

# Computational Load Reduction of STAP using a New Stage Processing

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## 1. Introduction

Space-Time Adaptive Processing (STAP)[1] is a promising technique for MTI on airborne radar system. Because of the two-dimensional filtering capability of STAP, the clutter rejection performance of STAP can be greatly improved in comparison with that of the conventional spatial-only adaptive processing. Multistage Wiener Filter (MWF)[2][3] is a reduce rank algorithm for STAP. In environments with low sample supports, the performance of the MWF is better than the full rank Wiener filter.

The MWF provides a stage-by-stage decomposition of the Wiener filter solution. At each stage, a “scalar weight” is computed for the MWF which weights the contribution of the residue in removing interference from the desired signal at the corresponding stage. The effect of interference rejection at the first stage is dominant. Successive stages work to remove the residue of interference which cannot remove by the previous stages.

In this report, we proposed a new stage processing. The proposed method skips some stages which processed in the conventional MWF. This is done by deleting several rows of a blocking matrix which is used for the weight derivation in the MWF. We evaluated the proposed method from the viewpoint of both the computational complexity and interference suppression effect.

## 2. Space-Time Signal Model

The spatial and temporal steering vector  $s_\theta, s_{f_d}$  using STAP are defined as follows

$$s_\theta = \left[ 1, e^{j\frac{2\pi d}{\lambda} \sin \theta}, \dots, e^{j(N-1)\frac{2\pi d}{\lambda} \sin \theta} \right]^T, \tag{1}$$

$$s_{f_d} = \left[ 1, e^{j2\pi \frac{f_d}{f_p}}, \dots, e^{j(M-1)2\pi \frac{f_d}{f_p}} \right]^T, \tag{2}$$

where  $T$  is transpose.  $N$  and  $M$  are the number of elements and pulses, respectively.  $\lambda$  is the wavelength,  $d$  is the element spacing, and  $\theta$  is the spatial angle. The frequencies,  $f_d$  and  $f_p$  are the doppler frequency and pulse repetition frequency (PRF), respectively. We can combine the spatial and temporal steering vectors into  $N \times M$  space-time steering vector  $s$  for STAP as follows

$$s = s_{f_d} \otimes s_\theta, \tag{3}$$

where  $\otimes$  is the Kronecker product. Clutter is modeled as zero-mean Gaussian random process. The space-time clutter covariance matrix  $R_c$  is given by

$$R_c = \sum_{i=1}^{N_c} \sigma_c^2(i) \left[ v_{f_i} v_{f_i}^H \right] \otimes \left[ v_{\theta_i} v_{\theta_i}^H \right], \tag{4}$$

where  $H$  is the conjugate transpose.  $N_c$  is the number of clutter patches uniformly distributed in azimuth,  $\sigma^2(i)$  is power of in the  $i^{\text{th}}$  clutter patch,  $\mathbf{v}_{\theta_i}$  and  $\mathbf{v}_{f_i}$  are the spatial and temporal steering vectors, respectively, associated with the  $i^{\text{th}}$  clutter patch.

All the undesired signals (clutter, jammer, and noise) for STAP are assumed to be uncorrelated each other. Therefore, the total interference-plus-noise covariance matrix  $\mathbf{R}$  can be given by sum of the individual covariance matrices

$$\mathbf{R} = \mathbf{R}_c + \sum_{i=1}^J \mathbf{R}_j(i) + \mathbf{R}_n, \quad (5)$$

where  $\mathbf{R}_j$  is the jammer covariance matrix,  $J$  is the number of jammers and  $\mathbf{R}_n$  is noise covariance matrix. Receiving data is given by

$$\mathbf{x}_i(k) = \frac{1}{\sqrt{2}} \mathbf{R}^{\frac{1}{2}} \mathbf{a}_k, \quad (6)$$

The  $\mathbf{x}_i(k)$  is the  $k^{\text{th}}$  training sample at the  $i^{\text{th}}$  element, where  $K$  is the total number of training samples. The  $N \times M$  matrix  $\mathbf{a}_k$  is generated from a complex vector of zero mean with unit variance.

