THEORETICAL PERFORMANCE OF BIT ERROR RATE IN TDMA/TDD TRANSMITTER DIVERSITY UNDER COCHANNEL INTERFERENCE

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1. Introduction

In mobile wireless communications, it is required to overcome the degradation of the transmission bit error rate (BER) performance due to Rayleigh fading. To cope with the degradation of the transmission quality, various space diversity methods [1] have been proposed and investigated. When the diversity methods are applied to small mobile stations (MS), the device configuration becomes larger because the plural modules from the antenna to the detection circuit are required. Therefore the diversity methods increase the cost and size of the MS. TDMA/TDD transmitter diversity method [2],[3] has been proposed to resolve these problems.

On the other hand, the reuse of the same frequency at spatially separated cells is a key technology to satisfy the increasing demand for the mobile communications. Moreover, if we adopt dynamic channel allocation (DCA) [4] to increase system capacity, the cochannel interference (CCI) is very severe. Under the CCI at the base station (BS), the transmitter diversity is not necessarily effective because the received signal power used as a criterion of the selection of downlink branch is not relative to the desired signal power. As for this problem, we could not find any method for deriving theoretical BER under the CCI at the BS.

This paper proposes the theoretical derivation of bit error performance in transmitter diversity under the CCI at the BS. It is confirmed from the coincidence of theoretical results with simulation ones that the proposed theoretical approach is applicable to a variety of system parameters, such as average SIR at the BS, normalized Doppler frequency, and so on.

2. Derivation of Average Bit Error Rate

2.1 TDMA/TDD Transmitter diversity under cochannel interference

Fig. 1 shows the concept of the TDMA/TDD transmitter diversity scheme. The uplink channel (MS (#k) to BS) and the downlink channel (BS to MS(#k)) are switched at the cycle of T_D and several users(M) are contained in each channel. Therefore, T_D is given by $T_D = MK/f_s$ where K is the number of the symbol in a frame and f_s is the transmission rate of a TDMA carrier. In TDD system, there is the reversible propagation characteristic between uplink channel and downlink channel, because a single carrier frequency is used to provide two way communication. Therefore the transmitter diversity is very effective

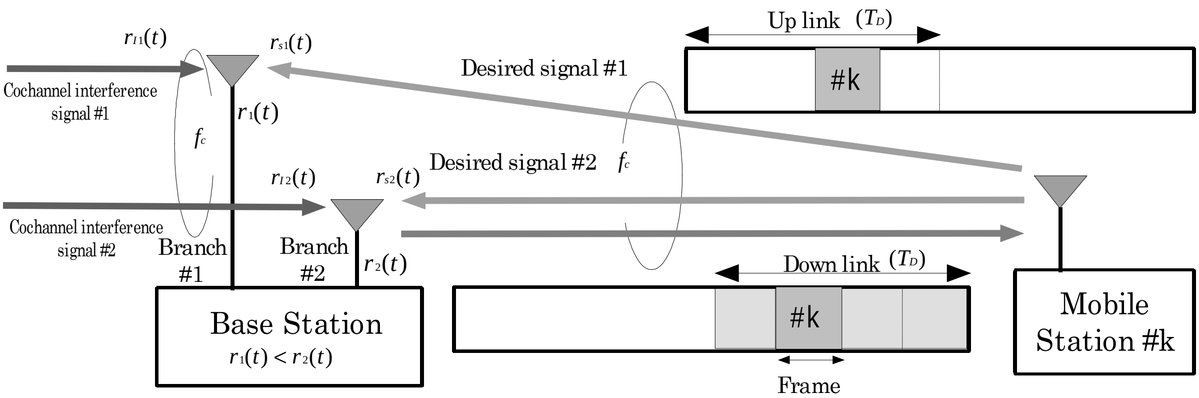

Fig. 1 Concept of the TDMA/TDD transmitter diversity scheme.

in TDD systems. However, under the CCI at the base station, the transmission quality at the downlink channel degrades as the average SIR at the base station is low. In such a case, the selection of branch which gives the largest uplink signal power sometimes fails because the uplink signal at the BS contains not only the desired signal but also the CCI signal.

