SCATTERING OF ELECTROMAGNETIC WAVE FROM A PLANAR WAVEGUIDE STRUCTURE WITH A SLIGHTLY RANDOM SURFACE

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Abstract

The present paper deals with the scattering of an incident wave from a waveguide structure with a slightly rough surface. The waveguide structure is a dielectric film on a planar, perfectly conductive surface, and the film top is a two-dementional (2D) homegeneous Gaussian random surface. The treatment is based on the stochastic functional theory where the random electromagnetic (EM) field is represented in terms of Wiener-Hermite functional of the random surface. Numerical calculations show that there occur enhanced backscattering and cross-polarization, but that no enhanced satellite peak appears for 2D random surface contrary to the case of 1D one. The enhanced backscattering is caused by the interference of two double-scattering processes and is attributed to the existence of guided waves in the scattering structure.

1 Introduction

The scattering of EM wave from rough surface has been studied by many authers, because it is closely related to practical problems such as propagation over rough sea and land, optical properties of a random metal surface, etc. The interesting feature in this problem is that in spite of random scattering there is an enhanced backscattering peak, as a consequence of constructive interference between multiple scattering processes.

Recently, we have studied the scattering of EM wave from a perfectly conductive 2D random surface [1]. We showed that there is enhanced backscattering in both co- and cross-polarized scattering, which does not occur in the case of 1D random surface. We also have studied the scattering from waveguide structure with 1D random surface [2], which shows that there are satellite peaks in incoherent scattering distribution in addition to the enhanced backscattering peak, when the waveguide structure supports two or more than two guided modes.

The present paper deals with EM scattering from a planar wave guide structure with 2D random surface and investigates co- and cross-polarized scattering characteristic by means of the stochastic functional approach. Using the Wiener-Hermite expansion technique in the probability theory, we represent the wave field as a stochastic functional of the Gaussian random surface, and then determine Wiener kernels. Incoherent scattering profiles are calculated numericaly. It is demonstrated that there occurs no satellite peak in the case of 2D random surface, a remarkable difference from the case of 1D one.

2 Representations for Random Surface and Random EM Field

Waveguide structure We consider the scattering of an EM wave, which is incident from air side, as shown in Fig. 1. EM field satisfies the boundary conditions on the air-dielectric interface $z = f(x)$ and the dielectric-metal interface $z = -a$. This structure can bear one or more guided modes.

Random Surface Let the 2D plane be denoted by R_2, a position vector by $\boldsymbol{x} = (x, y)$, and a sample point by ω. We assume the random surface $z = f(\boldsymbol{x}, \omega)$ to be a homogeneous Gaussian random field expressed by the spectral representation,

$$z = f(\boldsymbol{x}, \omega) = \int_{R_2} e^{i\boldsymbol{\lambda}\cdot\boldsymbol{x}} F(\boldsymbol{\lambda}) dB(\boldsymbol{\lambda}, \omega), \quad \overline{F(\boldsymbol{\lambda})} = F(-\boldsymbol{\lambda}) \tag{1}$$

$dB(\boldsymbol{\lambda})$ denotes the 2D complex Gaussian random measure with the property: $\left\langle \overline{dB(\boldsymbol{\lambda})} dB(\boldsymbol{\lambda}') \right\rangle = \delta(\boldsymbol{\lambda} - \boldsymbol{\lambda}') d\boldsymbol{\lambda} d\boldsymbol{\lambda}'$, the probability parameter ω being suppressed for brevity. The correlation function is given by

$$R(\boldsymbol{x}) = \int_{R_2} e^{i\boldsymbol{\lambda}\cdot\boldsymbol{x}} |F(\boldsymbol{\lambda})|^2 d\boldsymbol{\lambda}, \quad \boldsymbol{x} \in R_2, \quad \sigma^2 \equiv R(0) = \int_{R_2} |F(\boldsymbol{\lambda})|^2 d\boldsymbol{\lambda} \tag{2}$$

where $| F(\boldsymbol{\lambda}) |^2$ denotes the spectral density and σ^2 the variance, namely, the surface roughness. For numerical calculation we assume the isotropic Gaussian spectral density (ℓ : correlation distance):

$$R(\rho) = \sigma^2 e^{-\rho^2/(4\ell^2)}, 0 \le \rho < \infty, \quad | F(\lambda) |^2 = \sigma^2 \frac{\ell^2}{\pi} e^{-\ell^2 \lambda^2}, \quad 0 \le \lambda < \infty \tag{3}$$

