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SCATTERING AND CASCADING MATRICES OF THE LOSSLESS RECIPROCAL POLARIMETRIC TWO-PORT IN THEIR GENERAL FORMS

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1. Introduction

In the last year paper as in [1] the theory has been presented of the 4x4 complex scattering matrix of the lossless reciprocal polarimetric two-port using an 'engineering notation' based on the concept of the Poincare polarization sphere of tangent phasors proposed in the previously published papers [2, 3]. The scattering matrix under consideration consisted of two reflectance and two transmittance Sinclair matrices, the last two being bistatic scattering matrices in their canonical form corresponding to the characteristic orthogonal null-phase polarization basis. Applying that canonical form and neglecting two phase angles of the reflectance and transmittance matrices, the scattering matrix of the two-port has been developed as a function of five independent real parameters only but determining all polarimetric properties possible. They have been demonstrated geometrically on the Poincare sphere models of all four Sinclair matrices by the corresponding polarization forks of different size, shape and mutual orientations.

That theory will be further developed in this paper by presenting the scattering matrix of the two-port in a most general form by additing the two lacking phases and by rotating its characteristic orthogonal null-phase polarization basis of tangent phasors by three Euler angles to the position of the linear orthogonal polarization basis.

As the next step in developing the theory, the 4x4 complex cascading matrix of the two-port will be constructed, analogous to the corresponding 2x2 cascading matrix of the ordinary microwave two-port. In polarimetry, four types of such cascading matrices are possible. They depend on the choice as to which port the spatial propagation 'z-axis' is reversed by rotation of the xyz rectangular coordinate system by 180 degrees about, say, the vertical x-axis. When using cascading matrices, the cascaded connection of the two-ports can be considered, e.g., when analyzing the behaviour of an obstacle surrounded by a medium of known polarimetric properties.

2. General cases of scattering and cascading matrices satisfying the reciprocity condition

Scattering matrices of the polarimetric two-port consist of four component matrices: two reflectance and two transmittance matrices. Scattering matrices of a nonreciprocal two-port depend on 32 real parameters. They depend also on directions of the Oz axes along which the TEM waves, incoming and outgoing, are propagating. When the Oz axes at both ports are directed towards the two-port, then all four component matrices are scattering matrices (Sinclair or Kennaugh) of 'S1 type', according to definition in [4]. However, when one of these axes reverses its direction (by 180° rotation of the coordinate system about, say, 'vertical' Ox axis), then one of the reflectance matrices changes to the scattering matrix of 'S2 type', whereas two transmittance matrices become propagation matrices (Jones or Mueller), both of 'P1 or P2 type'. The type of amplitude matrices is essential because their transformation rules under change of basis and under change from amplitude to power representation depend on it. Changes of the Oz axes directions are necessary when considering a chain of the two-ports.

Scattering matrices of reciprocal two-ports are symmetrical what results in symmetry of their reflectance matrices, as of S and R beneath, whereas their two transmittance matrices, usually nonsymmetrical, are transposed to each other, thus reducing the number of independent real parameters from 32 to 20. In case both Oz axes are directed towards the two-port, and using amplitude component matrices, the two-port scattering matrix and its corresponding cascading matrix can be defined by the following transformation equations:

$$\begin{bmatrix} S & \tilde{T} \\ T & R \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \tilde{T} - ST^{-1}R & ST^{-1} \\ -T^{-1}R & T^{-1} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ a_2 \end{bmatrix} \qquad (1a, 2)$$

Here, $a_{1,2}$ and $b_{1,2}$ are complex amplitude column vectors of the incoming and outgoing waves at the port 1 or 2, respectively. No particular orthogonal null-phase (ONP) polarization basis has been specified in the above equations.

When reversing the Oz direction at the port 2, or at the port 1, the following new scattering matrices in their corresponding transformation equations, for the same two-port, become:

$$\begin{bmatrix} S & \tilde{T}^{o} \\ T^{o} & {}^{o}R^{o} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2^{o} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2^{o} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} {}^{o}S^{o} & {}^{o}\tilde{T} \\ {}^{o}T & R \end{bmatrix} \begin{bmatrix} a_1^{o} \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1^{o} \\ b_2 \end{bmatrix} \qquad (1b,\ 1c)$$

where the reversal transformations for amplitude vectors and matrices will depend on the ONP polarization basis. In the linear horizontal/verical basis we obtain:

$$a_{1,2}^{o} = C^{o} a_{1,2}, \qquad b_{1,2}^{o} = C^{o} b_{1,2}, \qquad T^{o} = C^{o}\, T \quad \text{(P1 type)},$$
$${}^{o}T = T\, C^{o} \quad \text{(P2 type)}, \qquad {}^{o}R^{o} = C^{o}\, R\, C^{o} \quad \text{(S2 type)}, \qquad {}^{o}S^{o} = C^{o}\, S\, C^{o} \quad \text{(S2 type)}$$

with

$$C^{o} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (3)$$

The transformation equations with the new cascading matrices will also change respectively.