2.2 Derivation of Theoretical equation for Average Bit Error Rate

A theoretical equation of the TDMA /TDD transmitter diversity under the CCI at the BS is derived for giving the average bit error rate(BER) of the downlink. Fig. 2 shows an example of the constellation diagram of the uplink signal of the *m*-th branch $r_m(t)$. Then $r_m(t)$ is given by

$$r_m(t) = r_{Sm}(t) + r_{\mathrm{Im}}(t) = R_{Sm}(t)e^{j\theta_{Sm}(t)} + R_{\mathrm{Im}}(t)e^{j\theta_{\mathrm{Im}}(t)} \tag{1}$$

where $R_{Sm}(t)$ is the envelope of the desired complex baseband signal $r_{Sm}(t)$, $R_{\mathrm{Im}}(t)$ is the envelope of the CCI complex baseband signal $r_{\mathrm{Im}}(t)$, $e^{j\theta_{Sm}(t)}$ is the phase of the desired complex baseband signal, $e^{j\theta_{\mathrm{Im}}(t)}$ is the phase of the CCI complex baseband signal. Therefore, the instantaneous uplink signal power of the *m*-th branch is given by

$$\begin{aligned}\left|r_m(t)\right|^2 &= \left(R_{Sm}(t)\cos\theta_{Sm}(t) + R_{\mathrm{Im}}(t)\cos\theta_{\mathrm{Im}}(t)\right)^2 + \left(R_{Sm}(t)\sin\theta_{Sm}(t) + R_{\mathrm{Im}}(t)\sin\theta_{\mathrm{Im}}(t)\right)^2 \\ &= R_{Sm}(t)^2 + 2R_{Sm}(t)R_{\mathrm{Im}}(t)\cos\varphi_m(t) + R_{\mathrm{Im}}(t)^2\end{aligned} \tag{2}$$

where $\varphi_m(t) = \theta_{\mathrm{Im}}(t) - \theta_{sm}(t)$.

The average uplink signal power of the *m*-th branch at each frame $u_m(2nT_D)$, that is used as a criterion of the selection of downlink branch, is obtained by the averaging the instantaneous uplink signal power in a period T_p much shorter than fade duration. The average uplink signal power $u_m(2nT_D)$ is given by

$$u_m(2nT_D) = \frac{1}{T_p}\int_{2nT_D}^{2nT_D+T_p}\left|r_m(t)\right|^2 dt \approx R_{sm}(2nT_D)^2 + R_{\mathrm{Im}}(2nT_D)^2 \tag{3}$$

where noncorrelation exists between $\theta_{Sm}(t)$ and $\theta_{\mathrm{Im}}(t)$ $(\langle\cos\varphi_m(t)\rangle = 0)$ in $T_{p.}$

In order to derive the BER of the downlink, we need the definition for the amplitude of the downlink signal in the transmission diversity. Let *t*=0 be the time at which the uplink signal is received and $z(\tau)$ be the amplitude of the downlink at $t=\tau$. Therefore, the BER of the downlink signal $\overline{P_e(\tau)}$ can be obtained by averaging the probability of error for an arbitrary modulation at a specific amplitude $P_e[z(\tau)]$ over the probability density function (p.d.f) of the downlink signal amplitude $p[z(\tau)]$. The BER of the downlink signal is given by

$$\overline{P_e(\tau)} = \int_0^\infty p[z(\tau)]P_e[z(\tau)]dz(\tau). \tag{4}$$

Let $u_m(0)$ is the average uplink signal power at *t*=0. Then the p.d.f of the downlink signal amplitude $p[z(\tau)]$ is given by

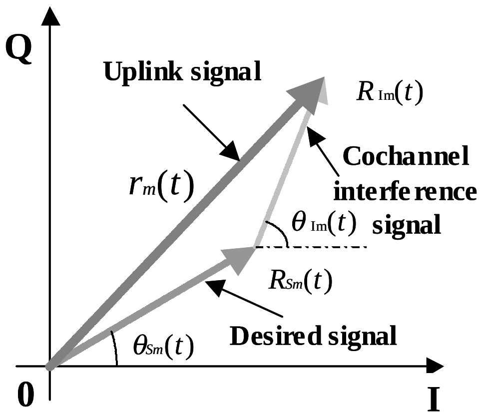

Fig. 2 An example of the constellation diagram of $r_m(t)$.

$$\begin{aligned}p[z(\tau)] &= p[u_1(0) > u_2(0), \cdots, u_1(0) > u_L(0), z(\tau) = R_{S1}(\tau)] \\ &+ \cdots + p[u_L(0) > u_1(0), \cdots, u_L(0) > u_{L-1}(0), z(\tau) = R_{SL}(\tau)]\end{aligned}. \tag{5}$$

The distributions of the uplink signal powers are assumed to be identical. Therefore, *L* probability terms all have the same value. It will therefore suffice only to evaluate *L* times the first term

$$\begin{aligned}p[z(\tau)] &= Lp[u_1(0) > u_2(0), \cdots, u_1(0) > u_L(0), z(\tau) = R_{S1}(\tau)] \\ &= L\int_0^\infty p[z(\tau) = R_{S1}(\tau), R_{S1}(0)]dR_{S1}(0)\int_0^\infty p[R_{I1}(0)]dR_{I1}(0) \\ &\int_0^{u_1(0)}\cdots\int_0^{u_1(0)} p[u_2(0)]\cdots p[u_L(0)]du_2(0)\cdots du_L(0)\end{aligned}. \tag{6}$$

