambiguity connected with the Jones vector notion. Since handedness information is not contained in a (pure) Jones vector it is most convenient to denote Jones vectors from the space $\mathcal{P}(+\hat{k}) \equiv \mathcal{P}_+$ by the attached symbol $\vec{E}|_+ \equiv$

$\vec{E}_+$ and Jones vectors from $\mathcal{P}(-\hat{k}) \equiv \mathcal{P}_-$ by the $\vec{E}|_- \equiv \vec{E}_-$. The vectors $\vec{E}_\pm$ are called directional Jones vectors. Such vectors have been introduced by Graves [4] as early as 1956 but have nearly never been used afterwards. With this notation one and the same polarization basis can be used without confusion for opposite directions of propagation and for backscattering. This notation can be carried over to the general bistatic scattering case, see [10].

The physically relevant real forms of the electric field representations read

$$\mathrm{Re}E_i(\hat{k},\hat{r},t) = |E_i|\cos\{\omega t + \phi_i - \vec{k}\cdot\vec{r}\} \quad (i = 1,2). \tag{3}$$

The substitutions $t \to -t$ and $\hat{k} \to -\hat{k}$ leave these expressions invariant with respect to $\hat{k} \to -\hat{k}$ if and only if $\phi_i \to -\phi_i$ $(i = 1,2)$. This is the essence of the time reversal operation. This operator effecting the correspondence between the original and the motion-reversed state is denoted by Θ. If E_+ is a directed Jones vector in $\mathcal{P}_+$ then the action of Θ on E_+ defines the directed Jones vector $E_- \in \mathcal{P}_-$:

$$\Theta\{\vec{E}_+|_{B_+}\} = \{\vec{E}^*_+|_{B_-}\} := \{\vec{E}_-|_{B_-}\} \quad \text{or} \quad \Theta E_+ := E_- \tag{4}$$

and correspondingly for the transition from $\mathcal{P}_-$ to $\mathcal{P}_+$. Θ maps any polarization state vector into its motion-reversed counterpart and consists of complex conjugation of the directional Jones vector and reversal of direction of propagation from the vector space $\mathcal{P}_+$ to $\mathcal{P}_-$ or vice versa, [9]. Θ is an involutory anti-unitary operator. It should be emphasized that the application of the operator Θ on a directed Jones vector changes the basis vectors as well as the components.

The directional Jones vectors $\vec{E}_+$ and $\vec{E}_-$, related by the time reversal operator Θ, describe the same state of polarization

$$\text{Polarization}\{\vec{E}_+\} = \text{Polarization}\{\vec{E}_-\}. \tag{5}$$

Hence, the time reversal operator is the ideal tool to analyse the radar backscattering case. The polarization spaces $\mathcal{P}_+$ and $\mathcal{P}_+$ are called conjugate spaces.

The following fundamental consequence should be noted: Polarization vectors in $\mathcal{P}_+$ and $\mathcal{P}_-$ that describe the same state of polarization transform conjugately under a (unitary) change of polarization basis, i.e.

$$\vec{E}_+ = U\vec{E'}_+ \quad \Longleftrightarrow \quad \vec{E}_- = U^*\vec{E'}_-. \tag{6}$$

Using the directional Jones vector notation the normalized radar equation reads

$$\vec{E}^s_- = S\,\vec{E}^i_+ \tag{7}$$

where the superscripts 'i' and 's' denote 'incident' and 'scattered', respectively. This equation connects the incident electromagnetic plane wave vector $\vec{E}^i_+$, propagating in the positive $\hat{k}$-direction towards the target, with the scattered far field plane wave vector $\vec{E}^s_-$, propagating in the negative $\hat{k}$-direction away from the target, by the Sinclair backscatter matrix S. The so-called voltage or power transfer equation reads

$$P = |V|^2 = |\vec{h}\cdot S\vec{E}^i|^2 \equiv |(\vec{h}, S\vec{E}^i)|^2 \tag{8}$$

where $\vec{h}$ is the receive antenna polarization. These equations are considered to be the cornerstones of radar polarimetry.

Under an unitary change of polarization basis $\vec{E}^i \to U\vec{E'}^i$ in $\mathcal{P}_+$ the Sinclair matrix S transforms according to

$$S \to S' = U^T S U \qquad (U^\dagger U = UU^\dagger = I). \tag{9}$$

This is nowadays called a (unitary) consimilarity transformation [5], [9] and should not be confused with ordinary similarity appropriate for optical transmission polarimetry.

3. CLASSIFICATION OF GENERAL SINCLAIR MATRICES

In the following it will be assumed that the Sinclair scattering matrix S is a general 2×2 complex matrix, i.e. it is not assunmed that S is always symmetric. This general case has only occasionally been considered so far, see [3] and [2], mainly in the context of bi-static scattering. In order to simplify notation the notation $x \equiv \vec{E}^i$ with $||x|| = 1$ is used throughout.

