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# LOCALIZATION OF SURFACE PLASMON WAVES ON A ROUGH METALLIC GRATING

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**Abstract** The random wave fields on the surface of a random rough metallic grating are expanded as the harmonics of the central spatial frequency of the grating with the stochastic amplitudes, and are numerically calculated for the Bragg incidence as the rough grating has a very narrow-band spectrum. The wave localization on the grating surface can be clearly observed from the spatial distribution of the random wave fields.

**Introduction** In recent years, there has been considerable interest in the study of enhanced backscattering and wave localization on scattering from random rough surfaces.[1] To investigate the localization effect, the random wave field scattered by a realization of rough surfaces has to be calculated. As pointed out by Saillard and Maystre,[2] the problem of rigorous computation of the field scattered by random rough surfaces is one of the most difficult to handle in the discipline of electromagnetism and optics, owing to the need for dealing with surfaces having a large number of illuminated asperities. Due to this difficulty, merely a few papers have been devoted to the study of the localization effect by the so-called beam-simulation method,[3, 4] in which only the surface with a finite length of about a few tens of wavelengths can be treated.

In a previous paper,[5] we have studied the phenomena of enhanced backscattering and wave localization, for a random rough grating on a silver film, by the stochastic functional approach based on the Wiener-Ito expansion. We have shown that the enhanced backscattering is mainly denominated by the "double" scattering process while the wave localization occurs as a result of "multiple" scattering process. To account for such multiple scattering, we had to evaluate the multiple-dimensional stochastic integrals with the higher-order Wiener kernels in the calculations of random wave fields. Unfortunately we have not yet obtained the convergent results, due to the poor convergence of the Wiener kernels at the resonant scattering and the accumulated errors as we go to the higher-order stochastic integration. To overcome such a divergent difficulty, in the present work we make use of an alternative way for the representation of random wave field, i.e., we expand the random wave fields as the harmonics of the central spatial frequency of the rough grating. Indeed, there is no divergence problem in this way, but on the other hand its application is limited only for the rough grating with a very narrow-band spectrum.

**Random Grating** As in the previous work,[5] we consider a random rough metallic grating represented by a narrow-band spectrum in the neighborhood of $\lambda = \pm 2\lambda_0$ where $\lambda_0$ is the real part of the plasmon pole in metal, such as

$$\begin{aligned} |F(\lambda)|^2 &= |F_0(\lambda + 2\lambda_0) + F_0(\lambda - 2\lambda_0)|^2 \\ &= \sigma^2 \frac{\ell}{2\sqrt{\pi}} \left[ e^{-\ell^2(\lambda+2\lambda_0)^2} + e^{-\ell^2(\lambda-2\lambda_0)^2} \right], \qquad 2\lambda_0 \ell \gg 1 \end{aligned} \tag{1}$$

The spectral representation for the random grating can therefore be written as a narrow-band homogeneous process with the carrier frequency $2\lambda_0$:

$$\begin{aligned} f(\mathrm{T}^x\omega) &= \mathrm{e}^{\mathrm{i}2\lambda_0 x}\int_U \mathrm{e}^{\mathrm{i}\mu x}F_0(\mu)\mathrm{d}B(2\lambda_0+\mu) + \mathrm{e}^{-\mathrm{i}2\lambda_0 x}\int_U \mathrm{e}^{-\mathrm{i}\mu x}\overline{F_0(\mu)\mathrm{d}B(2\lambda_0+\mu)} \\ &= \mathrm{e}^{\mathrm{i}2\lambda_0 x}f_1(x,\omega) + \mathrm{e}^{-\mathrm{i}2\lambda_0 x}\overline{f_1(x,\omega)} \end{aligned} \quad (2)$$

where $f_1(x)$ denotes a slowly-varying stochastic process. $\mathrm{d}B(\lambda)$ is the complex Gaussian random measure which can be simulated by Gaussian random numbers in numerical calculations.