Polarization Vectors As shown in Fig. 2 let the wave vector $\boldsymbol{k}_\iota$ be denoted by

$$\boldsymbol{k}_\iota = \boldsymbol{\lambda} + \mathrm{sign}([\boldsymbol{k}_\iota]_z) S_\iota(\lambda) \boldsymbol{e}_z \quad \iota = \begin{cases} a & \text{in air: } z > 0 \\ d & \text{in dielectric: } -a < z < 0 \end{cases} \tag{4}$$

$$S_\iota(\lambda) = \sqrt{k_\iota^2 - \lambda^2}, \quad k_\iota = |\boldsymbol{k}_\iota|, \quad \lambda = |\boldsymbol{\lambda}| \tag{5}$$

The horizontal (TE) and vertical (TM) polarization vectors corresponding to the wave vector $\boldsymbol{k}_\iota$ by can then be expressed by

$$\boldsymbol{e}_H^{\iota(\pm)}[\boldsymbol{\lambda}] \equiv \frac{\boldsymbol{\lambda} \times \boldsymbol{e}_z}{\lambda}, \quad \boldsymbol{e}_V^{\iota(\pm)}[\boldsymbol{\lambda}] \equiv \pm \frac{S_\iota(\lambda)}{k_\iota \lambda} \boldsymbol{\lambda} - \frac{\lambda}{k_\iota} \boldsymbol{e}_z \tag{6}$$

The signs (+) and (−) mean the outgoing wave and the incoming wave, respectively.

Winer-Hermite expansion for Random EM wave field Let $\boldsymbol{E}^\iota(\boldsymbol{x}, z; \omega|\boldsymbol{\lambda}_0)$ denote the stochastic electric field for the plane wave incidence in the direction $\boldsymbol{\lambda}_0$, where ω implies that $\boldsymbol{E}^\iota$ is a stochastic functional of (1). According to "the stochastic Floquet theorem", the electric field, which satisfies boundary conditions on the conductive surface $z = -a$, is expressible in terms of Wiener–Hermite expansion [1]:

$$\begin{aligned}
&\boldsymbol{E}^a(\boldsymbol{x}, z; \omega|\boldsymbol{\lambda}_0) = \\
&\{\boldsymbol{e}_H^{a(-)}[\boldsymbol{\lambda}_0] N^H(\boldsymbol{\lambda}_0) + \boldsymbol{e}_V^{a(-)}[\boldsymbol{\lambda}_0] N^V(\boldsymbol{\lambda}_0)\} e^{i\boldsymbol{\lambda}_0\cdot\boldsymbol{x} - iS^a(\boldsymbol{\lambda}_0) z} \\
&+ \{\boldsymbol{e}_H^{a(+)}[\boldsymbol{\lambda}_0] A_0^H(\boldsymbol{\lambda}_0) + \boldsymbol{e}_V^{a(+)}[\boldsymbol{\lambda}_0] A_0^V(\boldsymbol{\lambda}_0)\} e^{i\boldsymbol{\lambda}_0\cdot\boldsymbol{x} + iS^a(\boldsymbol{\lambda}_0) z} \\
&+ \sum_{n=1}^{\infty} \int \cdots \int \{\boldsymbol{e}_H^{a(+)}[\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 + \cdots + \boldsymbol{\lambda}_n] A_n^H(\boldsymbol{\lambda}_1, \cdots, \boldsymbol{\lambda}_n|\boldsymbol{\lambda}_0) \\
&\quad + \boldsymbol{e}_V^{a(+)}[\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 + \cdots + \boldsymbol{\lambda}_n] A_n^V(\boldsymbol{\lambda}_1, \cdots, \boldsymbol{\lambda}_n|\boldsymbol{\lambda}_0)\} e^{i(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 + \cdots + \boldsymbol{\lambda}_n)\cdot\boldsymbol{x} + iS^a(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 + \cdots + \boldsymbol{\lambda}_n) z} \\
&\quad \times \hat{\mathrm{h}}_n[dB(\boldsymbol{\lambda}_1), \cdots, dB(\boldsymbol{\lambda}_n)]
\end{aligned} \tag{7}$$

