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DISCRETE MULTIPOLE IMAGE THEORY OF SOMMERFELD HALF SPACE PROBLEM

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1. ABSTRACT

It was recognized from the outset by Sommerfeld that his solution to the half space problem could be interpreted as a bundle of plane waves reflected and refracted from the medium interface at various angles of incidence. This point of view was developed later by Weyl. It seems that Weyl's solution to the half space problem has not been investigated by virtue of Weyl integral representations of multipole fields in unbounded space. This paper points out that when the field points and the dipole are in the same half space, Weyl solution can be viewed as an angular spectrum representation of plane waves in unbounded space; and subsequently Weyl solution can be represented by the discrete multipoles at the mirror image. Although the Weyl path is infinite and complex, the numerical approximation of the expansion coefficient of discrete multipole images can be done in a unite spherical surface according to the principle of analytical continuation.

2. INTRODUCTION

Many numerical,asmpototic,and analytical techniques have been published to deal with the integrals arising from the sommerfeld half space problem[1-3]. The exact image theory introduced in 1983[2] is conceptually and computationally simple. The discrete complex image technique(DCIT)[3] has also been widely used[4-6] since its inception in 1988[7]. The main point of DCIT is to apply the Sommerfeld identity for a dipole in conjunction with complex-exponential-function approximation of the spectral function. Therefore,DCIT uses dipole images only.

The angular spectrum representation of plane waves for the half space problem was given by Weyl as early as in 1919[8]. And the relationship between the angular spectrum amplitude of plane waves and the Fourier transform of the source distribution was identified twenty years ago[9]. However, to the best of our knowledge, Weyl integral representations of multipole fields in unbounded space have never been explored to Weyl solution to the Sommerfeld half space problem. The objective of this paper is to show that Weyl solution ,if the field points and the source point are in the same half space, can be represented by multipoles at the mirror image of the dipole source. The amplitude of the discrete multipoles are determined by the angular spectrum amplitude on a unit spherical surface according to the principle of analytical continuation[9-14]. Also, it is noted that DCIT is of higher accuracy for smaller distance between the field point and the source point. So it is also possible to improve the result by extracting the discrete complex dipole images before the discrete multipole image theory is employed. Throughout this paper, time factor is $\exp(-i\omega t)$.

3. THEORY

As shown in Fig.1, an infititesimal three dimensional dipole is located at height z_0 over a planar interface at $z = 0$ separating two half spaces. The medium of the upper half space ($z \geq 0$) is the free space with permittivity ε_0, magnetic permeability μ_0 ,and wavenumber $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ .The medium of the lower half space ($z \leq 0$) is the lossy ground with the following parameters: permittivity $n^2\varepsilon_0$, magnetic permeability μ_0 ,and wavenumber $k_1 = nk_0$,where $n^2 = \varepsilon_r - \sigma/(\omega\varepsilon_0)$,$\varepsilon_r$ is the relative dielectric constant. and σ is the conductivity of the medium. In the following we will consider the Vertical Electric Dipole (VED) excitation to illustrate the method. The procedures for a horizontal magnetic dipole.horizontal electric dipole, and a vertical magnetic dipole are similar and omitted due to the space restriction. As shown in [8],the primary radiation fields of the VED located in $z = z_0$ can be derived by the following Hertz potential in the z direction