**Wave Equation and Boundary Conditions** Let $\psi_j, j = 1, 2$, denote the 2D scalar wave in the two media—silver and air, then they satisfy the Helmholtz equations and the boundary conditions on the random interface:

$$(\nabla^2 + \epsilon_j k^2)\psi_j(z, x; \omega) = 0, \quad j = 1, 2 \quad (3)$$

$$\psi_1(z, x; \omega) = \psi_2(z, x; \omega), \quad \frac{1}{\epsilon_1}\frac{\partial\psi_1}{\partial n} = \frac{1}{\epsilon_2}\frac{\partial\psi_2}{\partial n} \quad (\text{on } z = f(T^x\omega)) \quad (4)$$

where $\partial/\partial n$ denotes the normal derivative on the random interface. Assuming that the surface is slightly random and smooth enough, the boundary conditions of Eq.(4) can then be approximated at the flat interface $z = 0$ as

$$\psi_1 + f\frac{\partial\psi_1}{\partial z} = \psi_2 + f\frac{\partial\psi_2}{\partial z} \quad (5)$$

$$\frac{1}{\epsilon_1}\left(\frac{\partial\psi_1}{\partial z} - \frac{\mathrm{d}f}{\mathrm{d}x}\frac{\partial\psi_1}{\partial x} + f\frac{\partial^2\psi_1}{\partial z^2}\right) = \frac{1}{\epsilon_2}\left(\frac{\partial\psi_2}{\partial z} - \frac{\mathrm{d}f}{\mathrm{d}x}\frac{\partial\psi_2}{\partial x} + f\frac{\partial^2\psi_2}{\partial z^2}\right) \quad (6)$$

where we have neglected the second-order and much higher-order terms in $f$. However, the higher order terms in the boundary conditions can be incorporated if necessary.

**Form of Stochastic Wave Fields** Suppose a plane wave at the Bragg incidence written as $\mathrm{e}^{\mathrm{i}\lambda_0 x - \mathrm{i}\mu_1(\lambda_0)z}, \mu_1(\lambda_0) \equiv \sqrt{\epsilon_1 k^2 - \lambda_0^2}$, is incident from the silver side onto the silver-air interface. Then, the stochastic wave fields generated by the Gaussian random interface can be represented in the following forms, by virtue of the "stochastic Floquet theorem":

$$\psi_j(z, x; \omega) = \mathrm{e}^{\mathrm{i}\lambda_0 x}U^{(j)}(z, \mathrm{T}^x\omega), \quad j = 1, 2 \quad (7)$$

and $U^{(j)}(z, \mathrm{T}^x\omega)$ can be further written as the harmonic expansion of the central spatial frequency of the rough grating,

$$U^{(j)}(z, \mathrm{T}^x\omega) = \delta_{j1}\mathrm{e}^{-\mathrm{i}\mu_1(\lambda_0)z} + \sum_{n=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}2n\lambda_0 x}U_n^{(j)}(z, x; \omega) \quad (8)$$

where $U_n^{(j)}$ are slowly-varying, narrow-band stationary processes in $x$, such that

$$U_n^{(j)}(z, x; \omega) = \int \mathrm{e}^{\mathrm{i}\nu x + \mathrm{i}\mu_j(2n\lambda_0+\nu)z}\mathrm{d}M_n^{(j)}(\nu, \omega) \quad (9)$$

Here $\mu_j(\lambda) = \sqrt{\epsilon_j k^2 - \lambda^2}$. $\mathrm{d}M_n^{(j)}(\nu, \omega)$ are the orthogonal random measures satisfying the relation

$$\mathrm{d}M_n^{(j)}(\nu, \mathrm{T}^x\omega) = \mathrm{e}^{\mathrm{i}(2n\lambda_0+\nu)x}\mathrm{d}M_n^{(j)}(\nu, \omega) \quad (10)$$

**Approximation Solutions for Random Measures** The random measures $\mathrm{d}M_n^{(j)}(\nu,\omega)$ can be determined by inserting the stochastic wave fields given in Eq.(7) into the approximate boundary conditions of Eqs.(5) and (6), and in view of the relation (10), they need to be solved only at $x = 0$. In principle, however, for solving $\mathrm{d}M_n^{(j)}(\nu,\omega)$ we have to calculate numerically the inverse of very large matrixes (in the order as large as the number of the Gaussian random numbers used for generating the profile of the random grating, and it is usually over one thousand), and it is very difficult to do with the present computers. Fortunately, we find that if we deal with only the rough grating with a very narrow-band spectrum, we can make use of the approximation $\mu_j(m\lambda_0+\nu) \sim \mu_j(m\lambda_0)$ that permits us to calculate only for $M_n^{(j)} \equiv \int \mathrm{d}M_n^{(j)}(\nu,\omega)$. The final expressions for $\mathbf{M}_n = \left[M_n^{(1)}, M_n^{(2)}\right]^T$ are as follows:

