PROCEEDINGS OF ISAP '96, CHIBA, JAPAN

OPTIMIZATION PROCEDURES FOR POLARIMETRIC CONTRAST ENHANCEMENT IN MICROWAVE REMOTE SENSING

Harold MOTT†, Mitsuru TANAKA††, and Wolfgang-M. BOERNER†††
† Department of Electrical Engineering, University of Alabama, Tuscaloosa, AL 35487, USA
†† Department of Electrical and Electronic Engineering, Oita University, 700 Dannoharu, Oita-shi, Oita 870-11, Japan
††† Department of Electrical Engineering and Computer Science, University of Illinois at Chicago, Chicago, IL 60680, USA

1. Introduction

In microwave remote sensing, it is desirable to select radar antenna polarizations that maximize the contrast between two classes of scatterers or scatterer ensembles. A polarimetric radar measures complete polarization properties of a target and then provides a vector description of the resulting scattered wave through various target matrices [1-6]. Several optimization procedures for the completely and the partially polarized cases have been proposed based on the theory of radar polarimetry [7-12].

It is the purpose of this paper to present optimization procedures for the enhancement of polarimetric contrast between two time-varying targets. The targets are now characterized by the time-averaged Kronecker matrices, from which one can obtain the Graves and the Kennaugh matrices. The Graves matrices of the targets are used to find a transmitter polarization to maximize the ratio of scattered power densities at the receiver. Using the Lagrange multipliers method, the maximization problem is cast into the form of a generalized eigenvalue equation. The largest eigenvalue of the equation equals to the maximal power ratio, and the optimal effective length of the transmitting antenna is proportional to the corresponding eigenvector. The Kennaugh matrices of the targets are employed to obtain the Kennaugh vectors of partially polarized scattered waves from the two targets. Each of the scattered Kennaugh vectors is decomposed into a completely polarized and unpolarized parts. It is well known that [1] the power received from the unpolarized part is independent of the polarization characteristics of the receiving antenna. Then a receiver polarization is selected to maximize or minimize the completely polarized part scattered from the desired or the undesired target. As a numerical example, the optimal Stokes vectors of transmitting and receiving antennas are given to show the validity of the optimization procedures.

2. Basic Theory

Consider a time-varying target illuminated by a plane monochromatic wave in a general bistatic scattering geometry. Each instantaneous state of the target is completely characterized by the Sinclair matrix [1, 2]:

$$S=\begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix}. \tag{1}$$

For backscattering from the target in a reciprocal medium, the matrix is symmetric and $S_{xy}=S_{yx}$. The time-averaged Kronecker matrix of the target is given by

$$D=\langle S\otimes S^{*}\rangle=\left(D_{ij}\right) \quad (i,j=1,2,3,4), \tag{2}$$

where the angle bracket denotes a time average operation, $\otimes$ the Kronecker or direct product, and the asterisk being complex conjugation. With the Kronecker matrix elements, the time-averaged Graves matrix may be expressed as

$$\sigma = \langle S^{T*} S \rangle = \begin{bmatrix} D_{11}+D_{41} & D_{13}+D_{43} \\ D_{12}+D_{42} & D_{14}+D_{44} \end{bmatrix}, \tag{3}$$

where the superscript T indicates the transpose of a matrix.

The Stokes vectors of transmitting and receiving antennas are

$$\mathbf{G}_\alpha = Q\left(\mathbf{h}_\alpha \otimes \mathbf{h}_\alpha^*\right) = \begin{bmatrix} |h_{\alpha x}|^2 + |h_{\alpha y}|^2 \\ |h_{\alpha x}|^2 - |h_{\alpha y}|^2 \\ 2\,\mathrm{Re}\,(h_{\alpha x}^* h_{\alpha y}) \\ 2\,\mathrm{Im}\,(h_{\alpha x}^* h_{\alpha y}) \end{bmatrix} = \begin{bmatrix} G_{\alpha 0} \\ G_{\alpha 1} \\ G_{\alpha 2} \\ G_{\alpha 3} \end{bmatrix}, \tag{4}$$

where $\mathbf{h}_\alpha (\alpha = t, r)$ are the effective lengths of the transmitting (t) and receiving (r) antennas, Re and Im represent the real part and the imaginary part, and

$$Q = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}. \tag{5}$$

Neglecting multiplying factors, the power density of a partially polarized scattered wave at the receiver is

$$P_s = \mathbf{h}_t^{T*} \sigma \, \mathbf{h}_t . \tag{6}$$

The Kennaugh vector of the scattered wave can be written as

$$\mathbf{G}_s = \begin{bmatrix} G_{s0} \\ G_{s1} \\ G_{s2} \\ G_{s3} \end{bmatrix} = \frac{1}{2}\left(Q^* D\, Q^{T*}\right)\mathbf{G}_t = K\,\mathbf{G}_t , \tag{7}$$