where N^H and N^V denote TE and TM component of incident wave, respectively. $\boldsymbol{E}^d$ can be similarly expanded with the coefficients C_n^H and C_n^V. The n-th Winer kernels, A_n^H and C_n^H give partial scattering amplitude of TE wave. Similarly A_n^V and C_n^V represent that of TM wave.

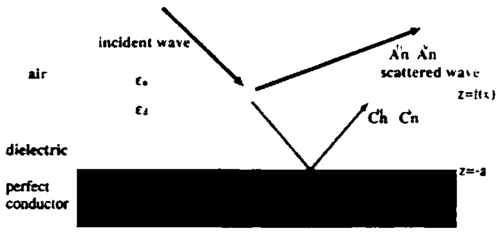
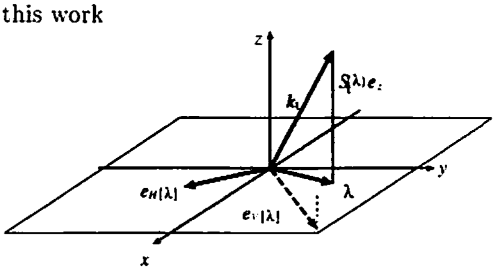


Figure 1: The scattring structure studied in this work



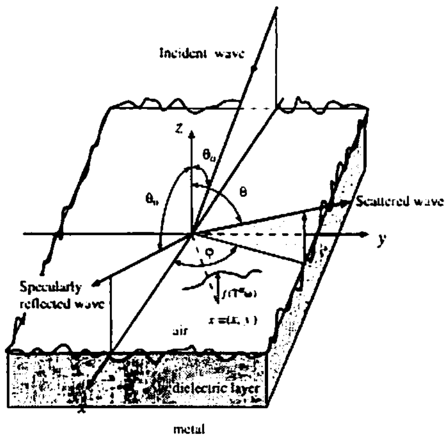


Figure 2: Scattering Vector and Horizontal and Vertical Polarization Vectors

Figure 3: Plane Wave Incidence on Random Surface

Incoherent scattering distribution Let $\theta \equiv (\theta, \phi)_{\rm pol}$ and $\theta_0 \equiv (\theta_0, \phi_0)_{\rm pol}$ stand for the spherical angles of scattering and incidence, respectively, as defined in Fig. 3 Then $P(\theta|\theta_0)$, the angular distribution of the incoherent scattering per unit area, is given by

$$\begin{aligned} P(\theta|\theta_0) &= \sum_{n=1}^{\infty}\{P_n^H(\theta|\theta_0) + P_n^V(\theta|\theta_0)\} \\ &= k^2\cos^2\theta\Big[|A_1^H(\lambda-\lambda_0|\lambda_0)|^2 + |A_1^V(\lambda-\lambda_0|\lambda_0)|^2 \\ &\quad +2!\int_{R_2}|A_2^H(\lambda-\lambda_0-\lambda_2,\lambda_2|\lambda_0)|^2 d\lambda_2 \\ &\quad +2!\int_{R_2}|A_2^V(\lambda-\lambda_0-\lambda_2,\lambda_2|\lambda_0)|^2 d\lambda_2\cdots\Big] \end{aligned} \quad (8)$$

where $P_n^H(\theta|\theta_0)$ and $P_n^V(\theta|\theta_0)$ denote the contributions to TE and TM polarized scattering from the n-th Wiener kernels A_n^H and A_n^V, respectively.

