HIGH-FREQUENCY DIFFRACTION BY A COMPOSED WEDGE

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INTRODUCTION

Diffraction of waves by a wedge is an important canonical problem in electromagnetic theory that serves as a building block when modeling electromagnetic scattering from a variety of large and complex objects of practical interest. This paper presents high frequency analysis of a novel closed form solution which has been obtained recently [*Osipov*, 1995] to describe scattering of a time harmonic H- polarized plane electromagnetic wave $\mathbf{H}^i = \hat{z} H_0 \exp(-ikr\cos(\varphi - \varphi_0))$ incident normally to a straight edge of a composed wedge-like configuration of external angle 2Φ, where r, φ, z are the cylindrical coordinates (Fig.1). The configuration may be either a perfectly conducting wedge with dielectric/ferrite coating on its sides or a homogeneous absorbing wedge whose conductivity is large enough so that the waves transmitted directly through the wedge can be ignored. In both cases it is possible to simulate its faces through higher order boundary conditions (see, for example, [*Hoppe and Rahmat-Samii*, 1995; *Senior and Volakis*, 1995])

$$\mp \left(a_0^\pm + a_2^\pm \frac{\partial^2}{\partial r^2} \right) \frac{1}{r} \frac{\partial}{\partial \varphi} H_z(r, \pm\Phi) + ikb_0^\pm H_z(r, \pm\Phi) = 0 \ , \tag{1}$$

where $a_0^\pm = 1$, $a_2^\pm = -1/(2k_\pm^2)$, $b_0^\pm = \sqrt{(\epsilon\mu_\pm)/(\mu\epsilon_\pm)}$ for a wedge with absorbing faces, and $a_0^\pm = 1$, $a_2^\pm = -\left(1 + (2k_\pm h_\pm)/\sin(2k_\pm h_\pm)\right)/(2k_\pm^2)$, $b_0^\pm = -i\sqrt{(\epsilon\mu_\pm)/(\mu\epsilon_\pm)}\tan(k_\pm h_\pm)$ for a perfectly conducting wedge covered with dielectric/ferrite layers of thicknesses $h_\pm$. Here, the plus and minus signs refer to the upper and lower sides of the configuration, and ϵ, μ, $\epsilon_\pm$, $\mu_\pm$ denote permittivities and permeabilities related to the surrounding medium and wedge faces, respectively.

SOLUTION

The modified Malyuzhinets' technique [*Osipov*, 1994] when applied to the third-order boundary conditions (1) leads to an explicit solution of the problem formulated above in the form of Sommerfeld integral

$$H_z(r, \varphi) = \frac{H_0}{2\pi i} \int_\gamma e^{-ikr\cos\alpha} S(\alpha + \varphi)\, d\alpha \tag{2}$$

where contour γ consists of two loops one of which goes from $\pi/2 + i\infty$ above all singularities of $S(\alpha + \varphi)$ to $-3\pi/2 + i\infty$ and the other is symmetrical to it about the origin $\alpha = 0$. The transform $S(\alpha)$ is given by the formula

$$S(\alpha) = \Psi(\alpha) \left(\frac{\sigma(\alpha, \varphi_0)}{\Psi(\varphi_0)} + C^+ \Lambda^+(\alpha) + C^- \Lambda^-(\alpha) \right) \ ,$$

where $\sigma(\alpha, \varphi_0) = \dfrac{\nu \cos(\nu\varphi_0)}{\sin(\nu\alpha) - \sin(\nu\varphi_0)}$, $\nu = \dfrac{\pi}{2\Phi}$, the auxiliary function $\Psi(\alpha)$ is a product of the special Malyuzhinets' functions $\psi_\Phi(\alpha)$ [*Malyuzhinets*, 1958]

$$\Psi(\alpha) = \frac{\Psi_1^+(\alpha)\Psi_3^+(\alpha)}{\Psi_2^+(\alpha)} \frac{\Psi_1^-(\alpha)\Psi_3^-(\alpha)}{\Psi_2^-(\alpha)} \ ,$$

$$\Psi_n^{\pm}(\alpha) = \psi_\Phi(\alpha \pm \Phi + \frac{\pi}{2} + (-1)^n\theta_n^{\pm})\psi_\Phi(\alpha \pm \Phi - \frac{\pi}{2} - (-1)^n\theta_n^{\pm}) \, , \qquad n = 1, 2, 3,$$