### 3. Multistage Wiener Filter

The conventional structure of the MWF is shown in Fig. 1. The MWF derives weight for interference rejection by successive signal processing at each stage. Each parameter used for the filter is derived by the recursive equations listed in Table 1, where  $*$  and  $E[\cdot]$  denote conjugate and ensemble average, respectively.  $\mathbf{r}_{x_i d_i}$  is the cross-correlation vector, and  $\mathbf{h}_i$  is the corresponding unit vector.  $\mathbf{B}_i$  is the blocking matrices. The operator  $null(\cdot)$  derives the orthogonal matrix for the argument. That is  $\mathbf{B}_i \mathbf{h}_i = \mathbf{0}$  in this case.  $w_i$  is the scalar weights, and  $\epsilon_i(k)$  is the error signals.

The adaptive weight vector  $\mathbf{w}_{mwf}$  by the MWF can be given by

$$\mathbf{w}_{mwf} = \mathbf{s} - w_1 \mathbf{B}_0^H \mathbf{h}_1 + w_1 w_2 \mathbf{B}_0^H \mathbf{B}_1^H \mathbf{h}_2 - w_1 w_2 w_3 \mathbf{B}_0^H \mathbf{B}_1^H \mathbf{B}_2^H \mathbf{h}_3 + \dots, \quad (7)$$

where  $\mathbf{B}_0$  is the  $null(\mathbf{s})$ .

A useful expression for estimating the rank of the clutter-only covariance matrix  $\mathbf{R}_c$ , can be obtained by Brennan's Rule [2] for the case of a side-looking uniform linear array (ULA) with fixed PRF. For ideal conditions, the Brennan's Rule can be expressed by

$$\text{rank}\{\mathbf{R}_c\} \approx [N + \beta \times (M - 1)], \quad (8)$$

where  $\beta$  is a parameter depending on the platform velocity  $v_p$ . It is defined by

$$\beta = \frac{2v_p}{d \cdot f_p}. \quad (9)$$

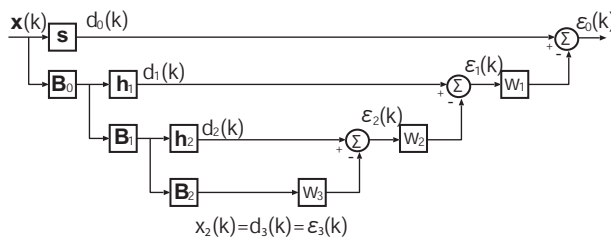


Fig.1: The filter structure of the MWF for  $N \times M=4$ .

Table 1: Recursion equations for the MWF

Forward Recursion	Backward Recursion
$\mathbf{r}_{x_i d_i} = E[\mathbf{x}_i(k) d_i^*(k)]$	$w_i = \delta_i / \xi_i$
$\delta_{i+1} = \sqrt{\mathbf{r}_{x_i d_i}^H \mathbf{r}_{x_i d_i}}$	$\epsilon_{i-1}(k) = d_{i-1}(k) - w_i \epsilon_i(k)$
$\mathbf{h}_{i+1} = \mathbf{r}_{x_i d_i} / \delta_{i+1}$	$\xi_{i-1} = E[ \epsilon_{i-1} ^2]$
$\mathbf{B}_{i+1} = null(\mathbf{h}_{i+1})$	$= \sigma_{d_{i-1}}^2 - \delta_i^2 / \xi_i$
$d_{i+1}(k) = \mathbf{h}_{i+1}^H \mathbf{x}_i(k)$	
$\sigma_{d_i}^2 = E[ d_i(k) ^2]$	
$\mathbf{x}_{i+1}(k) = \mathbf{B}_{i+1} \mathbf{x}_i(k)$	

## 4. Proposed Stage Processing Method

In stage processing of the MWF, the upper stages remove the interference more effectively than the lower stages. Performance of the upper stages greatly contribute to interference suppression. Computational complexity required by the processing at each stage highly depends on dimension of a vector  $\mathbf{h}$  and a Blocking matrix  $\mathbf{B}$ . Therefore, signal processing at the upper stage has a high computational complexity because of its dimension ( $\mathbf{h}$  and  $\mathbf{B}$ ). The dimension of  $\mathbf{h}$  and  $\mathbf{B}$  becomes smaller as the processing progresses to lower stage, hence the computational complexity also becomes low. As described in the above discussion, there is a “trade off” relation between interference rejection performance and computational complexity in the MWF. It can be said that performance of the MWF highly depends on the stage processing scheme.

