FUNDAMENTAL ACCURACY LIMITATIONS OF POLARISATION MEASUREMENT SYSTEMS CALIBRATED WITH PRACTICALLY AVAILABLE STANDARD ANTENNAS

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Introduction

Polarisation-sensitive measurements of wave or antenna properties are of interest in several fields of radio-wave research, microwave imaging and antenna design.
Practical measurement systems for these applications consist usually of dual-polarised antennas and devices for amplitude- and phase measurements [1, 2]. Due to inevitable errors in any practical measurement setup, the systems have to be calibrated before they can be used for measurements. This is usually performed with antennas of assumed polarisation state which are regarded as 'standards'. However, if the polarisation of these 'standards' has not been determined by absolute methods (e.g. [3]), the deviation between their assumed and actual polarization sets a fundamental limitation for the accuracy of the whole system.

This contribution gives a mathematical description of the measurement errors caused by a calibration with practically available 'standard' antennas of assumed polarisation state. For the most significant polarisations (linear and circular) complete diagrams for the occuring errors are presented.

Mathematical description

A mathematical description of the calibration problem will be formulated for a typical antenna measurement system represented in Fig.1.
The antenna-under-test (AUT) is used in the transmit-mode and is characterised by the two components $\underline{C}_A$, $\underline{C}_B$ of its farfield radiation referred to two arbitrary but orthogonal polarisations A and B. These components are measured by a dual-polarised receiving antenna with corresponding output ports A, B and a phase-sensitive receiver (NWA) giving two complex output values $\underline{S}_A$ and $\underline{S}_B$ which can be regarded as the reactions of the measurement systems on the radiation components $\underline{C}_A$ and $\underline{C}_B$. If $\underline{C}_A$ and $\underline{C}_B$ are known, any interesting polarization parameter can be determined by simple calculations. In case of radio-wave polarisation measurements the AUT has to be replaced by the incoming wave front, but the following consideration remains still valid.

The properties of the measurement system can be described by the system-matrix $[\underline{k}]$ which relates the output values $\underline{S}_A$, $\underline{S}_B$ with the input radiation components $\underline{C}_A$, $\underline{C}_B$.

$$\begin{bmatrix} \underline{S}_A \\ \underline{S}_B \end{bmatrix} = [\underline{k}] \begin{bmatrix} \underline{C}_A \\ \underline{C}_B \end{bmatrix} \quad \text{with} \quad [\underline{k}] = \begin{bmatrix} \underline{k}_{AA} & \underline{k}_{AB} \\ \underline{k}_{BA} & \underline{k}_{BB} \end{bmatrix} \tag{1}$$

If $[\underline{k}]$ is known, $\underline{C}_A$ and $\underline{C}_B$ for an AUT or an incoming wave can be calculated from the system outputs $\underline{S}_A$ and $\underline{S}_B$ by inversion of eq.(1).
The determination of the $[\underline{k}]$-matrix is performed by a calibration process using two standard antennas I and II with known (or usually assumed) polarisation components. If these components and the corresponding reactions of the system on the standard antennas I and II are given by the two matrices $[\underline{C}]$ and $[\underline{S}]$ as

$$[\underline{C}]=\begin{bmatrix} \underline{C}_A^I & \underline{C}_A^{II} \\ \underline{C}_B^I & \underline{C}_B^{II} \end{bmatrix} \quad [\underline{S}]=\begin{bmatrix} \underline{S}_A^I & \underline{S}_A^{II} \\ \underline{S}_B^I & \underline{S}_B^{II} \end{bmatrix}$$

the system matrix can be evaluated by the equation

$$[\underline{k}]=[\underline{S}][\underline{C}]^{-1} \tag{2}$$

In practice, however, $[\underline{C}]$ is not known exactly. If a matrix of assumed values $[\underline{C}']$ is used in eq.(2) instead of $[\underline{C}]$, the evaluated system-matrix $[k']$ does not describe the system correctly and the measurement accuracy deteriorates. The occuring errors can be calculated for given matrices $[\underline{C}]$ and $[\underline{C}']$ using a new 'transformation-matrix' $[\underline{T}]$ which transfers the actual polarisation components $\underline{C}_A$, $\underline{C}_B$ into those which were measured by the non-ideal calibrated system $\underline{C}_A'$, $\underline{C}_B'$.

