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MBAA-BFN DESIGN WITH A NEW DFT ALGORITHM

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1. Introduction

A multibeam array antenna (MBAA) simultaneously radiates multiple beams from a common antenna which consists of antenna elements and a beam forming network (BFN). The structure of the MBAA is conceptionally shown in Fig. 1. The Blass network, modified Blass network, and Butler-Matrix network[1],[2] are typical examples of a BFN. These BFNs are originally developed for linear MBAA, making it necessary for us to devise methods of feeds in the case of planar array antennas.

BFNs for a planar array can be synthesized starting from BFNs for a linear array in at least two ways, the cross cascade arrangement and the antenna ports rearrangement.

The latter implementation, first proposed by Shelton[3], is applicable for certain kinds of planar arrays which satisfy "covering condition". This condition has been found by Shelton empirically, and the mathematical reasoning had not been given. Recently, an inducement to provide a mathematical basis of Shelton's results led us to carry out theoretical research which transforms a two-dimensional discrete Fourier transform (2D-DFT) to a one-dimensional one (1D-DFT). We have introduced a new concept of "simultaneous sampling" and "discrete equivalence" in 2D-DFT, and the new concept has been explained by defining several terms and presenteing the algorithm in the form of theorems and several formulas[4].

This paper attempts to explain how our new DFT algorithm is applied in the design of DFTs for planar arrays. We first rewrite the definitions and theorems in [4] in terms of array antenna terminology, followed by description of an example for a 6×6 planar array.

2. Definitions

[Definition 1] *Element lattice vectors* (r_1, r_2) and *beam period vectors* (W_1, W_2) which are reciprocal to each other, see Figs. 2 and 3.

[Definition 2] *Simultaneous sampling* is defined as the sampling of beam and element planes simultaneously at N points whose positions are chosen in a mutually dependent manner so that the two planes have similar properties including periodicity.

[Definition 3] *Beam lattice vectors* (w_1, w_2) and *element period vectors* ($\boldsymbol{R}_1, \boldsymbol{R}_2$)

which are reciprocal to each other, see Figs. 2 and 3.

[Definition 4] *Sampling matrix* S, which satisfies

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = S \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}, \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = S^T \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

where s_{11}, s_{12}, s_{21} and s_{22} are integers.

[Definition 5] *2D lattice points vectors* (r_{mn}, w_{gh}) such that

$$\begin{aligned} r_{mn} &= mr_1 + nr_2 \\ w_{gh} &= gw_1 + hw_2 \end{aligned}$$

where m,n,g and h are integers.

[Definition 6] Two groups of N sampled points on an element (beam) plane is defined to be *discretely equivalent* if their 2D-DFT at N sampled points on beam (element) plane are equal.

[Definition 7] *Element base-line vector* and *beam base-line vector* (r_b, w_b) such that

$$\begin{aligned} r_b &= pr_1 + qr_2 \\ w_b &= rw_1 + sw_2 \end{aligned}$$

where p,q,r and s are appropriate integers.

3. Properties of sampling matrix

[Property 1] The number of elements N in one period is equal to the determinant of S.

[Property 2] The number of beams in one period is equal to the determinant of S^T, or S, which is equal to N.

[Property 3] Sampling matrix S is represented by beam and element period vectors as,

$$S = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \cdot \begin{bmatrix} W_1 & W_2 \end{bmatrix} = \begin{bmatrix} R_1 \cdot W_1 & R_1 \cdot W_2 \\ R_2 \cdot W_1 & R_2 \cdot W_2 \end{bmatrix}$$

[Property 4] The inverse matrix S^{-1} of sampling matrix S is represented by beam and element lattice vectors as,

$$S^{-1} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \cdot \begin{bmatrix} w_1 & w_2 \end{bmatrix} = \begin{bmatrix} r_1 \cdot w_1 & r_1 \cdot w_2 \\ r_2 \cdot w_1 & r_2 \cdot w_2 \end{bmatrix}$$

4. Theorems

[Theorem 1] It is possible to find a linear array of elements along an element base-line vector which is discretely equivalent to a given planar array of elements, if there exists at least one element of sampling matrix S which is prime to N.

[Theorem 2] For 2D lattice points vectors r_{mn} and w_{gh}, we can determine the integers $k(m,n)$ and $l(g,h)$ such that $k(m,n)r_b$ and $l(g,h)w_b$ are discretely equivalent to r_{mn} and w_{gh}, respectively, by the following linear congruences.

$$\begin{aligned} Mk(m,n) &\equiv mrs_{22} - mss_{12} - nrs_{21} + nss_{11} \pmod N \\ Ml(g,h) &\equiv pgs_{22} - phs_{12} - qgs_{21} + qhs_{11} \pmod N \end{aligned}$$

5. Application to 6×6 MBAA

An example of the element and the beam planes for an array with 6×6 BFN are shown in Figs. 2 and 3, respectively, where the element arrangement has been devised such that it satisfies the covering condition. In Figs. 2 and 3, one possible choice for a pair of r_b and w_b is $r_b = r_1$ and $w_b = w_2$. The numbering for the elements and the beams in this choice are as shown inside Figs. 2 and 3, respectively. The array may be manufactured as a low-profile antenna as shown in Fig. 4, where a BFN might be a modified Blass network and the antenna element a micro strip patch antenna, both producted on the same substrate.