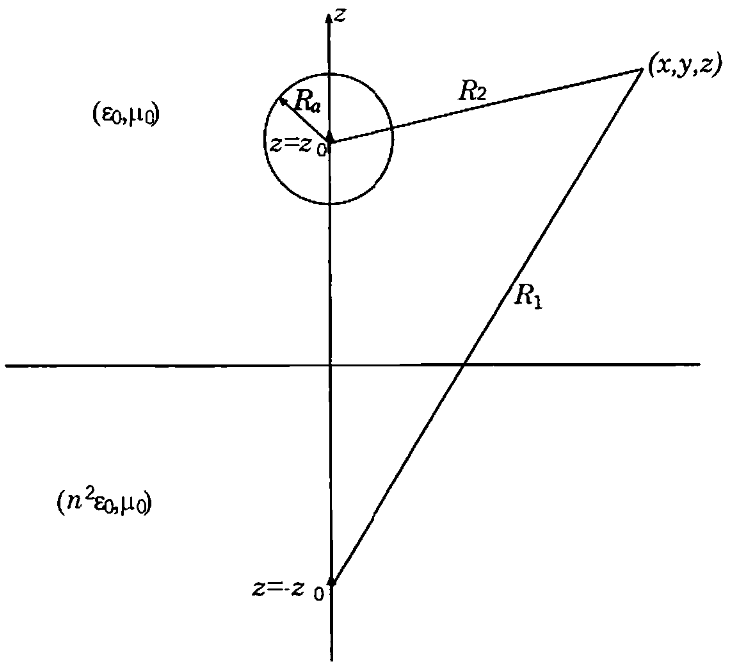

Figure 1: Discrete multipoles at the mirror image of dipole source

$$\Pi_0 = \frac{\exp[ik_0R_2]}{R_2} \tag{1}$$

$$R_2 = \sqrt{x^2 + y^2 + (z - z_0)^2} \tag{2}$$

The Hertz potential Π_r of the reflected field in the upper half space is

$$\begin{aligned}
\Pi_r &= \frac{ik_0}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} \sin\alpha d\alpha d\beta \\
&\quad f_r(\alpha)\exp[ik_0(x\sin\alpha\cos\beta + y\sin\alpha\sin\beta + (z + z_0)\cos\alpha)] && (3)\\
&= \frac{ik_0}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}-i\infty} \sin\alpha d\alpha d\beta f_r(\alpha)\exp[ik_0 s\cdot R_1] && (4)\\
s &= \sin\alpha\cos\beta\widehat{x} + sin\alpha\sin\beta\widehat{y} + \cos\alpha\widehat{z} && (5)\\
R_1 &= x\widehat{x} + y\widehat{y} + (z + z_0)\widehat{z} && (6)\\
f_r(\alpha) &= \frac{n^2\cos\alpha - \sqrt{n^2 - 1 + \cos^2\alpha}}{n^2\cos\alpha + \sqrt{n^2 - 1 + \cos^2\alpha}} && (7)
\end{aligned}$$

where $\widehat{x}$, $\widehat{y}$, and $\widehat{z}$ are unit vectors in the rectangular coordinate system. $f_r(\alpha)$ is the Fresnel reflection coefficient for a plane wave and α is the angle of incidence at which an elementary plane wave meets the medium interface $z = 0$. Most importantly, $f_r(\alpha)$ is the angular spectrum amplitude of the angular spectrum representation of the reflection field. Following the theory developed in [9], we next expand the spectral amplitude $f_r(\alpha)$ into a series of Legendre polynomial $P_l(\cos\alpha)$,viz.,

$$f_r(\alpha) = \sum_{l=0}^{\infty} a_l P_l(\cos\alpha)\sqrt{\frac{2l+1}{2}} \tag{8}$$

where the expansion coefficients are obtained by means of the orthogonality of Legendre polynomial $P_l(\cos\alpha)$:

$$a_l = \sqrt{\frac{2}{2l+1}}\int_0^{\pi} f_r(\alpha)P_l(\cos\alpha)\sin\alpha d\alpha \tag{9}$$

Substituting the expansion (8) into the representation(4) of Π_r and interchange the order of integration and summation. We then obtain the following series expansion for Π_r

$$\Pi_r(R_1) = k_0 \sum_{l=0}^{\infty} a_l \Pi_{rl}(R_1) \tag{10}$$

where

$$\Pi_{rl}(R_1) = \sqrt{\frac{2l+1}{2}}\frac{i}{2\pi}\int_0^{2\pi}\int_0^{\pi/2-i\infty} d\beta d\alpha \sin\alpha P_l(\cos\alpha)\exp[ik_0 s\cdot R_1] \tag{11}$$