$$\begin{bmatrix} \underline{C}_A' \\ \underline{C}_B' \end{bmatrix}=[\underline{k}']^{-1}\begin{bmatrix} \underline{S}_A \\ \underline{S}_B \end{bmatrix}=[\underline{T}]\begin{bmatrix} \underline{C}_A \\ \underline{C}_B \end{bmatrix} \tag{3}$$

with

$$[\underline{T}]=[\underline{k}']^{-1}[\underline{k}]=[\underline{C}'][\underline{C}]^{-1} \tag{4}$$

It is interesting to note that the second formulation for $[\underline{T}]$ does not depend on the properties of the system. The measurement errors are therefore only a function of assumed and actual polarisation of the standard antennas without any dependence from the actual measurement system.

Results for linearly and circularly polarised standard antennas

Fig. 2 shows possible calibration standards for an LP-calibration ($\underline{C}_A=\underline{C}_X$, $\underline{C}_B=\underline{C}_Y$). One horn-antenna is used in two different positions for the calibration procedure. If this antenna is assumed to be ideal but has in reality a non-vanishing crosspolarisation, the matrices $[\underline{C}]$ and $[\underline{C}']$ can be written as

$$[\underline{C}']=\underline{C}_0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [\underline{C}]=\underline{C}_{co}\begin{bmatrix} 1 & -\underline{e} \\ \underline{e} & 1 \end{bmatrix} \quad \text{with } \underline{e}=\frac{\underline{C}_{cr}}{\underline{C}_{co}}\Big|_{STD} \tag{5}$$

The corresponding antennas for a CP-calibration ($\underline{C}_A=\underline{C}_R$, $\underline{C}_B=\underline{C}_L$) are given in Fig. 3. If the standard for RHCP is the mirror-image of the standard for LHCP and both antennas are assumed to be ideal, their $[\underline{C}]$-matrices are

$$[\underline{C}']=\underline{C}_0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [\underline{C}]=\underline{C}_{co}\begin{bmatrix} 1 & -\underline{e} \\ -\underline{e} & 1 \end{bmatrix} \quad \text{with } \underline{e}=\frac{\underline{C}_{cr}}{\underline{C}_{co}}\Big|_{STD} \tag{6}$$

Choosing $\underline{C}_{CO}=\underline{C}_0$, the transformation matrices $[\underline{T}]$ for both calibrations are given by the inverse of the corresponding $[\underline{C}]$-matrix (c.f. eq.(4)). The virtual polarisation measured by the system can be related with the actual polarisation as follows

$$\begin{matrix} \underline{p}'=\frac{\underline{C}_x'}{\underline{C}_y'}=\frac{\underline{p}+\underline{e}}{1-\underline{e}\underline{p}} \\ \underline{w}'=\frac{\underline{C}_R'}{\underline{C}_L'}=\frac{1-j\underline{e}}{1+j\underline{e}}\cdot\underline{w} \end{matrix} \quad \text{LP-Cal.} \qquad \begin{matrix} \underline{p}'=\frac{1-\underline{e}}{1+\underline{e}}\cdot\underline{p} \\ \underline{w}'=\frac{\underline{w}-\underline{e}}{1-\underline{w}\underline{e}} \end{matrix} \quad \text{CP-Cal.} \tag{7}$$

It can be seen from the equations that the LP-calibration yields a vanishing measurement error for incoming CP ($\underline{p}=\pm j$, $\underline{w}=0,\infty$) whereas the same effect occurs for CP-calibration if the incoming radiation is linearly x- or y-polarised ($\underline{p}=0,\infty$, $\underline{w}=\pm 1$). If nearly linear (x, y) or nearly circular polarised antennas (or waves) have to be analysed, the accuracy of the system can therefore be improved by a proper choice of the standard polarisation even in case of available i.e. non-ideal calibration antennas. Calculated results for both calibrations are given in Fig. 4 for three cases: the measurement of the crosspolarisation discrimination for LP-components XPDL ($\underline{C}_{co}=\underline{C}_X$; $\underline{C}_{cr}=\underline{C}_Y$) and CP-components XPDC ($\underline{C}_{co}=\underline{C}_R$, $\underline{C}_{cr}=\underline{C}_L$) and the measurement of the polarisation tilt-angle τ referred to the x-axis.

