SCATTERING OF LIGHT FROM ONE-DIMENSIONAL RANDOM METAL SURFACE

— 45°-LINEARLY POLARIZED INCIDENCE, BACKSCATTERING ENEHANCEMENT AND DEGREE OF POLARIZATION —

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Introduction The enhanced backscattering from rough surfaces or rough waveguide strucuture has been intensively investigated in many papers with various methods, and it has been clarified that the mechanism responsible for this is the coherent interference in the multiple scattered waves. and can be particularly attributed to the presence of guided wave modes on the random surface[1-3]. The state of polarization of scattered light or EM waves is also perturbed by the random scattering. For comparison with experimental observations the one-dimensional rough surface [4] is suitable for separating TE and TM-polarization. The present paper treats the optical scattering from a random metal surface, which supports a surface guided wave mode called surface plasmon, by means of the stochastic functional approach.[3,5] To clarify the plasmon's association with scattering characterisctics, we treat a random surface with a normal spectrum and a random grating with twin spectral peak at plasmon spatial-frequency. The "dressed" plasmon mode is obtaind for several rough surfaces, and the enhanced backscattering is studied in connection with the enhanced peak width and the mass operator of the dressed plamon. The Stokes parameters and the degree of polarization are numerically calculated from polarized components of the scattering distribution.

Stochastic Wave Field for 45°-Polarized Plane Wave Incidence Let the coordinate system be (x, y, z), and $z = 0$ be the plane between air ($z > 0$) with the dielectric constant $\epsilon_1 = 1$ and silver ($z < 0$) with $\epsilon_2 = -17.55 + i0.404$. Aassume that 45°-polarized electromagnetic (light) plane wave with wave vector $k_0 = (\lambda_0, 0, -\mu_0), \mu_0 = \sqrt{k^2 - \lambda_0^2}$ is incident on the 1D random surface $z = f(x)$, which is a homogeneous Gaussian random surface with variance $\langle f^2 \rangle = \sigma^2$.

Then the stochastic EM field in the two media can be represented in terms of Wiener-Itô expansion,

$$E_1(x, z; \omega|\lambda_0) = e^{i\lambda_0 x}\left[\frac{e_{TE}[\lambda_0] + e_{TM}[\lambda_0]}{\sqrt{2}} e^{-i\mu_1(\lambda_0)z} + \sum_{n=0}^{\infty} U_1{}^n(T^x\omega, z|\lambda_0)\right] \quad (1)$$

$$E_2(x, z; \omega|\lambda_0) = e^{i\lambda_0 x}\sum_{n=0}^{\infty} U_2{}^n(T^x\omega, z|\lambda_0) \quad (2)$$

$$\begin{aligned} U_1{}^n(T^x\omega, z|\lambda_0) = & \int\cdots\int \{e_{TE}^{(+)}[\lambda_0 + \cdots + \lambda_n] A_n^{TE}(\lambda_1, \cdots, \lambda_n|\lambda_0) \\ & + e_{TM}^{(+)}[\lambda_0 + \cdots + \lambda_n] A_n^{TM}(\lambda_1, \cdots \lambda_n|\lambda_0)\} \\ & e^{i(\lambda_1 + \cdots + \lambda_n)x} e^{i\mu_1(\lambda_0 + \cdots + \lambda_n)z} \hat{h}_n[dB(\lambda_1), \cdots, dB(\lambda_n)] \end{aligned} \quad (3)$$

$$U_2{}^n(T^x\omega, z|\lambda_0) = \int\cdots\int \{e_{TE}^{(-)}[\lambda_0 + \cdots + \lambda_n] C_n^{TE}(\lambda_1, \cdots, \lambda_n|\lambda_0)$$

$$+e^{(-)}_{\mathrm{TM}}[\lambda_0+\cdots+\lambda_n]C^{\mathrm{TM}}_n(\lambda_1,\cdots\lambda_n|\lambda_0)\}$$
$$e^{i(\lambda_1+\cdots+\lambda_n)x}e^{-i\mu_2(\lambda_0+\cdots+\lambda_n)z}\hat{\mathrm{h}}_n[dB(\lambda_1),\cdots,dB(\lambda_n)] \tag{4}$$