$$\mathbf{M}_0 = \left[\mathbf{\Delta}_0(\lambda_0) - \mathbf{P}(-\lambda_0)\left[\mathbf{\Delta}_{-1}(\lambda_0)\right]^{-1}\mathbf{Q}(\lambda_0)\right]^{-1}\left[\mathbf{E}_0 - \mathbf{P}(-\lambda_0)\left[\mathbf{\Delta}_{-1}(\lambda_0)\right]^{-1}\mathbf{E}_{-1}\right] \tag{11}$$

$$\mathbf{M}_{-1} = \left[\mathbf{\Delta}_{-1}(\lambda_0) - \mathbf{Q}(\lambda_0)\left[\mathbf{\Delta}_0(\lambda_0)\right]^{-1}\mathbf{P}(-\lambda)\right]^{-1}\left[\mathbf{E}_{-1} - \mathbf{Q}(\lambda_0)\left[\mathbf{\Delta}_0(\lambda_0)\right]^{-1}\mathbf{E}_0\right] \tag{12}$$

$$\mathbf{M}_{+1} = \left[\mathbf{\Delta}(3\lambda_0) - \mathbf{Q}(5\lambda)\left[\mathbf{\Delta}(5\lambda)\right]^{-1}\mathbf{P}(3\lambda)\right]^{-1}\left[\mathbf{E}_{+1} - \mathbf{P}(\lambda_0)\mathbf{M}_0\right] \tag{13}$$

$$\mathbf{M}_{m+1} = -\left[\mathbf{\Delta}((2m+3)\lambda_0)\right]^{-1}\mathbf{P}((2m+1)\lambda_0)\mathbf{M}_m, \quad m = +1, +2, +3, \cdots \tag{14}$$

$$\mathbf{M}_{m-1} = -\left[\mathbf{\Delta}((2m-1)\lambda_0)\right]^{-1}\mathbf{Q}((2m+1)\lambda_0)\mathbf{M}_m, \quad m = -1, -2, -3, \cdots \tag{15}$$

where the symbols are defined as

$$\mathbf{\Delta}_0(\lambda) = \mathbf{\Delta}(\lambda) + \mathbf{Q}(3\lambda)\left[\mathbf{\Delta}(3\lambda)\right]^{-1}\mathbf{P}(\lambda), \quad \mathbf{\Delta}_{-1}(\lambda) = \mathbf{\Delta}(\lambda) + \mathbf{P}(-3\lambda)\left[\mathbf{\Delta}(3\lambda)\right]^{-1}\mathbf{Q}(-\lambda) \tag{16}$$

$$\mathbf{E}_0 = \begin{bmatrix} -1 \\ \dfrac{\mathrm{i}\mu_1(\lambda_0)}{k\epsilon_1} \end{bmatrix}, \quad \mathbf{E}_{\pm 1} = \begin{bmatrix} \mathrm{i}\mu_1(\lambda_0)f_1 \\ \eta_1^{\pm}(\lambda_0) \end{bmatrix}, \quad \mathbf{\Delta}(\lambda) = \begin{bmatrix} 1 & -1 \\ \dfrac{\mathrm{i}\mu_1(\lambda)}{k\epsilon_1} & \dfrac{\mathrm{i}\mu_2(\lambda)}{k\epsilon_2} \end{bmatrix} \tag{17}$$

$$\mathbf{P}(\lambda) = \begin{bmatrix} \mathrm{i}\mu_1(\lambda)f_1 & \mathrm{i}\mu_2(\lambda)f_1 \\ -\eta_1^{+}(\lambda) & \eta_2^{+}(\lambda) \end{bmatrix}, \quad \mathbf{Q}(\lambda) = \begin{bmatrix} \mathrm{i}\mu_1(\lambda)\overline{f_1} & \mathrm{i}\mu_2(\lambda)\overline{f_1} \\ -\eta_1^{-}(\lambda) & \eta_2^{-}(\lambda) \end{bmatrix} \tag{18}$$

$$\eta_j^{+}(\lambda) = \frac{1}{k\epsilon_j}\left[\left(\mu_j^2(\lambda) - 2\lambda_0\lambda\right)f_1 + \mathrm{i}\lambda\frac{\mathrm{d}f_1}{\mathrm{d}x}\right], \quad \eta_j^{-}(\lambda) = \frac{1}{k\epsilon_j}\left[\left(\mu_j^2(\lambda) + 2\lambda_0\lambda\right)\overline{f_1} + \mathrm{i}\lambda\frac{\mathrm{d}\overline{f_1}}{\mathrm{d}x}\right] \tag{19}$$

