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ELECTROMAGNETIC CHARACTERIZATION OF AN INHOMOGENEOUS GYROTROPIC PLASMA EXHIBITING STRONG-PROPERTY FLUCTUATIONS

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1. Background

Studies of the propagation of electromagnetic waves in a turbulent ionized gas or plasma is of interest in many engineering and scientific applications such as radio communication through the ionosphere, simulation of missile exhaust plumes, and laser-fusion devices. One prominent feature of plasmas which strongly affects wave propagation is that the relative intensity of permittivity fluctuations in the plasma grows considerably as the circular frequency ω of electromagnetic wave approaches a value at which the permittivity of plasma tends to zero. For instance, for an isotropic cold electron plasma where the ion currents and collisions may be neglected the permittivity vanishes when ω equals the plasma frequency. The previous attempts at accounting for this phenomenon have employed the renormalization approach grounded on extracting the delta-function constituent of the space-domain Green's function. This approach is well documented in the literature, and we refer the reader to works [1]–[2] as representative references.

In many instances the electromagnetic field is essentially shaped by spatial inhomogeneity in the mean electrical properties of the plasma. This is the case, e. g., for a confined plasma if the dimensions of plasma region are comparable to or smaller than the wavelength. Thus, a strong-fluctuation theory accounting for random medium's statistical inhomogeneity is needed to describe the interaction of electromagnetic waves with such plasmas accurately. Unfortunately, however, most previous strong fluctuation approaches are applicable to statistically homogeneous media only. Among the exceptions are the works [3] and [4]–[5] referring to random media which are, respectively, plane stratified or cylindrically inhomogeneous in a statistical sense. The renormalization approach employed therein takes into proper account a delta function singularity of the spectral Green's function rather than that of its spatial counterpart used in the past.

This paper can be considered as an extension of our previous analysis [4]–[5], dealing with the cylindrically inhomogeneous plasma with strong permittivity fluctuations, to the case where the plasma is bestowed by electromagnetic anisotropy due to an external static magnetic field. More particularly, a locally gyrotropic model is employed, where the turbulent plasma filling a cylindrical region $V = \{0 \le \rho < a, 0 \le \varphi < 2\pi, -\infty < z < +\infty\}$

of radius a is characterized by the random permittivity matrix of the form

$$\bar{\varepsilon}_e = skew\,[\varepsilon_1^r, \varepsilon_2^r, \varepsilon_3^r, if^r] \equiv \begin{bmatrix} \varepsilon_1^r & if^r & 0 \\ -if^r & \varepsilon_2^r & 0 \\ 0 & 0 & \varepsilon_3^r \end{bmatrix}, \qquad (1)$$

relative to the unit vectors ρ_0, φ_0, z_0. Here ε_j^r, $(j = 1, 2, 3)$, and f^r are random Gaussian functions of $x = (\rho, \varphi, z)$, with ε_1^r being identically equal to ε_2^r. A permittivity matrix of this type is realized in an applied magnetic field which is directed along the z axis and may vary in the radial direction.

We examine a situation in which the expectation values of the permittivity matrix entries (1) are constant with respect to φ, z and may depend upon ρ variable only, and all the correlation functions $\langle\varepsilon_1^r(x)\varepsilon_1^r(x')\rangle$, $\langle\varepsilon_1^r(x)f^r(x')\rangle$, ... , $\langle\varepsilon_3^r(x)\varepsilon_3^r(x')\rangle$ depend on spatial variables through ρ, ρ' and the differences $\varphi - \varphi'$, $z - z'$. These requirements reflect the assumption of statistical invariance of the random medium with respect to arbitrary rotations about the z axis and translations along this axis.

In this report it is shown how the effective permittivity for such statistically inhomogeneous medium can be calculated in the strong fluctuation case, and some effects of multiple scattering on the mean electromagnetic field propagation are revealed.