where $\mu_j(\lambda) = \sqrt{\epsilon_j k^2-\lambda^2}, j = 1,2, e^{(\pm)}_J[\lambda], J = \mathrm{TE,TM}$, denotes the polarization vector, $A^J_n(\lambda_1,\cdot,\lambda_n|\lambda_0), C^J_n(\lambda_1,\cdot\cdot\lambda_n|\lambda_0)$ n-th Wiener kernels for TE, TM-polarized components, and $\hat{\mathrm{h}}_n[\ ]$ n-order Wiener-Hermite orthogonal functional of Gaussian random measure $dB(\lambda)$.

Solving the stochastic boundary condition the vector Wiener kernel $\boldsymbol{A}_n \equiv (A_n, C_n)$ can approximately be obtained in the following form

$$\boldsymbol{A}_0(\lambda_0) \simeq \frac{1}{\boldsymbol{\Delta}^*(\lambda_0)}\left[\boldsymbol{E}_0(\lambda_0)-\int Q(\lambda_1|\lambda_0+\lambda_1)\frac{1}{\boldsymbol{\Delta}(\lambda_0+\lambda_1)}\boldsymbol{E}_1(\lambda_1|\lambda_0)d\lambda_1\right] \tag{5}$$
$$\boldsymbol{A}_1(\lambda_1|\lambda_0) \equiv \frac{1}{\boldsymbol{\Delta}^*(\lambda_0+\lambda_1)}\boldsymbol{E}(\lambda_1|\lambda_0) \tag{6}$$
$$\boldsymbol{E}(\lambda_1|\lambda_0) \equiv \boldsymbol{E}_1(\lambda_1|\lambda_0)-\boldsymbol{P}(\lambda_1|\lambda_0)\frac{1}{\boldsymbol{\Delta}^*(\lambda_0)}\boldsymbol{E}_0(\lambda_0) \tag{7}$$
$$\boldsymbol{A}_2(\lambda_1,\lambda_2|\lambda_0) \simeq -\frac{1}{2\boldsymbol{\Delta}^*(\lambda_0+\lambda_1+\lambda_2)}\Big[\boldsymbol{P}(\lambda_1|\lambda_0+\lambda_2)\frac{1}{\boldsymbol{\Delta}^*(\lambda_0+\lambda_2)}\boldsymbol{E}(\lambda_2|\lambda_0)$$
$$+\boldsymbol{P}(\lambda_2|\lambda_0+\lambda_1)\frac{1}{\boldsymbol{\Delta}^*(\lambda_0+\lambda_1)}\boldsymbol{E}(\lambda_1|\lambda_0)\Big] \tag{8}$$
$$M(\lambda) \equiv \int Q(\lambda_1|\lambda+\lambda_1)\frac{1}{\boldsymbol{\Delta}^*(\lambda+\lambda_1)}P(\lambda_1|\lambda)d\lambda_1 \tag{9}$$
$$\boldsymbol{\Delta}^*(\lambda) \equiv \boldsymbol{\Delta}(\lambda)-M(\lambda) \tag{10}$$
$$\boldsymbol{\Delta}^{\mathrm{TE}}(\lambda) \equiv \begin{bmatrix}1 & -1\\ i\mu_1(\lambda) & i\mu_2(\lambda)\end{bmatrix} \qquad \boldsymbol{\Delta}^{\mathrm{TM}}(\lambda)\equiv\begin{bmatrix}\frac{\mu_1(\lambda)}{k_1} & \frac{\mu_2(\lambda)}{k_2}\\ -ik_1 & ik_2\end{bmatrix} \tag{11}$$

and $1/\boldsymbol{\Delta}(\lambda)$ implies the inverse matrix $\boldsymbol{\Delta}(\lambda)^{-1}$, which we call propagator. $\boldsymbol{P}, \boldsymbol{Q}$ are 2×2-matrices proportional to the rough surface kernel, the detail being omitted. (9) corresponds to the Dyson equation in the multiple scattering theory.