and functions $\Lambda^{\pm}(\alpha)$ in the strip $|\mathrm{Re}\ \alpha| \leq \Phi$ can be expressed as follows

$$\Lambda^{\pm}(\alpha) = \mp\frac{1}{4\Phi i}\int_{-i\infty}^{+i\infty} \tan\left(\frac{\pi}{4\Phi}(\alpha+\beta\pm\Phi)\right)\frac{\sin\beta\, \mathrm{d}\beta}{L_{\pm}(\beta)\Psi(\beta\pm\Phi)} \, .$$

Here, $L_{\pm}(\beta) = \pm\sin\beta\left(1 - k^2 a_2^{\pm}\cos^2\beta\right) + b_0^{\pm}$ are symbols of the boundary condition operators; the Brewster angles $\theta_n^{\pm}$ are defined by $L_{\pm}(\mp\theta_n^{\pm}) = 0$, $n = 1, 2, 3$, and numbered so that $\mathrm{Re}\ \theta_{1,3}^{\pm} \in (0, \pi/2)$, $\mathrm{Re}\ \theta_2^{\pm} \in (-\pi/2, 0)$. Constants $C^{\pm}$ can be found from the relations

$$C^{\pm} = \frac{g_{\pm}\Lambda^{\mp}(\alpha_{\mp}^{f}) - g_{\mp}\Lambda^{\mp}(\alpha_{\pm}^{f})}{\Lambda^{+}(\alpha_{+}^{f})\Lambda^{-}(\alpha_{-}^{f}) - \Lambda^{+}(\alpha_{-}^{f})\Lambda^{-}(\alpha_{+}^{f})} \, ,$$

where $g_{\pm} = -\dfrac{\sigma(\alpha_{\pm}^{f}, \varphi_0)}{\Psi(\varphi_0)}$ and $\alpha_{\pm}^{f}$ identify the forbidden poles of the transform $S(\alpha)$ (see [*Osipov*, 1994])

$$\alpha_{\pm}^{f} = \begin{cases} \pm(\Phi - \theta_2^{\pm}), & \text{if} \quad \mathrm{Im}\ \theta_2^{\pm} > 0 \\ \pm(\Phi + \pi + \theta_2^{\pm}), & \text{if} \quad \mathrm{Im}\ \theta_2^{\pm} < 0 \end{cases} .$$

The solution (2) satisfies a pair of contact conditions $\lim\limits_{r\to 0}\dfrac{1}{r}\dfrac{\partial}{\partial\varphi}H_z(r, \pm\Phi) = 0$ which ensures the uniqueness, reciprocity, and its being an asymptotical one when $|\epsilon_{\pm}/\epsilon| \to \infty$ (absorbing wedge) or $|k_{\pm}h_{\pm}| \to 0$ (coated wedge) [*Osipov*, 1995].

HIGH FREQUENCY ASYMPTOTICS

For $kr \gg 1$, the integral (2) can be evaluated asymptotically by deformation of the contour γ into a pair of steepest descent paths $\Gamma_{\pm} = \{\alpha : \mathrm{Re}\,\alpha = \pm\pi - \mathrm{gd}(\mathrm{Im}\,\alpha)\}$ passing through the saddle points $\alpha = \pm\pi$, respectively. Here, $\mathrm{gd}(x) = -\frac{\pi}{2} + 2\arctan(e^{x})$. This yields a representation

$$H_z(r,\varphi) = H_z^{g}(r,\varphi) + H_z^{d}(r,\varphi) + H_z^{+}(r,\varphi) + H_z^{-}(r,\varphi),$$

in which

$$H_z^{d}(r,\varphi) = \frac{H_0}{2\pi i}\int_{\Gamma_+\cup\Gamma_-} \mathrm{e}^{-ikr\cos\alpha}\, S(\alpha+\varphi)\,\mathrm{d}\alpha \, , \tag{3}$$

whereas the other terms result from residues at poles of the integrand that may by located in the region between the steepest descent contours. The geometrical optics field is given therefore by the formula

