

A NEW METHOD TO OBTAIN THE OPTIMAL POLARIZATION STATES IN THE CO-POLARIZED CHANNEL

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1. Introduction

As regards the characteristic polarization states of a radar target for the completely polarized wave case, Boerner et al. [1],[2] have already derived eight characteristic polarization states based on the polarization transformation ratio, for which a radar receives optimum power. These states are two co-polarization maximums (CO-POL Maxs), two co-polarization nulls (CO-POL Nulls), two cross-polarization maximums (X-POL Maxs), two cross-polarization nulls (X-POL Mins), and two cross-polarization saddles (X-POL Saddles). Since the pair of X-POL Nulls and CO-POL Maxs is identical, there exists a total of eight physical characteristic polarization states. To obtain the CO-POL Maxs and CO-POL Nulls, some authors used the Lagrangian multiplier method and then solved nonlinear equations. So, it is not easy to obtain the characteristic polarization states in the co-polarized radar channel.

In this paper, a new method is provided to obtain the CO-POL Maxs and CO-POL Nulls very easily for the symmetric/asymmetric scattering matrix case. By use of this method, the formula of the CO-POL Maxs is presented. Finally, one numerical example is given, showing the results are identical with [2], [7].

2. The Formula for CO-POL Maxs

Let

$$T = \begin{bmatrix} t_1 & t_2 \\ t_2 & t_3 \end{bmatrix} \quad (1)$$

denote a scattering matrix of a radar target, \vec{a} denote the polarization state of the transmitter as given by

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

and $\|\vec{a}\|=1$. Then the received power in the co-polarized radar channel can be expressed as

$$P = |T\vec{a} \cdot \vec{a}|^2 \quad (2)$$

Using Cauchy-Schwarz inequality, we know from (2) that P will be maximal if and only if

$$\vec{a} = (T\vec{a})^* / \|T\vec{a}\| \quad (3)$$

where * denotes conjugation. Substituting (3) into (2), we have

$$P = \|T\vec{a}\|^2 = T^\dagger T \vec{a} \cdot \vec{a}^* \quad (4)$$

where \dagger denotes conjugate transpose.

Let $T_p = T^\dagger T$, then the matrix is so-called power matrix [3]. From (1), we have

$$T_p = \begin{bmatrix} |t_1|^2 + |t_2|^2 & t_2 t_1^* + t_3 t_2^* \\ t_1 t_2^* + t_2 t_3^* & |t_2|^2 + |t_3|^2 \end{bmatrix} \quad (5)$$

It can be proved easily that

$$T_p = \frac{1}{2} ((|t_1|^2 + 2|t_2|^2 + |t_3|^2) I - i \operatorname{Re}(t_1 t_2^* + t_2 t_3^*) K + \frac{1}{2} i (|t_1|^2 - |t_3|^2) L + i \operatorname{Im}(t_1 t_2^* + t_2 t_3^*) J) \quad (6)$$

where I, J, K and L are Pauli matrices [4], and

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \\ K = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \quad L = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$$

Let

$$\vec{g} = (1, g_1, g_2, g_3)^t \quad (7)$$

denotes the Stokes vector of \vec{a} , where t denotes transpose, then we know

$$g_1 = i L \vec{a} \cdot \vec{a}^*, \quad g_2 = -i K \vec{a} \cdot \vec{a}^*, \quad g_3 = i J \vec{a} \cdot \vec{a}^* \quad (8)$$

Substituting (5)-(8) into (4), we can obtain

$$P = \frac{1}{2} (|t_1|^2 + 2|t_2|^2 + |t_3|^2) + \frac{1}{2} (|t_1|^2 - |t_3|^2) g_1 + \operatorname{Re}(t_1 t_2^* + t_2 t_3^*) g_2 + \operatorname{Im}(t_1 t_2^* + t_2 t_3^*) g_3 \\ = \frac{1}{2} (|t_1|^2 + 2|t_2|^2 + |t_3|^2) + v_1 g_1 + v_2 g_2 + v_3 g_3 \quad (9)$$

where

$$v_1 = \frac{1}{2} (|t_1|^2 - |t_3|^2) \quad v_2 = \operatorname{Re}(t_1 t_2^* + t_2 t_3^*) \\ v_3 = \operatorname{Im}(t_1 t_2^* + t_2 t_3^*) \quad v = \sqrt{v_1^2 + v_2^2 + v_3^2} \quad (10)$$