Free and Dressed Plasmons The pole of the propagator $1/\boldsymbol{\Delta}^{\mathrm{TM}}(\lambda)$, that is, the root of $\mathrm{Det}\boldsymbol{\Delta}^{\mathrm{TM}}(\lambda)=0$ is called the free plasmon which is $\lambda_{p0}=(1.02975+i0.000716)k$. Similarly the pole λ_p of the dressed propagator $1/\boldsymbol{\Delta}^{*\mathrm{TM}}(\lambda)$, is called the dressed plasmon which is the plasmon perturbed by the random surface.

$$\lambda_p \simeq \pm\lambda_{p0}\left(1+\lambda_{p0}\frac{1}{\epsilon_1-\epsilon_2}\xi/k^2\right) \tag{12}$$
$$\xi \equiv k_1\left(M_{12}-\frac{1}{k}\sqrt{\frac{-1}{\epsilon_1+\epsilon_2}}M_{22}\right)+k_2\left(M_{11}-\frac{1}{k}\sqrt{\frac{-1}{\epsilon_1+\epsilon_2}}M_{21}\right) \tag{13}$$

where M_{ij} denotes the matrix element of the mass operator $M(\lambda)$ which is of the order of σ^2.

Incoherent Scattering Distribution In terms of Wiener kernels for the air side, A_1, A_2, the polarized component of the incoherently scattered intensity can be written

$$P^J(\theta_s|\theta_0) = k_1\cos^2\theta_s|A^J_1(\lambda-\lambda_0|\lambda_0)|^2+2k_1\cos^2\theta_s\int_{-\infty}^{\infty}|A^J_2(\lambda-\lambda_1,\lambda_1-\lambda_0|\lambda_0)|^2d\lambda_1 \tag{14}$$

where the pair of polarization index J can be taken in three ways: $J=(\mathrm{TE,TM})$; $(+45°,-45°)$; $(\mathrm{R,L})$, R,L indicating right- and left-hand circular polarization. There are following relations:

$$A^{\pm 45°}_n = \frac{1}{\sqrt{2}}\left[A^{\mathrm{TE}}_n \pm A^{\mathrm{TM}}_n\right] \qquad A^{R,L}_n = \frac{1}{\sqrt{2}}\left[A^{\mathrm{TE}}_n \pm iA^{\mathrm{TM}}_n\right], n=1,2,\cdots \tag{15}$$

The total scattering intensity S_0 is given by the sum of polarized component pairs as shown below.

Stokes Parameters and Degree of Polarization The four Stokes parameters are given by

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} P^{\rm TM} + P^{\rm TE} \\ P^{\rm TM} - P^{\rm TE} \\ P^{+45^\circ} - P^{-45^\circ} \\ P^{\rm R} - P^{\rm L} \end{bmatrix}, \qquad \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}^{+45^\circ} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{16}$$

$S_0 = P^{\rm TM} + P^{\rm TE} (= P^{+45^\circ} + P^{-45^\circ} = P^{\rm R} + P^{\rm L})$ denotes the total power flow. The Stokes parameters for $+45^\circ$-linearly polarized incident wave with normalized intensity is given in the righthand side. The degree of polarization P_{ol} can be defined by the ratio of the completely polarization to the total intensity:

$$P_{ol} = \sqrt{{S_1}^2 + {S_2}^2 + {S_3}^2}/S_0 \tag{17}$$

Scattering Distribution, Stokes Parameters and Degree of Polarization for Random Grating Scattering characteristics have been studied for 4 kinds of rough surface. We show here some numerical results for a random grating with twin spectral peaks at the plasmon frequency λ_g:

$$|F(\lambda)|^2 = \sigma^2(\ell/2\sqrt{\pi})\left[e^{-\ell^2(\lambda+\lambda_g)^2} + e^{-\ell^2(\lambda-\lambda_g)^2}\right] \tag{18}$$

with surface parameters ($kl = 3.0, k\sigma = 0.1$). Fig.1 shows the incoherent scattering distribution for the vertical incidence $\theta_0 = 0^\circ$, where $+45^\circ$-, -45°- and TM-polarized components show enhanced peaks at the backscattering angle. The enhancement is explained by the interference of two reciprocal processes involved in 2nd Wiener kernel that describes "double" scattering processes of "dressed" plasmons [6,7]. Fig.2 shows (a) the Stokes parameters (b) the incoherent scattering intensity (completely polarized and unpolarized components) and (c) the degree of polariztion. The degree of polarization is reduced around $|\theta_s| \simeq 30^\circ$, which can be explained as follows. The peak of completely polarized component around $\theta_s \simeq 0^\circ$ of Fig.(b) is due to 2nd Wiener kernel, and the weaker peaks around $|\theta_s| \simeq 60^\circ$ due to 1st Wiener kernel; thus mixed contributions from 1st and 2nd kernels around $|\theta_s| \simeq 30^\circ$ reduce the degree of polarization.