Here, both the desired signal and the CCI signal are influenced by the Rayleigh fading independently. Therefore, $p[R_{Sm}(0)]$ and $p[R_{\mathrm{Im}}(0)]$ are given by

$$p[R_{Sm}(0)] = \frac{2R_{Sm}(0)}{\sigma(0)^2}\exp\left(-\frac{R_{Sm}^2(0)}{\sigma(0)^2}\right) \quad p[R_{\mathrm{Im}}(0)] = \frac{2\Lambda R_{\mathrm{Im}}(0)}{\sigma(0)^2}\exp\left(-\frac{\Lambda R_{\mathrm{Im}}^2(0)}{\sigma(0)^2}\right) \tag{7}$$

where Λ is the average SIR at the BS and $\sigma(0)^2$ is the average uplink desired signal power at t=0.

Next, $p[z(\tau) = R_{S1}(\tau), R_{S1}(0)]$ is obtained by a time dependent Rician density function that tends to be original Rayleigh density function as τ tends to infinity. Therefore, $p[z(\tau) = R_{S1}(\tau), R_{S1}(0)]$ is given by

$$p[z(\tau) = R_{S1}(\tau), R_{S1}(0)] = \left\{\frac{2z(\tau)}{\sigma^2(\tau)}\exp\left(-\frac{z^2(\tau)+k^2R_{S1}^2(0)}{\sigma^2(\tau)}\right)I_0\left(\frac{2kz(\tau)R_{S1}^2(0)}{\sigma^2(\tau)}\right)\right\}$$
$$\frac{2R_{S1}(0)}{\sigma(0)^2}\exp\left(-\frac{R_{S1}^2(0)}{\sigma(0)^2}\right) \tag{8}$$

where I_0 is the modified Bessel function of the first kind and zero order, $\sigma^2(\tau) = \sigma^2(0)(1-k^2)$, $k = J_0(2\pi f_D\tau)$, J_0 is the Bessel function of the first kind of zero order, and f_D is the maximum Doppler frequency. Moreover, $p[u_m(0)]$ is given by

$$p[u_m(0)] = P[u_m(0) = R_{sm}(0)^2 + R_{\mathrm{Im}}(0)^2]$$
$$= \frac{\Lambda}{\sigma(0)^2(1-\Lambda)}\left\{\exp\left(-\frac{\Lambda u_m(0)}{\sigma(0)^2}\right) - \exp\left(-\frac{u_m(0)}{\sigma(0)^2}\right)\right\}. \tag{9}$$

Therefore, Eq.(6) is therefore rewritten by using Eq.(7)~(9) as follows:

$$p[z(\tau)] = \frac{2Lz(\tau)}{\sigma^2}\sum_{l=0}^{L-1}\binom{L-1}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{(-1)^m\Lambda^{l-m+1}}{(1-\Lambda)^l\{\Lambda+(\Lambda-1)m+l\}\left[(1-k^2)\{(\Lambda-1)m+l+1\}+k^2\right]}$$
$$\exp\left(-\frac{(\Lambda-1)m+l+1}{\sigma(0)^2\left[(1-k^2)\{(\Lambda-1)m+l+1\}+k^2\right]}z(\tau)^2\right) \tag{10}$$

The BER of the downlink can be calculated by using $P_e[z(\tau)]$, Eq.(4), and Eq.(10). In the case that $\pi/4-$QPSK differential detection is adopted as the modem, $P_e[z(\tau)]$ is given by

$$P_e[z(\tau)] = \frac{1}{2}erfc\left\{2\sqrt{\frac{1}{2}\frac{z(\tau)^2}{2N}}\sin\left(\frac{\pi}{8}\right)\right\} \tag{11}$$

where N is the power spectral density of the noise.

Therefore, the BER of the transmission diversity $\overline{P_e(\tau)}$ is given by

$$\overline{P_e(\tau)} = \frac{L}{2}\sum_{l=0}^{L-1}\binom{L-1}{l}\sum_{m=0}^{l}\binom{l}{m}\frac{(-1)^m\Lambda^{l-m+1}}{(1-\Lambda)^l\{\Lambda+(\Lambda-1)m+l\}\{(\Lambda-1)m+l+1\}}$$
$$\left(1-\sin\left(\frac{\pi}{8}\right)\Big/\sqrt{\frac{(\Lambda-1)m+l+1}{2\Gamma\left[(1-k^2)\{(\Lambda-1)m+l+1\}+k^2\right]}+\sin^2\left(\frac{\pi}{8}\right)}\right) \tag{12}$$

where Γ is the average CNR ($\Gamma = \sigma(0)^2/2N$) at the MS.

