PROCEEDINGS OF ISAP '92, SAPPORO, JAPAN

MODELING OF SUPERCONDUCTIVITY FOR EM BOUNDARY VALUE PROBLEMS

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1 Introduction

High-temperature superconductivity has spawned a wide search for electronics applications. Various experiments on microwave circuits, including resonators, filters, phase shifters, and small antennas have been performed with promising results. The design of these components will necessarily involve numerical simulation, which requires the solution of superconductive EM boundary value problems. In this paper, several approaches to this solution are presented. In the first approach, superconductors are treated as negative dielectric materials. Secondly, the superconductor surface impedance condition is used at the boundaries. Finally, the problem is solved using perfectly conducting boundaries, and perturbation is used to approximate the desired parameters. The above methods are used to solve the scattering by a superconducting cylinder, propagation in a superconducting parallel-plate waveguide, and radiation by a superconducting short dipole. Very good agreement is found between the negative dielectric model and the surface impedance method. The perfect conductor approximation also gives very good estimates of the fields, but fails to predict some important effects, such as the change in resonant frequency of the dipole with temperature.

2 Superconductors as Negative Dielectrics

In earlier work we have shown how a superconductor operating below its critical temperature can be modeled as a negative dielectric material [1], [2], and [3]. This approach makes analyzing these materials much easier for antenna and microwave engineers. In this paper we shall calculate superconductive boundaries as an interface of air and a negative dielectric, and compare with other known approaches, such as perturbation and surface impedances.

3 Approximate Solution to Superconductive Boundary Value Problems

Even though negative dielectric materials can be treated in a similar way as conventional dielectrics in the solution of boundary value problems, much effort can be spared if an approximate solution is obtained. The most common approximation is to use the surface impedance boundary condition at the interface

$$\vec{E} \times \hat{n} = Zs \cdot \vec{H} \times \hat{n} \tag{1}$$

where Zs is the surface impedance of the material, and $\hat{n}$ is the unit vector normal to the boundary. This approximation is only valid when the radius of curvature of the interface is much larger than a wavelength. Moreover, for frequencies low enough so that $\epsilon_r \gg 1$, and for $T \ll T_c$, we may obtain an approximate solution by perturbing the solution found using perfectly conducting boundaries. In the following sections, we will present several examples using both rigorous and approximate approaches.

3.1 Scattering By An Infinitely Long Superconducting Circular Cylinder

The scattering by a circular cylinder has an exact solutions which can be applied to negative dielectrics. Therefore, we can compare the approximated results with them.

3.1.1 Numerical Results

The following results have been computed for a typical YBCO high-T_c superconductor ($T_c = 88$ K). We assume a surface resistance of 0.3 mΩ at 77 K and 10 GHz, and a penetration depth $\lambda(0) = 140$ nm. The surface impedance can then be expressed as

$$Zs = RS_{10\mathrm{GHz}} \left(\frac{f(\mathrm{GHz})}{10} \right)^2 + j\omega\mu_o \frac{\lambda(0)}{\sqrt{1 - \bar{T}^4}} \tag{2}$$

which at 77 K results in

$$Zs(f) \simeq 3 \cdot 10^{-6} f^2(\mathrm{GHz}) + j1.1 \cdot 10^{-3} f(\mathrm{GHz}) \quad (\Omega) \tag{3}$$

From this expression it is easy to find the alternative material parameters mentioned earlier to be $f_o \simeq 550$ THz, $f_r \simeq 250$ GHz, and $\bar{T} = 0.875$. These values are used for the following simulations, unless otherwise indicated. Figure 1 shows the ϕ component of the TMz surface magnetic field for the exact simulation and the two approximate methods. It is observed that the fields obtained with the three different methods cannot be distinguished even for cylinders with a small radius. It is interesting to see the variation of the cylinder skin depth with frequency. Figure 2 shows the skin depth versus frequency for two different temperatures, and for a copper cylinder. We see that for the superconducting cylinder it is a very weak function of frequency, and for the copper cylinder it is proportional to $f^{-1/2}$.