In the conventional MWF, the stage-by-stage decomposition is employed from the first and the last stages. Therefore degree of rank-reduction is one by one-step. For the interference suppression point of view, this would be an optimum approach, however computational load is extremely high. Therefore, we consider an approach for reduction of computational load without loss of MWF performance. We propose a new method of the stage processing to reduce computational loads of the MWF algorithm. Since the number of required stages is determined by Brennan’s Rule, a selection of the stages can be changeable. The concept of the proposed method is selection of adequate stages for decreasing computational loads without loss of interference suppression performance. Fig. 2 shows the structure of the proposed MWF processing. As shown in the figure, the proposed method skips some steps which processed in the conventional MWF. In the followings, we use the word “ $n$  skip” which means that we skip  $n$  stage by every stage-processing. For example, actual processed stages for “one skip” are the 1st, the 3rd, the 5th stages, and so forth stages at the conventional MWF. Because of the proposed method works with skipping some steps, size of matrices becomes smaller at each stage than the conventional MWF. As a result, we can obtain reduction of computational load of the MWF.

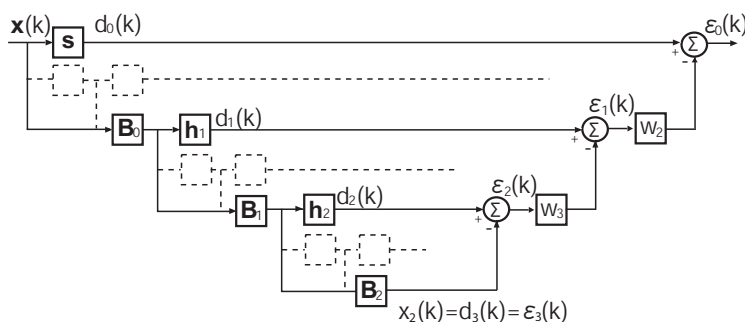


Fig.2: Processing image of the improved stage processing (example by one skip).

This new stage processing can be realized by generating  $\mathbf{B}$  matrix at each stage. At each stage, generate process of the  $\mathbf{B}$  matrix is as Fig. 3. Thus the  $\mathbf{B}$  matrix of  $i^{\text{th}}$  stage is generated by deleting lower (skip number+1) rows of the  $(\mathbf{I} - \mathbf{h}_i \mathbf{h}_i^H)$  matrix, where  $\mathbf{I}$  is the identity matrix.

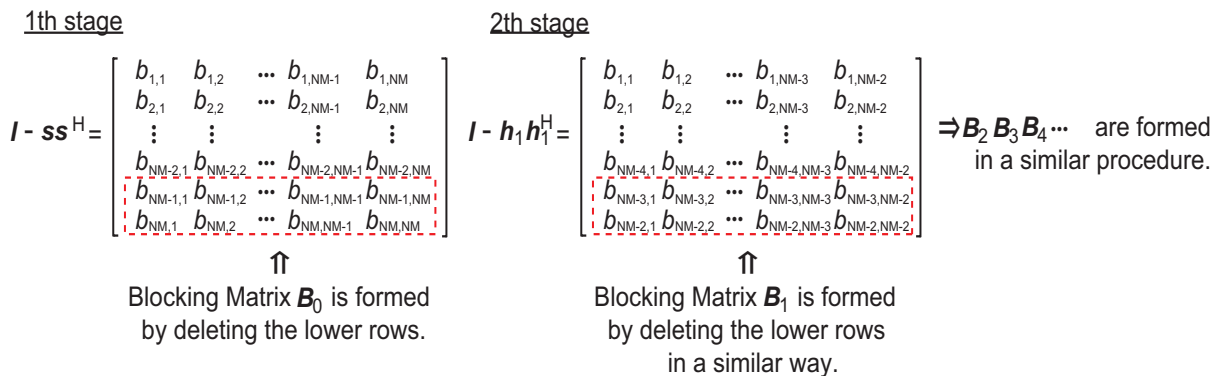


Fig.3: Generate process of the  $\mathbf{B}$  matrix at each stage (example by one skip).