$$\begin{aligned} H_z^{g}(r,\varphi) &= \chi(\pi - |\varphi-\varphi_0|)H_0\mathrm{e}^{-ikr\cos(\varphi-\varphi_0)} \\ &+\chi(\pi - |2\Phi-\varphi-\varphi_0|)H_0R_{+}(\Phi-\varphi_0)\mathrm{e}^{-ikr\cos(\varphi+\varphi_0-2\Phi)} \\ &+\chi(\pi - |2\Phi+\varphi+\varphi_0|)H_0R_{-}(\Phi+\varphi_0)\mathrm{e}^{-ikr\cos(\varphi+\varphi_0+2\Phi)} \, , \end{aligned}$$

where $R_{\pm}(\alpha)$ are the planar reflection coefficients and $\chi(x) = 1$ if $x > 0$, 0 if $x < 0$.

The terms $H_z^{\pm}(r,\varphi)$ denote the residues at captured poles of the function $\Psi(\alpha+\varphi)$. The dominant contributions to $H_z^{\pm}(r,\varphi)$ (if any) are due to the residues at poles $\alpha = \pm\Phi - \varphi \pm\pi\pm\theta_1^{\pm}$ that describe the surface waves excited at the edge by the incident field and then traveling outwards along both sides of the wedge (without dissipation if the material parameters of the faces are assumed to be entirely real, that is $\mathrm{Re}\,\theta_1^{\pm} = 0$). The corresponding expressions may be summarized as follows

$$H_z^{\pm}(r,\varphi) = \chi(\pm\varphi - \Phi - \mathrm{Re}\theta_1^{\pm} - \mathrm{gd}(\mathrm{Im}\theta_1^{\pm}))H_0A_{\pm}\mathrm{e}^{ikr\cos(\Phi\mp\varphi+\theta_1^{\pm})} \, ,$$

where

$$A_{\pm} = \mp\frac{2b_0^{\pm}\Psi(\pm\Phi\mp\pi\mp\theta_1^{\pm})}{\cos\theta_1^{\pm}\left(1-k^2a_2^{\pm}+3k^2a_2^{\pm}\sin^2\theta_1^{\pm}\right)}\left(\frac{\sigma(\pm\Phi\pm\pi\pm\theta_1^{\pm},\varphi_0)}{\Psi(\varphi_0)}+\right.$$
$$\left.+\ C^+\Lambda^+(\pm\Phi\mp\pi\mp\theta_1^{\pm})+C^-\Lambda^-(\pm\Phi\mp\pi\mp\theta_1^{\pm})\mp\frac{C^{\pm}\sin\theta_1^{\pm}}{b_0^{\pm}\Psi(\pm\Phi\mp\pi\mp\theta_1^{\pm})}\right),$$

The integrals (3) can be evaluated by virtue of the steepest descent method. Retaining the leading terms of the asymptotic expansion yields

$$H_z^d(r,\varphi)\approx H_0D(\varphi,\varphi_0)\frac{\exp(ikr-i3\pi/4)}{\sqrt{2\pi kr}},$$

where $D(\varphi,\varphi_0)=S(\varphi+\pi)-S(\varphi-\pi)$ is the diffraction coefficient. This term describes the cylindrical wave arising due to diffraction of the incident field by the edge. On account of reciprocity the diffraction coefficient can be expressed as follows

$$D(\varphi,\varphi_0)=\frac{\mu\cos(\mu\varphi)}{\Psi(\varphi)}\sum_{p=1}^{4}\frac{D_p(\varphi_0)}{\sin(\mu\varphi)-\sin(\mu\varphi_p)}\quad, \tag{4}$$

where $\varphi_1=\varphi_0+\pi$, $\varphi_2=\varphi_0-\pi$, $\varphi_3=\Phi+\theta_2^+$, $\varphi_4=-\Phi-\theta_2^-$. The coefficients $D_p(\varphi_0)$ are specified by equating both sides of the equation (4) at the poles $\varphi=\varphi_p$, $p=1,2,3,4$. This gives $D_1(\varphi_0)=-\Psi(\varphi_1)$, $D_2(\varphi_0)=\Psi(\varphi_2)$, $D_3(\varphi_0)=B_+\Psi(\varphi_3)$, $D_4(\varphi_0)=B_-\Psi(\varphi_4)$, where