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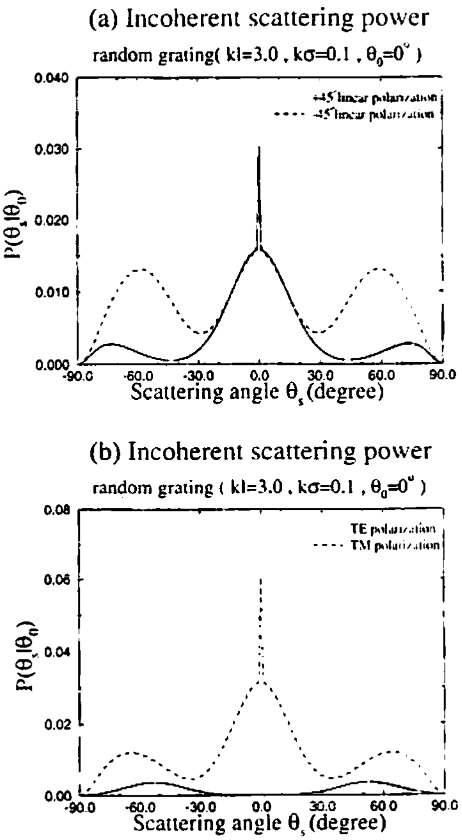

Figure 1: Incoherent scattering from random grating with incident angle $\theta_0 = 0°$. (a)+45°- and −45°-linearly polarized components (b)TE- and TM-polarized components

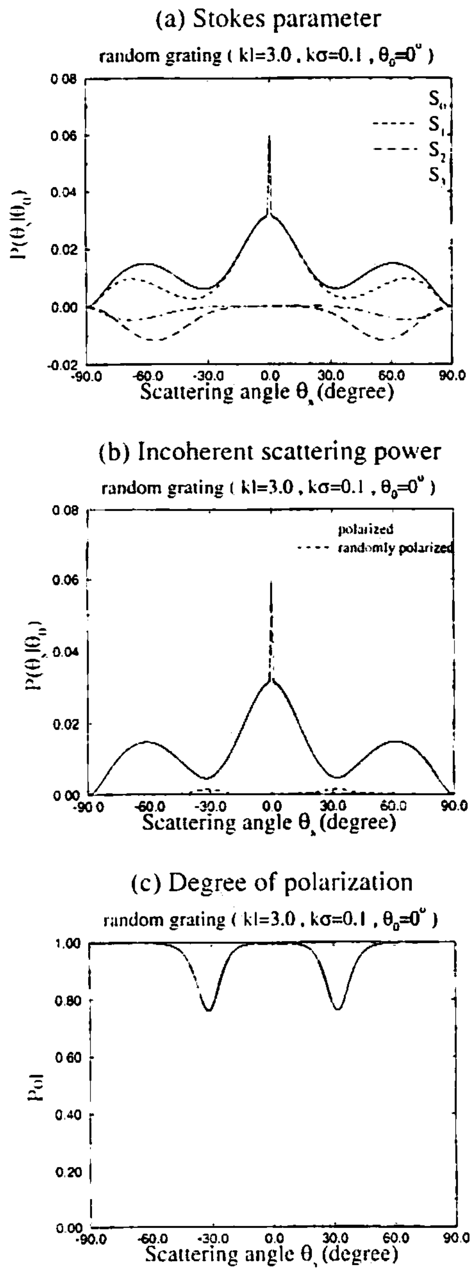

Figure 2: Random grating with $\theta_0 = 0°$: (a)Stokes parameters (b)completely polarized and unpolarized components (c)degree of polarization.