where $K = Q^* D Q^{T*}/2$ is called the Kennaugh matrix of the target. The scattered Kennaugh vector $\mathbf{G}_s$ is uniquely represented as a sum of the two Kennaugh vectors of a completely polarized wave and an unpolarized wave [1]. Thus

$$\mathbf{G}_s = \mathbf{G}_s^{(1)} + \mathbf{G}_s^{(2)} = \begin{bmatrix} (1-R)G_{s0} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} R\,G_{s0} \\ G_{s1} \\ G_{s2} \\ G_{s3} \end{bmatrix}, \tag{8}$$

where $R = \sqrt{(G_{s1})^2 + (G_{s2})^2 + (G_{s3})^2}\,/G_{s0}$ is the degree of polarization of the scattered wave, and the superscript (1) refers to the unpolarized part and (2) to the completely polarized part.

The received power is given by the product of the scattered Kennaugh vector with the Stokes vector of the receiving antenna [1], thus

$$W = \mathbf{G}_r^T \mathbf{G}_s = \mathbf{G}_r^T \mathbf{G}_s^{(1)} + \mathbf{G}_r^T \mathbf{G}_s^{(2)} . \tag{9}$$

As is well known [1], the power $\mathbf{G}_r^T \mathbf{G}_s^{(1)}$ received from the unpolarized wave does not depend on the polarization characteristics of the receiving antenna. Then the receiving antenna polarization is found by optimizing the power $\mathbf{G}_r^T \mathbf{G}_s^{(2)}$ from the completely polarized wave. The quantity $\mathbf{G}_r^T \mathbf{G}_s^{(2)}$ will be a maximum or a minimum if

$$\mathbf{G}_r = \begin{bmatrix} G_{r0} \\ G_{r1} \\ G_{r2} \\ G_{r3} \end{bmatrix} = C_1 \begin{bmatrix} R\,G_{s0} \\ G_{s1} \\ G_{s2} \\ G_{s3} \end{bmatrix} \quad \text{or} \quad \mathbf{G}_r = \begin{bmatrix} G_{r0} \\ G_{r1} \\ G_{r2} \\ G_{r3} \end{bmatrix} = C_2 \begin{bmatrix} R\,G_{s0} \\ -G_{s1} \\ -G_{s2} \\ -G_{s3} \end{bmatrix}, \tag{10}$$

where C_1 and C_2 are arbitrary constants.

3. Optimization Procedures

Now consider the enhancement of polarimetric contrast between two time-varying targets A and B. The ratio of scattered power densities at the receiver

$$F_{AB} = \frac{P_{sA}}{P_{sB}} = \frac{\mathbf{h}_t^{T*} \sigma_A \mathbf{h}_t}{\mathbf{h}_t^{T*} \sigma_B \mathbf{h}_t} \tag{11}$$

is maximized by choosing the transmitting antenna effective length $\mathbf{h}_t$. We use the Lagrange multipliers method and maximize $\mathbf{h}_t^{T*} \sigma_A \mathbf{h}_t$ with the constraint $\mathbf{h}_t^{T*} \sigma_B \mathbf{h}_t = \text{constant}$. Then the maximization problem is cast into the form of a generalized eigenvalue equation

$$\left(\sigma_A - \lambda \sigma_B\right) \mathbf{h}_t = 0. \tag{12}$$

It is readily shown that the largest eigenvalue λ_{max} equals to the maximal power ratio, and the optimal effective length $\mathbf{h}_{t,opt}$ of the transmitting antenna is proportional to the corresponding eigenvector. Then we can get the optimal transmitted Stokes vector $\mathbf{G}_{t,opt}$ from (4).

The Kennaugh vectors $\mathbf{G}_{sA}$ and $\mathbf{G}_{sB}$ of the waves scattered from targets A and B are obtained from (7) with the time-averaged Kennaugh matrices K_A and K_B of the targets. Each of the scattered Kennaugh vectors is separated into a completely polarized and unpolarized parts as (8). The optimal receiver Stokes vector $\mathbf{G}_{r,opt}$ is found by maximizing the power $\mathbf{G}_r^T \mathbf{G}_{sA}^{(2)}$ from the desired target A (Method I) or minimizing the power $\mathbf{G}_r^T \mathbf{G}_{sB}^{(2)}$ from the undesired target B (Method II).