From (9), we know that P will be maximal if

$$g_i = v_i / v \quad i = 1, 2, 3 \quad (11)$$

Notice that, if \vec{g} denotes one state of the CO-POL Maxs, the other one is

$$(1, -g_1, -g_2, -g_3)^t$$

So, the CO-POL Maxs are

$$\vec{g} = (1, \pm v_1 / v, \pm v_2 / v, \pm v_3 / v)^t \quad (12)$$

where v_i and v are given by (10).

Note that for the asymmetric scattering matrix (denoted by S) case, the power expression in the co-polarized radar channel remains the same if the scattering matrix S is replaced by a symmetric scattering matrix $T = (S + S^T)/2$. In this way, we can also obtain the CO-POL Maxs by the above method.

3. The method for obtaining the CO-POL Nulls

Let S denote a scattering matrix of a radar target, where S may be a asymmetric matrix. From (2), we know that $P = 0$ if and only if

$$S \vec{a} = \lambda \begin{bmatrix} a_2 \\ -a_1 \end{bmatrix} = \lambda \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{a}$$

or

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} S \vec{a} = \lambda \vec{a} \quad (13)$$

Obviously, this eigenvalue problem can be solved easily. The two eigenvectors are so-called formulations of the CO-POL Nulls.

4. Numerical Example

If a scattering matrix is

$$T = \begin{bmatrix} 2i & \frac{1}{2} \\ \frac{1}{2} & -i \end{bmatrix}$$

Using the formula (12), we obtain the two CO-POL Maxs (expressed by Stokes vectors) are

$$\vec{g} = (1, \pm \frac{\sqrt{2}}{2}, 0, \pm \frac{\sqrt{2}}{2})'$$

From (13), we can obtain the two CO-POL Nulls are

$$\vec{a} = (i, -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i)'$$

or (expressed by Stokes vectors)

$$\vec{g} = (1, -\frac{1}{3}, \pm \frac{\sqrt{7}}{3}, -\frac{1}{3})'$$

These results are identical with [6][7]. Fig.1 shows the CO-POL Maxs and CO-POL Nulls on the Poincaré sphere.

5. Conclusion

In this paper, a new method is provided for obtaining the characteristic polarization states in the co-polarized channel. It is pointed out that, the CO-POL Maxs can be obtained very easily by our formula. Finally, one numerical example is given, and the results are identical with [2],[7]. This shows the validity of our formula.

References

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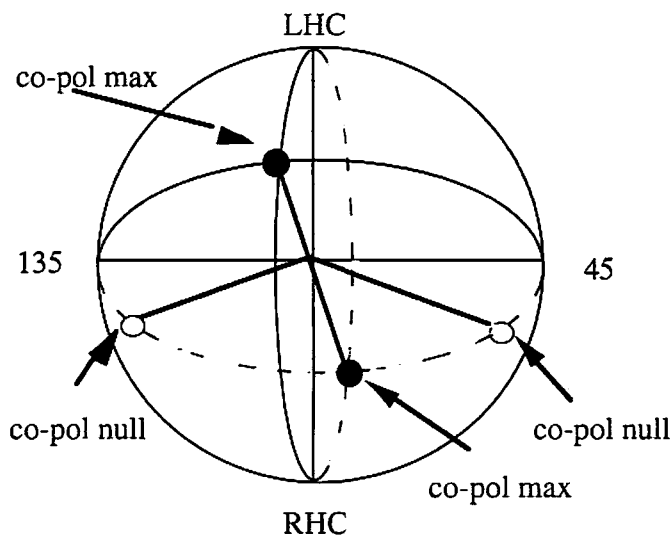


Fig.1 The CO-POL Maxs and CO-POL Nulls on the Poincaré sphere