3 Numerical calculation for incoherent scattering distribution

Approximate calculation We determine the Wiener kernels up to 2nd order by applying the boundary conditions on the random surface $z = f(x)$. Using the Wiener kernels, P_1^H, P_1^V, P_2^H and P_2^V are numerically calculated. For TE polarized incidence, Fig.4 shows the scattering distribution for $\phi = 0°$ and $90°$ with incident angles $\theta = 30°$, where the contributions from the 2nd order scattering are separately shown in Fig.5. The thickness of dielectric film is such that the waveguide structure bears three modes.

Backscattering enhencement and cross polarization As shown in Fig.4 and Fig.5, the backscattering enhencement apperars in both co- and cross-polarized scattering. But there

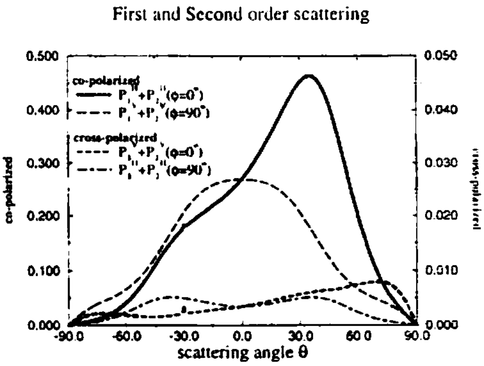


Figure 4: Scattering distribution:$P_1 + P_2$ $l = 1.0, \sigma = 0.3, a = 3.0, \theta_0 = 30°$,TE incidence

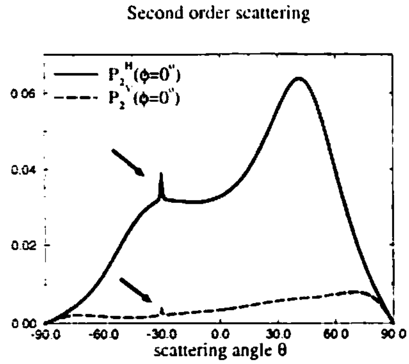


Figure 5: Scattering distribution:P_2 $l = 1.0, \sigma = 0.3, a = 3.0, \theta_0 = 30°$,TE incidence

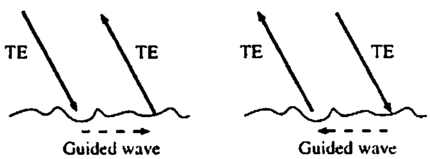


Figure 6: Second order co-polarized backscattering process

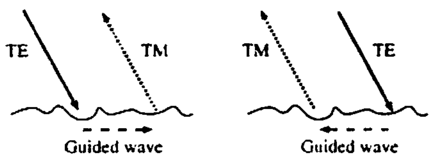


Figure 7: Second order cross-polarized backscattering process

is no satellite peak in spite of the fact that the structure supports more than one mode, whereas there are some satellite peaks in the case of 1D random surface associated with multimode propagation [2, 3]. This is because in the present case the integration along angular direction cancels interference between different modes.

The backscattering enhancement is usually interpreted as a constructive interference of two time reversal scattering processes [3]. The 2nd order scattering (A_2^H, A_2^V) consists of two processes, which represent double scattering respectively. Each of these processes is a time revesal process of another, only if the 2nd order scattering represents co-polarized backscattering (see Fig. 6). In the present case the enhanced backscattering appears in cross-polarized 2nd order scattering, due to the reciprocal symmetry among cross-polarized scattering processes, where the two co- or cross-polarized processes have the same optical path length (see Fig. 7).

References

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