3.2 Superconducting Dipole

One of the most interesting features of a superconducting dipole is the dependence of the resonant frequency on temperature. At higher temperatures the surface impedance becomes more and more inductive, making the dipole electrically longer, and the resonant frequency smaller. Figure 3 shows the resonant frequency shift with respect to temperature. This information can be very important for narrowband antennas and circuits, where the precise knowledge of the operating frequency is critical. It is also interesting to compare the input resistance of a superconducting dipole with that of a copper dipole. Figure 4 shows the input resistance of a superconducting dipole. Since the radiation resistance is dominant, the input resistance is very similar to that of a conventional dipole. Figure 5 shows the change in the input resistance if a YBCO dipole at 77 K is replaced by a copper dipole at 77 K. Finally, Fig. 6 shows the efficiency of a superconducting dipole, compared to that of a copper dipole. We see that very short superconducting dipoles would be efficient radiators at frequencies at which the loss resistance would dominate over the radiation resistance in a copper dipole.

4 Conclusion

A simple characterization of superconductors based on the classical two-fluid model has been presented, which allows to solve EM boundary value problems involving superconductors by treating them as negative dielectrics. First, these methods are tested in the scattering by a superconductive cylinder. The solutions found using the surface impedance is almost identical to that found using the negative dielectric model. Also, a perturbational solution for the dissipated power is computed, with very small error. Secondly, two methods are used to find the attenuation constant of a parallel-plate waveguide, also showing very good agreement. Finally, a short dipole is analyzed. Both the negative dielectric model and the surface impedance approximation succeed in predicting the most important features of superconductor dipoles, such as the variation of resonant frequency with temperature, and their high efficiency at small frequencies.

References

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[2] K. K. Mei and G. C. Liang, "Electrodynamics of Superconductors," submitted to the *IEEE Trans. on Microwave Theory and Tech.*, Special Issue on Superconductivity, September 1991.

[3] G. C. Liang, Y. W. Liu, and K. K. Mei, "Propagation Properties of a Superconductive Stripline," *IEEE AP-S Int. Symp.*, Dallas, TX, pp. 728–731, May 1990.

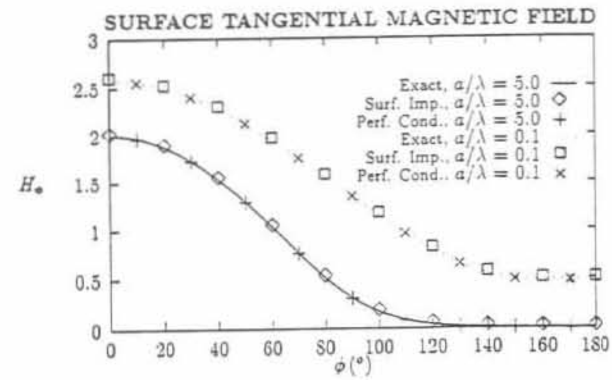

Figure 1: Angular component of the surface magnetic field (H_ϕ) for two cylinder radii. obtained by three different methods: (1) Negative dielectric model. (2) Surface impedance condition, and (3) Perfect conductor approximation ($a/\lambda = 5$, and $a/\lambda = 0.1$, $f = 50\,GHz$, TMz incidence).

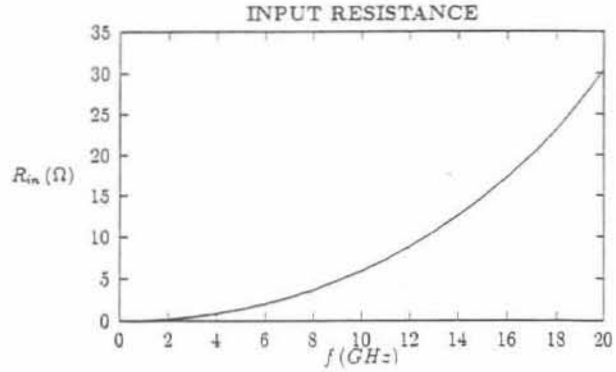

Figure 4: Input resistance of a 5-mm YBCO dipole with radius $a = 0.1\,mm$ versus frequency (T = 77 K).