The BER of the downlink frame $\overline{P_e}$ is obtained by averaging the $\overline{P_e(\tau)}$ in a frame period. If t=0 is the time at which the uplink signal is received at the BS, the BER of the downlink frame $\overline{P_e}$ is given by

$$\overline{P_e} = \frac{M}{T_D}\int_{T_D}^{T_D(1+1/M)}\overline{P_e(\tau)}d\tau. \tag{13}$$

3. Evaluation of Average Bit Error Rate

The theoretical equation is evaluated by comparing theoretical results with simulation ones. The number of traffic channels contained in a TDMA carrier (*M*) is set to 4.

3.1 BER performance versus E_b/N_0

Fig. 3 shows the BER performance versus E_b/N_0 with a parameter of average SIR, where f_sT_D=4.8×10^2, f_DT_D=2.5×10^{-2}, and *L*=2, 4. From Fig. 3, it is confirmed from the coincidence of theoretical results with simulation ones that the proposed theoretical equation is applicable to the parameters of E_b/N_0 and the average SIR. Regardless of L, the BER performance is degraded significantly as average SIR decreases. This means that selection of branch fails in the case that the influence of the CCI signal on the received signal at the BS becomes significant.

3.2 BER performance versus f_D

Fig. 4 shows the BER performance versus f_D with a parameter of average SIR, where E_b/N_0=15dB, f_sT_D=4.8×10^2, and *L*=2, 4. From Fig. 4, it is also confirmed from the coincidence of theoretical results with simulation ones that the proposed theoretical equation is applicable to the parameters of f_DT_D and the average SIR. Regardless of *L*, the BER performance is degraded significantly as average SIR decreases. This reason is the same as we discussed in the previous section. As f_DT_D increases, the BER performance is degraded significantly. The reason is that the low correlation between the uplink and downlink.

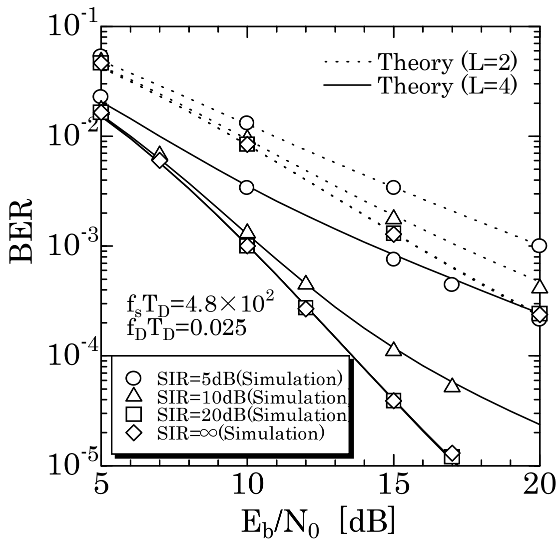

Fig. 3 BER performance versus E_b/N_0 with a parameter of average SIR, where f_sT_D=4.8 × 10^2, f_DT_D=2.5 × 10^{-2} and *L*=2, 4.

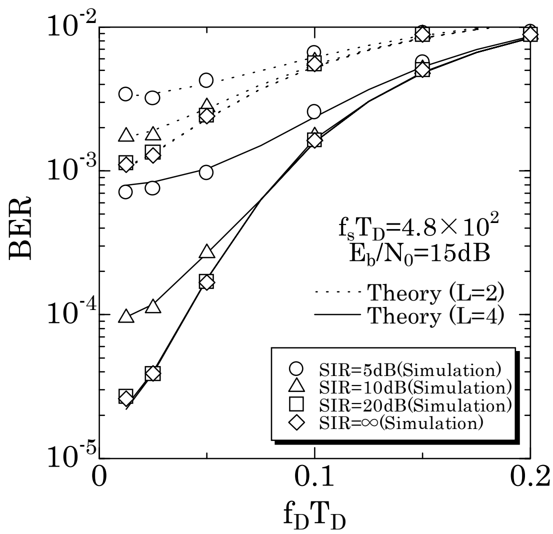

Fig. 4 BER performance versus f_D with a parameter of average SIR, where E_b/N_0=15dB, f_sT_D=4.8 × 10^2 and *L*=2, 4.

4. Conclusion

We proposed the theoretical derivation method of the BER performance in transmitter diversity under the CCI at the base station (BS). It was confirmed from the coincidence of theoretical results with simulation ones that the proposed theoretical approach is applicable to a variety of system parameters.

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