**Results and Discussion** Based on the above approximate solutions, we have carried out the numerical calculations for random wave fields on the surface of a rough grating for the Bragg incidence. Fig.1 and Fig.2 show the modulus of the random wave fields and the corresponding grating surfaces for the normalized roughness $k\sigma = 0.05$, and the different values of the normalized correlation length $k\ell = 100$ and $k\ell = 200$ respectively, while the other parameters are same as those used in [5]. The wave localization can be clearly observed from the spatial distributions of the random wave fields. It can also be seen that the locations of the localized modes correspond quite directly to the regions of the surface where the amplitudes of the rough grating are very small compared with those in the neighborhoods. Mathematically, this is because the amplitude of the random wave field on a point of the surface is inversely proportional to the square of the grating amplitude at the same point for the Bragg incidence. And Physically, it may be explained as a result of a cavity-like structure formed by a surface region with very small amplitudes and its neighborhoods with large amplitudes, because the surface plasmon waves are easily excited and propagating on the portions of the surface with very small amplitudes but are cutoff on the other portions of the surface with large amplitudes.

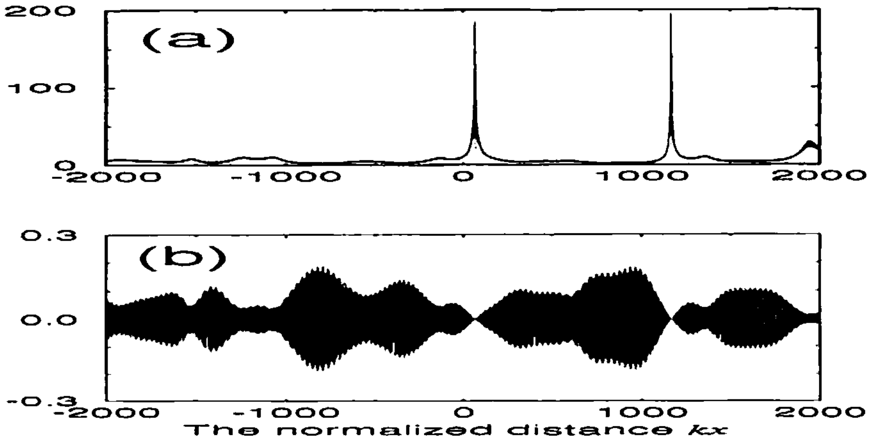


Figure 1: Modulus of (a) the random wave field and (b) the corresponding grating surface as $k\sigma = 0.05$ and $k\ell = 100$.

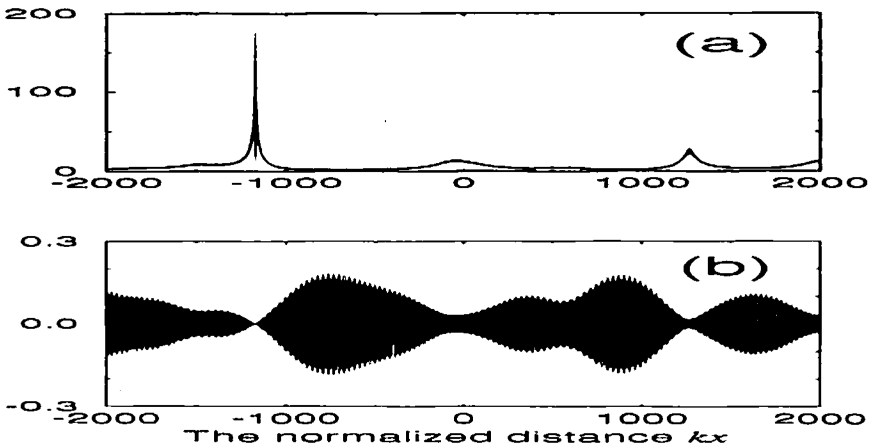


Figure 2: Same as Fig.1 but for $k\ell = 200$.

## References

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