3. General form of scattering matrices for reciprocal and lossless two-ports

Scattering matrices of reciprocal and lossless two-ports are not only symmetrical but also unitary what reduces the number of their independent real parameters from 20 to 10. The transmittance complex matrices, as nonsymmetrical, depend on 8 real parameters, whereas two symmetrical reflectance matrices together depend on 12 real parameters. From a practical point of view most interesting are the transmittance matrices, thus in [1] all of their 8 parameters have been assumed as independent and only 2 remaining independent parameters have been associated with the reflectance matrices. An analysis performed in [1] has shown that two of 10 independent parameters are general phases of transmittance and reflectance matrices, and that 7 remaining parameters of the transmittance matrix can be further reduced to 5 by proper rotation of its ONP polarization basis, by three Euler angles, from e.g. H , horizontal/vertical linear basis, to K , the characteristic basis. In the K basis the transmittance matrix takes the most simple canonical form and its Poincare sphere model can be constructed indicating inside the sphere an inversion point for the incoming polarizations, as well as an axis of rotation after inversion with an angle of rotation about that axis. Such a model, envisaged by Eduard Morton Kennaugh in his short communication [5] in 1981 and further developed by Czyż in [6] in 1985 (see also [7] - [9]), made possible a lucid geometrical interpretation of variation of scattered power and polarization in case of bistatic scattering or nonsymmetrical transmittance matrices and also enabled the synthesis of such matrices for desired polarization properties of the polarimetric two-port.

The following fundamental equalities result from the unitarity condition applied to the two-port scattering matrix defined in (1a):

$$Span\, S = Span\, R \qquad (4a)$$
$$Span\, S + Span\, T = 2 \qquad (4b)$$
$$|\det S|^2 = |\det R|^2 = 1 - Span\, T + |\det T|^2 \qquad (4c)$$
$$arg \det S + arg \det R = 2\, arg \det T \qquad (4d)$$

They determine e.g. diameters of the Poincare spheres serving as models of the reflectance and transmittance matrices. Squares of these diameters are:

$$\sigma_{oS} = \sigma_{oR} = Span\, S + 2|\det S|$$
$$\sigma_{oT} = \sigma_{o\tilde{T}} = Span\, T + 2|\det T|$$

Introducing four real parameters, $A_{1,2}$ and $B_{1,2}$, we obtain the following canonical form of the transmittance matrix in its characteristic K basis:

$$T_K = \begin{bmatrix} A_2 & B_1 + jB_2 \\ -B_1 - jB_2 & A_2 \end{bmatrix} e^{j\mu} \tag{5}$$

with μ denoting its 'canonical phase' and with matrix elements satisfying conditions

$$A_2 \geq A_1 \geq 0 \quad \text{and} \quad B_2 \geq 0$$

necessary for obtaining the unambiguously defined K basis, usually represented by its first vector as phasor tangent to the Poincare sphere. Now, using results presenting in [1], we can compute the reflectance matrices in the same K basis:

$$S_K = \begin{bmatrix} S_2 & S_3 \\ S_3 & S_1 \end{bmatrix} e^{j\sigma} \qquad \text{and} \qquad R_K = \begin{bmatrix} R_2 & R_3 \\ R_3 & R_1 \end{bmatrix} e^{j(2\mu-\sigma)} \tag{6}$$

when introducing an additional independent parameter $S_3 = S_3{}^*$ taken from an interval between limits which are known functions of matrix elements in (5). The phases σ and μ can be taken arbitrarily as independent parameters, with μ denoting the total increase of the two-port electrical length equal to $\sigma/2$ increase at one port plus $\mu-\sigma/2$ increase at the other, according to equation [4d].