$$B_{\pm} = \frac{2b_0^{\pm}\Psi(\pm\Phi\mp\pi\mp\theta_2^{\pm})}{\cos\theta_2^{\pm}\left(1-k^2a_2^{\pm}+3k^2a_2^{\pm}\sin^2\theta_2^{\pm}\right)}\left(\frac{\sigma(\pm\Phi\pm\pi\pm\theta_2^{\pm},\varphi_0)}{\Psi(\varphi_0)}+C^+\Lambda^+(\pm\Phi\mp\pi\mp\theta_2^{\pm})+\right.$$
$$\left.+\ C^-\Lambda^-(\pm\Phi\mp\pi\mp\theta_2^{\pm})\pm\frac{C^{\pm}\sin\theta_2^{\pm}}{b_0^{\pm}\Psi(\pm\Phi\mp\pi\mp\theta_2^{\pm})}\right).$$

The asymptotic expressions above have been used to compute various scattering characteristics of the composed wedges. Fig.2 presents an example of such computations, showing the dependence of the surface wave diffraction coefficients $|A_{\pm}|$ on the incidence angle φ_0 for a perfectly conducting wedge of an external angle $2\Phi=5\pi/4$ covered by a dielectric/ferrite layer with parameters $h_{\pm}=0.12\lambda$, $\epsilon_{\pm}/\epsilon=4.55$, $\mu_{\pm}/\mu=3$.

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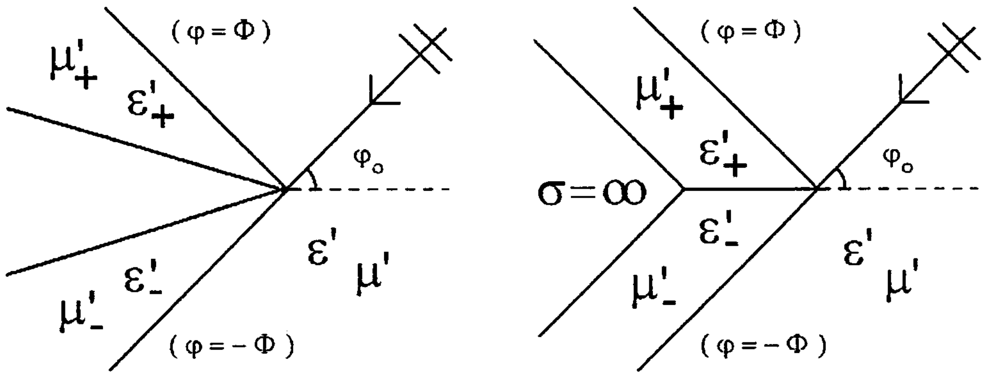

Figure 1 : The geometry

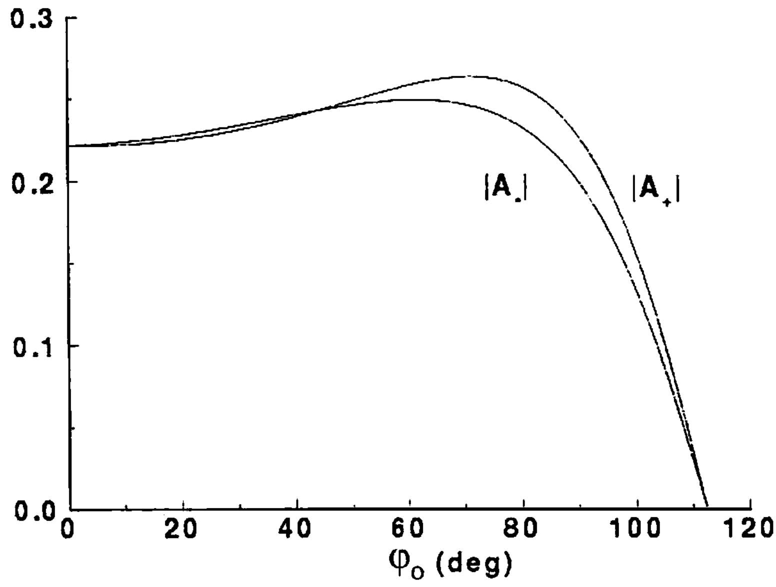

Figure 2 : The coated wedge surface wave diffraction coefficients