## 5. Simulation Results

In this section, we evaluate performance of the proposed method by computer simulation. The simulation parameters are as follows: 8 elements ULA ( $N = 8$ ) with element spacing of half-wavelength, 8 pulse ( $M = 8$ ) coherent processing interval, 64 training data samples ( $K = 64$ ), noise floor is 0 dB, clutter-to-noise ratio (CNR) is 20 dB at each element and each pulse, the conditions of clutter aliasing  $\beta = 1.0$  and the results are evaluated by 100 Monte Carlo trials.

Fig. 4 shows normalized signal-to-interference-plus-noise ratio (SINR) versus number of processing stages with the conventional and proposed MWF. In this figure, we plotted the results of conventional one and three types of the proposed ones with several skips (one skip, two skips and three skips). From these results, the performance of the normalized SINR changes by each processing. However, maximum SINR performance can be obtained at 15 stages (the number of processed stages is 15) in all methods. This number corresponds to that estimated by Brennan's Rule. Therefore even if stage-by-stage decomposition of the MWF is processed by skipping some steps as proposed method, we will be able to obtain high SINR performance with the optimum number of processed stages determined by Brennan's Rule.

Fig. 5 shows computational complexity required by each methods. Compared with complexity of the conventional MWF at 15 processed stages, that of the proposed method at 15 stages reduces about 25 % by one skip, about 45% by two skips, and about 60% by three skips.

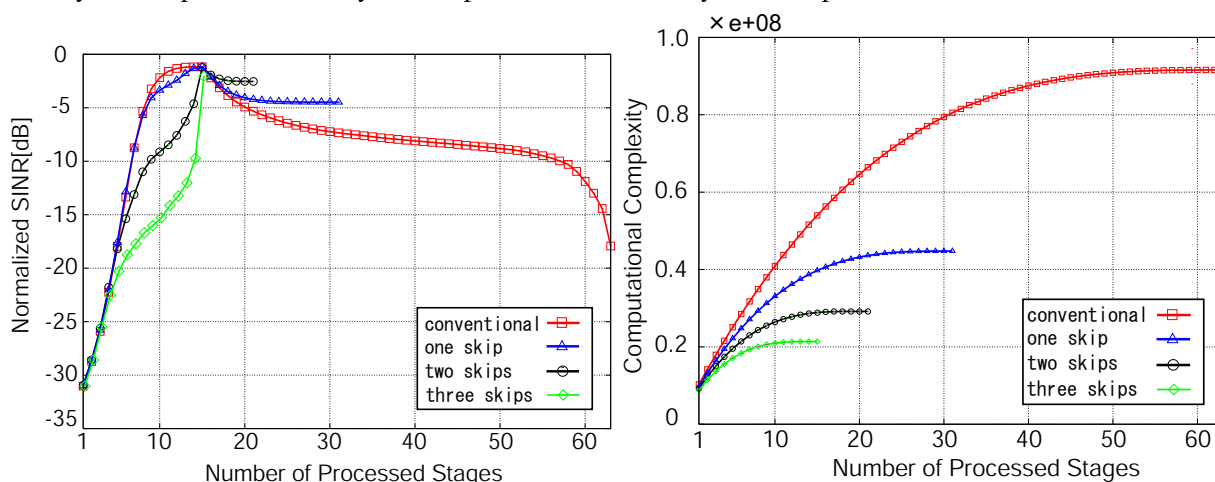


Fig.4: Number of processed stages vs. SINR

Fig.5: Number of processed stages vs. computational complexity

## 6. Conclusions

In this paper, we proposed a new stage processing for computational load reduction of the conventional MWF algorithm. Computer simulation results show that the proposed method has almost the same interference suppressing performance at the optimum rank determined by Brennan's Rule. Therefore, it can be said that we can realize computational-effective MWF with a same performance by the proposed method.

## References

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