It is well known that the expression on the right hand side of (11) is exactly the angular spectrum representation of scalar multipole field of degree l[9],i.e.,

$$\Pi_{rl}(R_1) = \sqrt{\frac{2l+1}{2}} h_l^{(1)}(k_0 R_1)P_l(\cos\theta_1) \tag{12}$$

where $h_l^{(1)}$ is the spherical Hankel function of the first kind of order l. (R_1, θ_1, ϕ_1) are the spherical coordinates of the field point at the spherical coordinate system centered at the mirror image $z = -z_0$ as shown in Fig.1, and

$$\cos\theta_1 = \frac{z+z_0}{R_1} \tag{13}$$

As emphasized in [9], although the the multipole moment, defined by (9),depend explicitly only on those spectral amplitude $f_r(\alpha)$ which are associated with real s,the expansion (10) is valid for all unit vectors associated with complex contour $O \to \pi/2 - i\infty$; this result is a consequence of the fact that the real angular spectrum is the boundary value of an entire functions[9]. Namely, although the angular spectrum representation of reflection waves which include both homogeneous plane waves (corresponding to real s) and inhomogeneous/evanescent plane waves(corresponding to complex s), we can use the spectral amplitude of homogeneous plane waves only to determine the multipole moment a_l and the approximation to the spectral amplitude of inhomogeneous plane waves is guaranteed, theoretically, by the analytic properties of the spherical harmonics. Actually, since the multipole field includes the contribution of inhomogeneous plane waves, it is should be understandable that the spectral amplitude of inhomogeneous is also approximated in some sense. This is the main advantage of present method (and any numerical methods based on the principle of continuation[10,11]). We have benifited a great deal from such a real spectrum approximation of the complex spectral representation[12-14]. On the other hand,this is major disadvantage of our method: it is just the enough smoothness of spherical harmonics that may deteriorate the numerical stability. Numerically, the accuracy depends on the accuracy of the expansion (8). There are two means to diminish the numerical error. One is lessen the numerical error of the a_l. This can be done by using the higher accuracy quadrature for this kind of functions[13]. Another is to reduce the range of analytical continuation[11]. To this end, we need a better way to approximate the spectral amplitude corresponding the far terminal of complex contour $O \to \pi/2 - i\infty$. As shown by Stratton[8],

$$k_\rho = k_0 \sin\alpha, k_z = k_0 \cos\alpha = i\sqrt{k_\rho^2 - k_0^2} \tag{14}$$

where k_ρ and k_z are the wave numbers in a circular cylinder coordinate system[3]. So when $\alpha = 0, k_\rho = 0$, and $\alpha = \pi/2 - i\infty$,$k_\rho = \infty$. Returning to the k_ρ complex plane, this means that we need to extract the quasi-dynamic image first. And next ,note that the numerical results based on complex image methods is only accurate in the region of $| r - z_0\hat{z} | \leq R_a$ (see Fig.1) as shown in [3.7]. Furthermore,according to a lot of numerical results of DCIT we notice that the accuracy is better if the R_a become smaller. This means that DCIT gives a good approximation to larger k_ρ. Therefore, the accuracy can be further improved if we extract the discrete complex images before this paper's method is employed. So we have

$$f_r(\alpha) = [f_r(\alpha) - \sum_{i=1}^{N} c_i \exp(b_i\sqrt{1-\cos^2\alpha})] + \sum_{i=1}^{N} c_i \exp[b_i\sqrt{1-\cos^2\alpha}] \tag{15}$$

$$\begin{aligned} \Pi_r(R_1) &= k_0 \sum_{l=0}^{\infty} a_l' \sqrt{\frac{2l+1}{2}} h_l^{(1)}(k_0 R_1) P_l(\cos\theta_1) \\ &+ \sum_{i=1}^{N} c_i \frac{\exp[ik_0\sqrt{x^2+y^2+(z+z_0-b_i/k_0)^2}]}{\sqrt{x^2+y^2+(z+z_0-b_i/k_0)^2}} \end{aligned} \tag{16}$$

$$a_l' = \sqrt{\frac{2}{2l+1}} \int_0^{\pi} [f_r(\alpha) - \sum_{i=1}^{N} c_i \exp(b_i\sqrt{1-\cos^2\alpha})] P_l(\cos\alpha) \sin\alpha d\alpha \tag{17}$$

4. CONCLUSION

Due to the fundamental importance of Sommerfeld half space problem, any new idea on it should be welcome. A bright idea of utilizing the Weyl integral representation of multipole field to evaluate the Sommerfeld integral is reported. We will present the numerical application of the method proposed here in a separate paper.

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