4. Numerical Example

A numerical example is presented for the following time-averaged Kronecker matrices for targets A and B:

$$D_A = \begin{bmatrix} 5.0 & 0.2+j0.1 & 0.2-j0.1 & 1.0 \\ 0.2+j0.06 & 0.3+j0.1 & 0.9-j0.2 & 0.1-j0.03 \\ 0.2-j0.06 & 0.9+j0.2 & 0.3-j0.1 & 0.1+j0.03 \\ 1.2 & 0.1-j0.01 & 0.1+j0.01 & 3.0 \end{bmatrix}, \tag{13}$$

$$
D_B = \begin{bmatrix} 2.0 & 0.1-j0.02 & 0.1+j0.02 & 0.5 \\ 0.09-j0.03 & 0.8-j0.1 & 0.45-j0.08 & 0.2+j0.1 \\ 0.09+j0.03 & 0.45+j0.08 & 0.8+j0.1 & 0.2-j0.1 \\ 0.7 & 0.16+j0.12 & 0.16-j0.12 & 4.0 \end{bmatrix}. \qquad (14)
$$

The corresponding Graves and Kennaugh matrices can be obtained from the Kronecker matrices through (3) and (7).

The maximum value of F_{AB} is 2.303, and the normalized Stokes vectors describing the optimal transmitter and receiver polarizations are given by

$$
\mathbf{G}_{t,opt} = \begin{bmatrix} 1.000 \\ 0.995 \\ -0.094 \\ -0.044 \end{bmatrix}, \quad \mathbf{G}_{r,opt} = \begin{bmatrix} 1.000 \\ 0.997 \\ 0.072 \\ 0.027 \end{bmatrix} \text{ (Method I)}, \quad \mathbf{G}_{r,opt} = \begin{bmatrix} 1.000 \\ -0.999 \\ -0.049 \\ 0.021 \end{bmatrix} \text{ (Method II)}. \qquad (15)
$$

5. Conclusion

Optimization procedures for the enhancement of polarimetric contrast between two time-varying targets have been presented. Optimal transmitting antenna polarization is found by maximizing the ratio of scattered power densities at the receiver. Using the Lagrange multipliers method, the maximization problem is cast into the form of a generalized eigenvalue equation. The largest eigenvalue of the equation equals to the maximal power ratio, and the optimal effective length of the transmitting antenna is proportional to the corresponding eigenvector. A receiver polarization is selected to maximize or minimize the power received from the completely polarized part from the desired or the undesired target. The optimal Stokes vectors of transmitting and receiving antennas are given to confirm the validity of the optimization procedures.

References

[1] Mott H., "Antennas for Radar and Communications: A Polarimetric Approach," John Wiley & Sons, New York (1992).

[2] Boerner W.-M. et al., eds., "Direct and Inverse Methods in Radar Polarimetry," Part 1 and Part 2, Kluwer Academic Pub., Dordrecht (1992).

[3] Ulaby F. T. and Elachi C., eds., "Radar Polarimetry for Geoscience Applications," Artech House, Norwood (1990).

[4] Zebker H. A. and van Zyl J. J., *Proc. IEEE*, **79**, 11, pp. 1583-1606 (Nov. 1991).

[5] Kostinski A. B. and Boerner W.-M., *IEEE Trans. Antennas Propagat.*, **AP-34**, 12, pp. 1395-1404 (Dec. 1986).

[6] Huynen J. R., "Phenomenological Theory of Radar Targets," Ph.D. Dissertation, Drukkerij Bronder-Offset, N. V., Rotterdam (1970).

[7] Tanaka M. and Boerner W.-M., *1995 Asia-Pacific Microwave Conf. Proc.*, **1**, pp. 403-406, Taejon, Korea (Oct. 1995).

[8] Yamaguchi Y., Takayanagi Y., Boerner W.-M., and Sengoku M., *Proc. 1994 Int. Symp. Noise and Clutter Rejection in Radars and Imaging Sensors*, pp. 315-320, Kawasaki, Japan (Nov. 1994).

[9] Boerner W.-M., Liu C.-L., and Zhang X., *EARSeL Advances in Remote Sensing*, **2**, 1-I, pp. 55-82 (1993).

[10] Tanaka M. and Boerner W.-M., *Proc. 1992 Int. Symp. Antennas and Propagat.*, **2**, pp. 545-548, Sapporo, Japan (Sept. 1992).

[11] Kostinski A. B. and Boerner W.-M., *IEEE Trans. Antennas Propagat.*, **AP-35**, 8, pp. 988-991 (Aug. 1987).

[12] Ioannidis G. A. and Hammers D. E., *IEEE Trans. Antennas Propagat.*, **AP-27**, 3, pp. 357-363 (May 1979).