$$
\begin{aligned}
\Delta XPDL &= XPDL' - XPDL = 20\lg|\underline{p}'/\underline{p}| \quad [dB] \\
\Delta XPDC &= XPDC' - XPDC = 20\lg|\underline{w}'/\underline{w}| \quad [dB] \\
\Delta\tau &= \tau' - \tau = \tfrac{1}{2}\operatorname{arc}(\underline{w}'/\underline{w}) \quad [°]
\end{aligned}
\tag{8}
$$

Figs. 4b, c, and e show cases where the measurement error does not depend on the actual polarisation state and can therefore be represented for different possible values of the standard antenna error $\underline{e}$. In Figs. 4a, d and f the measurement error is given as a function of the actual polarisation for an assumed calibration standard error of $\underline{e}$ = -30 dB$\angle$0°. Equivalent diagrams for other values of $\underline{e}$ can be calculated easily from eqs.(7) and (8).
A more detailed discussion of the results will be given during the presentation.

References

[1] Hollis, J.S., Lyon, T.J., Clayton, L. (eds): Microwave Antenna Measurements, Scientific Atlanta Inc., Atlanta, Georgia, USA, 1970, Chapter 10

[2] Kummer, W.H., Gillespie, E.S.: Antenna Measurements-1978, Proc. IEEE, vol. 66, No. 4 (1978), pp. 483-507

[3] Newell, A.C., Kerns, D.M.: Determination of both polarisation and power gain of antennas by a generalised 3-antenna measurement method, Electron. Lett., vol. 7, No. 3 (1971), pp. 68-70