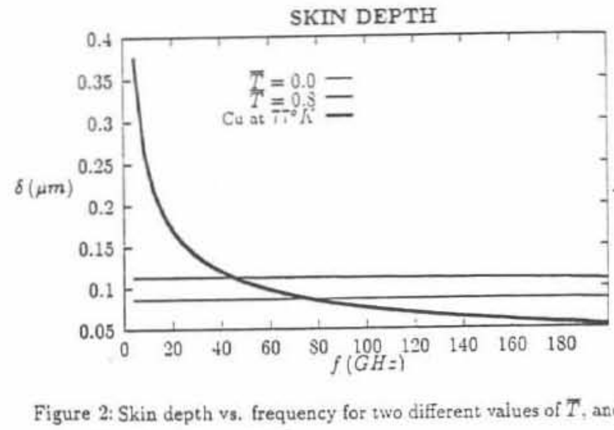

Figure 2: Skin depth vs. frequency for two different values of $\overline{T}$, and for a copper cylinder at 77 K.

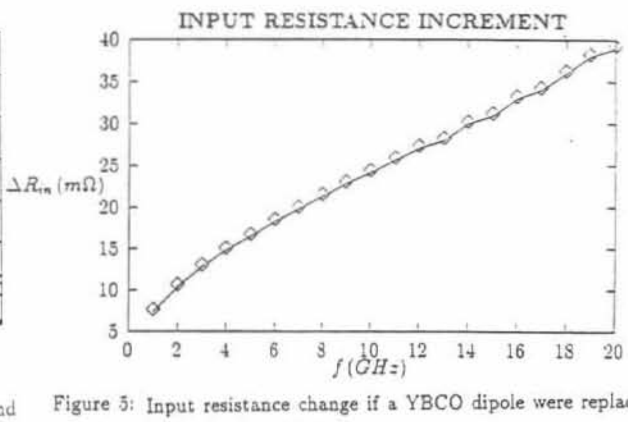

Figure 5: Input resistance change if a YBCO dipole were replaced by a copper dipole (both at 77 K).

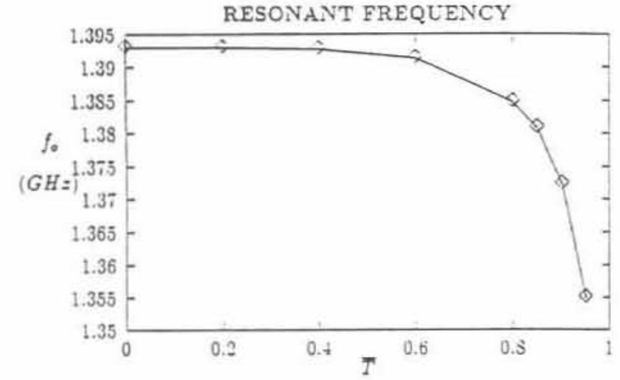

Figure 3: Resonant frequency of a 10-cm YBCO dipole of radius $a = 1\,mm$ versus normalized temperature.

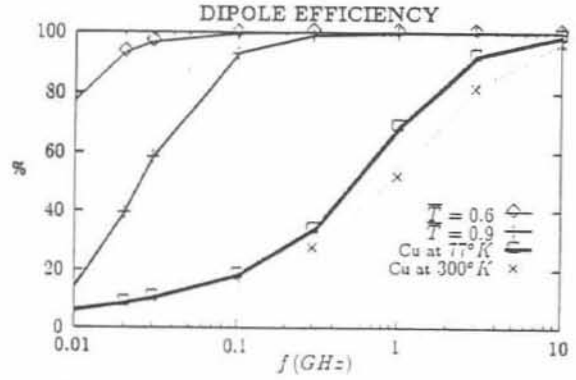

Figure 6: Efficiency of a superconductor and a copper dipole versus frequency for different temperatures ($l = 5\,mm$, $a = 0.05\,mm$).