The scattering matrix of the form as in (1a), considered as being defined in the K basis, i.e. with the component Sinclair matrices as in (5) and (6), can now easily be transformed to the H basis when using the unitary unimodular rotation matrix $C_H{}^K$. The following matrix equation holds:

$$\begin{bmatrix} S & \tilde{T} \\ T & R \end{bmatrix}_H = \begin{bmatrix} \tilde{C}_K{}^H & 0 \\ 0 & \tilde{C}_K{}^H \end{bmatrix} \begin{bmatrix} S & \tilde{T} \\ T & R \end{bmatrix}_K \begin{bmatrix} C_K{}^H & 0 \\ 0 & C_K{}^H \end{bmatrix} \tag{7}$$

The scattering matrix of a reciprocal lossless two-port, thus obtained, is of the most general form employing all 10 independent real parameters. The corresponding cascading matrix is as in (2) with all Sinclair matrices taken in the H basis. Complex amplitude vectors in (1a) and (2) should also be transformed accordingly. We obtain

$$a_{1,2H} = C_H{}^K a_{1,2K} \qquad \text{and} \qquad b_{1,2H} = C_H{}^{K*} b_{1,2K}$$

where $C_H{}^K$ is conjugate transposed $C_K{}^H$ matrix. The difference in the above transformation rules is obvious because $a_{1,2}$ directly correspond to the polarization and phase (PP) vectors, whereas $b_{1,2}$ are conjugate values of the PP vectors.

3. An example of a cascade of three two-ports

Consider a cascade of three two-ports, No. 1, 2 and 3, with Oz axes at the input and output ports of the cascade directed to the cascade and between the two-ports directed to the cenrtal two-port. Scattering matrices of the successive two-ports are as in equations (1b), (1a) and (1c), respectively. Wave amplitudes between the two-ports are then:

$$a_2{}^{o}{}_{(1)} = b_{1(2)} \qquad a_{2(2)} = b_1{}^{o}{}_{(3)}$$
$$b_2{}^{o}{}_{(1)} = a_{1(2)} \qquad b_{2(2)} = a_1{}^{o}{}_{(3)}$$

with two-port numbers in brackets. The cascading matrix of such a cascade of the two-ports equals the product of cascading matrices of the successive two-ports. When considering transformations from K to H basis, the following additional transformation rules should be observed:

$a_{1,2}{}^{o}{}_H = C_H{}^{K*}\, a_{1,2}{}^{o}{}_K$ $\qquad$ $b_{1,2}{}^{o}{}_H = C_H{}^K\, b_{1,2}{}^{o}{}_K$

$T^{o}{}_H = C_H{}^K\, T^{o}{}_K\, C_K{}^H$ (P1 type Jones matrix) $\qquad$ ${}^{o}T_H = C_H{}^{K*}\, {}^{o}T_K\, C_K{}^{H*}$ (P2 type Jones matrix)

${}^{o}R^{o}{}_H = C_H{}^K\, {}^{o}R^{o}{}_K\, C_K{}^{H*}$ (S2 type Sinclair matrix) $\qquad$ ${}^{o}S^{o}{}_H = C_H{}^K\, {}^{o}S^{o}{}_K\, C_K{}^{H*}$ (S2 type Sinclair matrix)

All the above formulae can easily be derived when analyzing the corresponding transmission equations.

In practical applications we are usually interested in computation of the two-port 2 scattering matrix having measured matrices of the entire cascade and those of the two-ports 1 and 3. Or we may be interested in the synthesis of the scattering matrix of the entire cascade having a scattering matrix of the two-port 2. In the last case, of the synthesis, special attention should be given to the problem of the characteristic bases, different for each two-port and for the entire cascade.

4. Concluding remarks on Sinclair and Jones matrices models of the two-port

For the Sinclair scattering matrices as in (1a), sizes, shapes and handle positions of their polarization forks have been shown in [1] to be functions of 4 real parameters of the T matrix only. An additional real parameter S_3 is required for the determination of mutual orientations of the polarization forks associated with the reflectance versus transmittance matrices.

For the Jones matrices as in (1b) or (1c), implementing transmittance matrices only, a similar dependence can be observed. Also diameters of their Poincare sphere models remain unchanged. The only but significant difference is that Jones matrices always possess two (or one doubled) eigenpolarizations, usually not orthogonal with respect to each other, what has been explained in [10] or [9].

Both Sinclair and Jones nonsymmetrical transmittance matrices have their polarization forks also being nonsymmetrical, with an inflected (broken) handle pointing to the polarization of maximum transferred (minimum reflected) power. The prongs of Jones matrices' forks are pointing to the eigenpolarizations, not to the copolarization nulls as in the case of Sinclair matrices' forks.

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