General Butler matrices with $N \times N$ ports are characterized by the scattering matrix $S = [S_{nm}]$, where the (n,m)th element is given by

$$S_{nm} = \frac{1}{\sqrt{N}} \exp\left[-j\left(\theta_n + \theta_m + \frac{2\pi nm}{N}\right)\right]. \qquad (1)$$

In Equation (1), n and m are the indices of antenna and beam ports, respectively, and θ_n and θ_m are optional phase shift to the n-th antenna port and the m-th beam port which can be specified arbitrarily and independently. As it is clear from Equation (1), the general Butler matrix supplies outputs with equal amplitude and linear phase shift with respect to the number of elements. If θ_n is chosen appropriately, all the 6 beams lie with in the visible region. The directive pattern of this planar MBAA has been calculated as the product of array factor and the approximate element factor. Due to the space limitation, the radiation patterns for the $m = 0$ and $m = 1$ beam ports as examples are shown in Figs. 5 and 6 respectivity, where the circular region represents the visible region and is the projection of the upper spherical surface of unit radius $(0 \leq \theta \leq \pi/2)$ to the $x - y$ plane. The contours of equal amplitudes are shown, where the increment of the contour is 2.0dB.

References

[1] J.Butler and R.Lowe, "Beam-forming matrix simplifies design of electrically scanned antennas", Electron. Design, Vol.9, pp.170-173, Apr. 1961.

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[3] J.P.Shelton, "Multibeam Planar Array", Proc. IEEE, Vol.56, pp.1818-1821, Nov. 1968.

[4] N.Inagaki, "2D-DFT to 1D-DFT Transformation via Simultaneous Sampling", to appear in Trans. IEICE Japan, 1996 (Japanese).

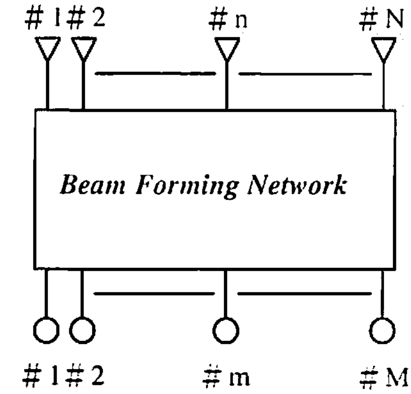

Fig.1 : The structure of the MBAA.

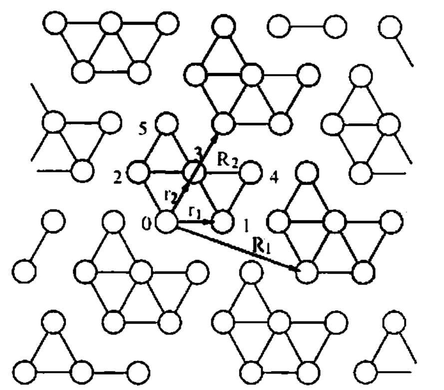
Fig.2 : The element plane for an array with 6 × 6 BFN.

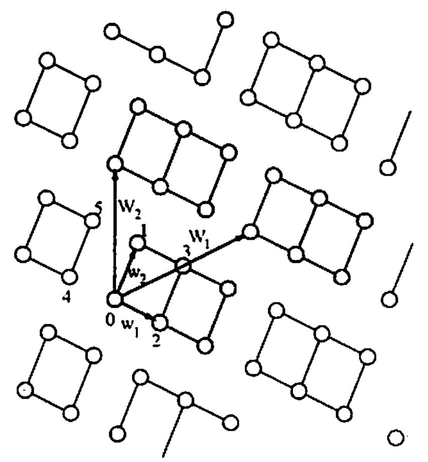
Fig.3 : The beam plane of an array with 6 × 6 BFN.

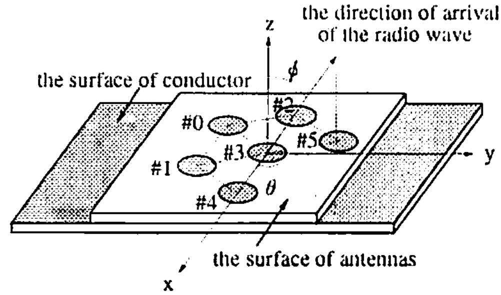

Fig.4 : The configuration of 6 × 6 planar array.

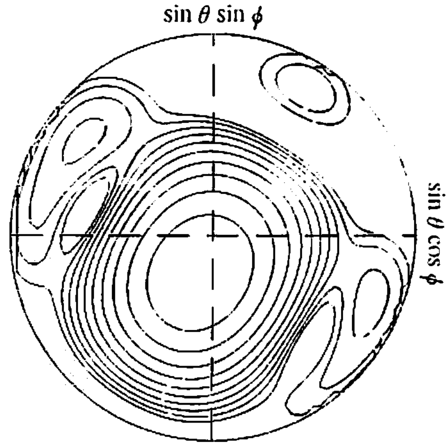
Fig.5 : The radiation patterns for the $m = 0$ beam port. The contour of equal amplitudes with increment 2.0dB.

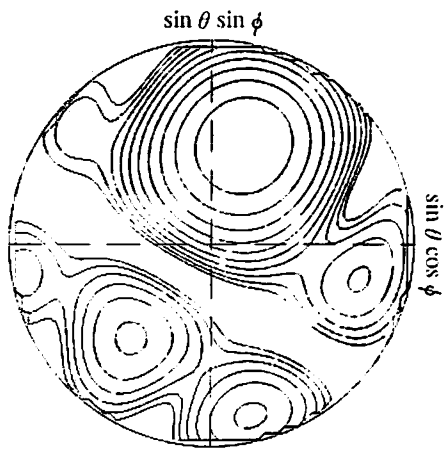
Fig.6 : The radiation patterns for the $m = 1$ beam port. The contour of equal amplitudes with increment 2.0dB.