2. Content of Report

We assume that the impressed sources with a time dependence $\exp(-i\omega t)$ have the form of a spatial harmonic $\underline{J}(x) = \underline{J}(\rho, n, h)\exp[i(n\varphi + hz)]$ determined by arbitrary integer number n and complex value h, and a correlative spectral amplitude $\underline{J}(\rho, n, h)$. Here $\underline{A}$ designates a column matrix $[A_\rho, A_\varphi, A_z]^{\mathrm{T}}$, and T denotes the matrix transposition operation. We note that a general distribution of the impressed sources can be represented as a proper superposition of spatial harmonics with the help of double Fourier transform.

Owing to the statistical properties of the random medium under consideration the ensemble average value of random electric field $\underline{E}_r(x)$ and that of random electric flux density $\underline{D}_r(x)$ have to be of a spatially harmonic form as well, viz.

$$\langle\underline{D}_r(x)\rangle = \underline{D}(\rho, n, h)\exp[i(n\varphi + hz)], \quad \langle\underline{E}_r(x)\rangle = \underline{E}(\rho, n, h)\exp[i(n\varphi + hz)]. \qquad (2)$$

The spectral amplitudes $\underline{E}(\rho, n, h)$, $\underline{D}(\rho, n, h)$ are linked by the spectral effective permittivity operator (EPO) $\bar{\varepsilon}_e(n, h)$:

$$\underline{D}(\rho, n, h) = \bar{\varepsilon}_e(n, h) \circ \underline{E}(\rho, n, h) \equiv \int_0^a \bar{\varepsilon}_e(\rho, \rho' \mid n, h) \circ \underline{E}(\rho', n, h)\, \rho' d\rho', \qquad (3)$$

which fully accounts for the random medium's properties with respect to the mean electromagnetic field.

To ascertain the EPO in terms of statistical characteristics of the random medium, we resort to a modified renormalization approach proposed in [4]–[5]. We start with the familiar volumetric integral equation of scattering

$$\underline{E}_r(x) = \underline{E}_b(x) + k_0^2 \int \overline{G}(x, x') \circ [\bar{\varepsilon}_r(x') - \bar{\varepsilon}(\rho')] \circ \underline{E}_r(x')\, dx', \qquad (4)$$

where k_0 is the free-space wavenumber, $dx' = \rho' d\rho' \, d\varphi' \, dz'$, $\underline{E}_b(x)$ and $\overline{G}(x, x')$ are, respectively, the electric field and the Green's matrix referring to a deterministic background medium of permittivity $\overline{\varepsilon} = skew\,[\varepsilon_1, \varepsilon_2, \varepsilon_3, if]$, with ε_j, $(j = 1, 2, 3)$, and f being functions of variable ρ. The matrix Green's function can be stated as

$$\overline{G}(x, x') = (2\pi)^{-2} \sum_{n=-\infty}^{n=+\infty} \int_{-\infty}^{+\infty} \exp\{i[n(\varphi - \varphi') + h(z - z')]\} \overline{G}(\rho, \rho', n, h) \, dh, \tag{5}$$

where $\overline{G}(\rho, \rho', n, h)$ is the spectral Green's matrix. The singular behaviour of the latter as a generalized function of variables ρ, ρ' may be expressed through [6]

$$\overline{G}(\rho, \rho', n, h) = \overline{G}'(\rho, \rho', n, h) - \overline{\mathcal{I}}_1 \frac{\delta(\rho - \rho')}{k_0^2 \rho' \varepsilon_1(\rho')}, \tag{6}$$

where the first term is an integrable function of each of the variables ρ, ρ', $\delta(\rho - \rho')$ is a Dirac's delta function, and $\overline{\mathcal{I}}_1$ stands for a matrix $diag\,[1, 0, 0]$. Using the space-domain counterpart of (6) in (4), we transform the latter equation to a renormalized form

$$\underline{F}(x) = \underline{E}_b(x) + k_0^2 \int \overline{G}'(x, x') \circ \overline{\xi}(x') \circ \underline{F}(x') \, dx', \tag{7}$$

with a new field variable $\underline{F}(x)$ and a random perturbation matrix $\overline{\xi}(x) = skew\,[\xi_1, \xi_2, \xi_3, ih]$ defined by

$$\begin{aligned} F_\rho &= \frac{\varepsilon_1}{\varepsilon_1^r} E_\rho^r - i \frac{f^r - f}{\varepsilon_1^r} E_\varphi^r, \\ F_\varphi &= E_\varphi^r, \quad F_z = E_z^r; \end{aligned} \tag{8}$$