The total power contained in the scattered wave field for incident x is given by the expression $P_G(x) = x^\dagger Gx$ where $G = S^\dagger S$ is the Hermitian positive semi-definite Graves power matrix. The set of all power values is the field of values, [6],

$$F^1(G) = \{x^\dagger Gx|x^\dagger x = 1\} \equiv \{x^\dagger S^\dagger Sx|x^\dagger x = 1\} = [\mu_2, \mu_1]; \tag{10}$$

μ_1 and μ_2 are the eigenvalues of G: $Gx_i = \mu_i x_i$ with $0 \leq \mu_2 \leq \mu_1$, say, and the orthonormal eigenvectors x_1 and x_2. † denotes complex conjugation and transposition.

The 2-field of values is given by, [6],

$$F^2(G) = \left\{\frac{1}{2}(x^\dagger Gx + x_\perp^\dagger Gx_\perp)|X^\dagger X = I, X = [x, x_\perp]\right\} = \frac{1}{2}(\mu_2 + \mu_1) \subset F^1(G). \tag{11}$$

$F^1(G)$ and $F^2(G)$ are unitarily invariant. $F^2(G)$ reduces to a single point, i.e. is polarization invariant: $F^2(G) = \frac{1}{2}\text{span}S = \frac{1}{2}\sum|s_{ij}|^2$.

Introducing the co- and crosspolar power terms

$$P_{co}(x) := |x^T Sx|^2 \quad \text{and} \quad P_x(x) := |x_\perp^T Sx|^2 \tag{12}$$

it can be shown that

$$P_G(x) = P_{co}(x) + P_x(x), \tag{13}$$

leading to the power tree concept sketched in [9]. The following notion will be introduced:

A nonzero vector x such that $Sx = \lambda x^*$ is said to be a coneigenvector of the matrix S; the scalar λ is a coneigenvalue of S. Coneigenvectors are those polarization vectors that do not change their state of polarization undergoing backscattering, [9] and [10].

The theory of coneigenvalues/-vectors has some similarities with the theory of ordinary eigenvalues/-vectors but differs fundamentally in some other respects. Thus a matrix may have an infinite number of distinct coneigenvalues or it may have no coneigenvalues at all, [5], since if x ic coneigenvector with coneigenvalue λ, then $\exp\{i\phi\}x$ is coneigenvector with coneigenvector $\exp\{2i\phi\}\lambda$ for arbitrary phase values ϕ. This phase undeterminacy can be used to take the coneigenvalues as real nonnegative. The coneigenvectors agree with the pseudo-eigenvectors introduced by Kennaugh [8] within the more elementary concept of optimal polarizations.

The Kennaugh-Huynen co-pol null vectors are solutions of the equation $Sy = \nu y_\perp^*$ and are intimately related to the coneigenvectors forming the well-known Huynen fork, [7] and [9].

The coherent Sinclair scattering matrices S may be classified as follows:

Class 1. S can be reduced to diagonal form by unitary consimilarity

$$U^T SU = D = \text{diag}[\lambda_1, \lambda_2] \quad \text{with} \quad U = [x_1, x_2] \tag{14}$$

if and only if S is symmetric: $S = S^T$. This is Takagi's theorem, [12]. Also

$$y_{1,2} = \frac{1}{\sqrt{\lambda_1^2 + \lambda_2^2}}\left\{\sqrt{\lambda_2}x_1 \pm i\sqrt{\lambda_1}x_2\right\}. \tag{15}$$

In this case

$$P_x(x_i) = 0 \quad \text{and} \quad P_G(x_i) = P_{co}(x_i) = |\lambda_i|^2 = \mu_i \quad (i = 1, 2), \tag{16}$$

i.e. the maximal available power in the scattered wavefield can be captured by the single-measurement copolar antenna set-up. A typical example is the unit matrix I.

Class 2. The scatter matrix S can be condiagonalized but not by unitary consimilarity: $S = ADA^{-1*}$, A nonunitary. This is the case if and only if S^*S is a diagonalizable matrix with real nonnegative eigenvalues and rank$S$ = rankS^*S, [5]. This is the most general case with two independent coneigenvectors.

Class 3. The Sinclair matrix S can be reduced to triangular form by (general or unitary) consimilarity if and only if all the eigenvalues of the matrix S^*S are real nonnegative (T upper triangular with real nonnegative digonal elements):

$$U^T SU = T \quad \text{with} \quad U = [x_1, x_2]. \tag{17}$$

Here, the vector x_1 is still a coneigenvactor to the eigenvalue t_{11} but in general $t_{11} < \sqrt{\mu_1}$ and x_2 is not a coneigenvector anymore. In this case in general

$$P_{co}(x_i), P_x(x_i) < P_G(x_i) \quad (i = 1, 2), \tag{18}$$

i.e. the maximal power of the scattered wave cannot completely be received by a co- or crosspolar single measurement. A typical example is

$$S = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 2i \\ 0 & 0 \end{pmatrix}, \quad U = \frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{19}$$

Class 4. The Sinclair matrix cannot be reduced to triangular form since the eigenvalues of the matrix S^*S are not real nonnegative. In this case there do not exist coneigenvectors at all. A typical example is the matrix

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad S^*S = -I, \quad G = S^\dagger S = I \tag{20}$$

with $P_{co}(x) = 0$ for all x.

The relations between coneigenvectors, co- and cross-polar extrema and null eigenvectors and their representations as generalized Huynen forks are presently under study.

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