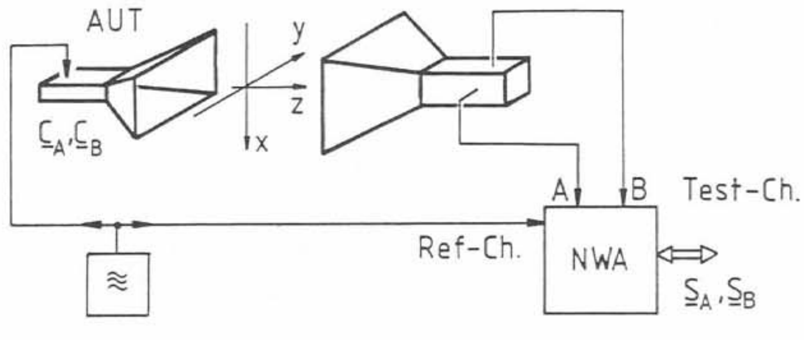

Fig. 1: Typical measurement system for the determination of antenna polarisation

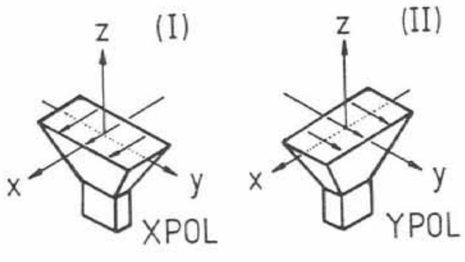

Fig. 2: Standard antennas for LP-calibration

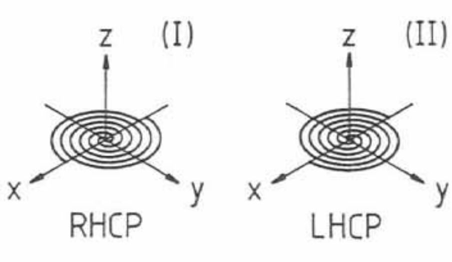

Fig. 3: Standard antennas for CP-calibration

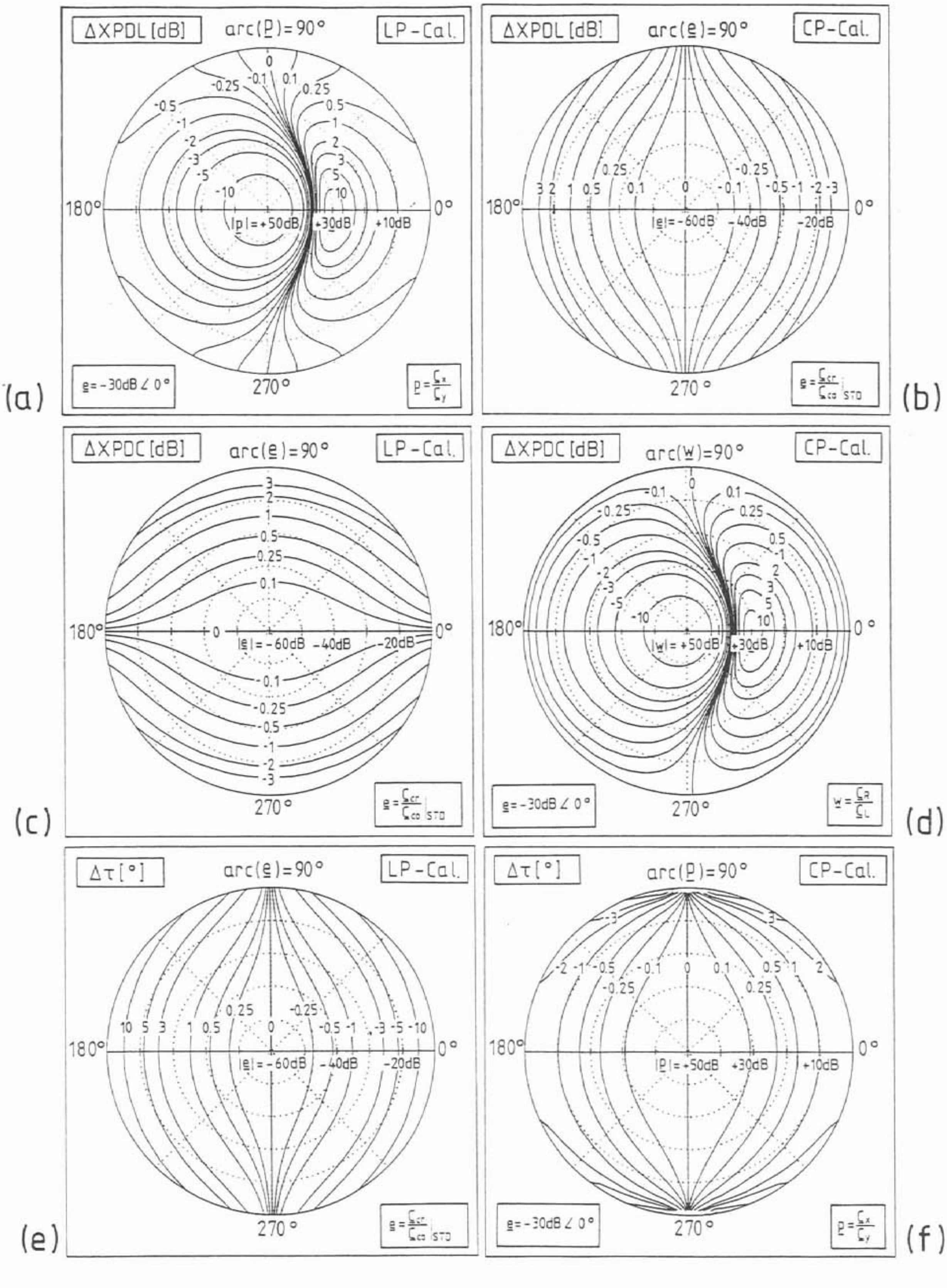

Fig. 4: Calculated measurement errors for LP- und CP-calibration