$$\begin{aligned} \xi_1 &= \varepsilon_1 \left(1 - \varepsilon_1/\varepsilon_1^r\right), \quad \xi_2 = \varepsilon_2^r - \varepsilon_2 - \left(f^r - f\right)^2/\varepsilon_1^r, \\ \xi_3 &= \varepsilon_3^r - \varepsilon_3, \quad h = \left(f^r - f\right)^2 \varepsilon_1/\varepsilon_1^r. \end{aligned} \tag{9}$$

Carrying out one iteration in (7), taking the ensemble average of the relation arising in the bilocal approximation, and comparing it with the averaged version of an original equation (7) leads to an explicit expression for the effective perturbation operator $\overline{\xi}_e$ defined by the identity $\langle \overline{\xi} \circ \underline{F}(x) \rangle \equiv \overline{\xi}_e \circ \langle \underline{F}(x) \rangle$. The EPO is then obtainable from the equation

$$\overline{\varepsilon}_e - \overline{\varepsilon} = \overline{\xi}_e \circ \left(\frac{1}{\varepsilon_1} \overline{\varepsilon}_e \circ \mathcal{I}_1 + \mathcal{I}_t \right), \tag{10}$$

($\mathcal{I}_t = diag\,[0, 1, 1]$), which can be solved with standard perturbation techniques. A solution to this equation which is consonant with the bilocal approximation has the following spectral-domain characterization:

$$\begin{aligned} &\overline{\varepsilon}_e(\rho, \rho' \mid n, h) \approx \overline{\varepsilon}(\rho) \delta(\rho - \rho')/\rho \\ &+ (k_0/2\pi)^2 \sum_{n'=-\infty}^{n'=+\infty} \int_{-\infty}^{+\infty} dh' \int_{-\infty}^{+\infty} dz \int_0^{2\pi} d\varphi \, \exp\{-i[(n - n')\varphi + (h - h')z]\} \\ &\langle \overline{\xi}(\rho, \varphi, z) \circ \overline{G}'(x, x') \circ \overline{\xi}(\rho', 0, 0) \rangle, \end{aligned} \tag{11}$$

where $\bar{\xi}(\rho, \varphi, z) \equiv \bar{\xi}(x)$. Expression (11) has been obtained under assumption that the average value of $\bar{\xi}$ equals identically zero, $\langle \bar{\xi}(x) \rangle \equiv 0$. This provides a means to determining a permittivity matrix for the background medium whose entries are given by:

$$\frac{1}{\varepsilon_1(\rho)} = \langle \frac{1}{\varepsilon_1^r(x)} \rangle, \quad f(\rho) = \varepsilon_1(\rho) \left\langle \frac{f^r(x)}{\varepsilon_1^r(x)} \right\rangle, \tag{12}$$
$$\varepsilon_2(\rho) = \langle \varepsilon_2^r(x) \rangle + \frac{f^2(\rho)}{\varepsilon_1(\rho)} - \left\langle \frac{(f^r(x))^2}{\varepsilon_1^r(x)} \right\rangle, \quad \varepsilon_3(\rho) = \langle \varepsilon_3^r(x) \rangle.$$

Relations (11), (12) are generalizations of similar results derived previously for an isotropic random medium [4], [5]. It is important to note that the right-hand members in eq. (11) can be assigned the status of first terms in an asymptotic expansion for $\bar{\varepsilon}_e(\rho, \rho' \mid n, h)$ in powers of $(\sigma k_0 l)^2$ where σ is a positive constant of the order of rms value of random perturbations, and l is their correlation length in the ρ direction. The requirement of smallness of said parameter is met even in the case of strong fluctuations ($\sigma \gg 1$) provided their characteristic scale l is sufficiently small.

A strong fluctuation approach reported in the present paper is capable of handling the continuous random media, as well as the discrete composite media. The latter point is illustrated in the report by considering the effective properties of a mixture prepared by distributing electrically small dielecric spheres in a cylindrically inhomogeneous plasma column.

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