UNIFORM ASYMPTOTICS OF THE FAR FIELD IN THE PROBLEM OF DIFFRACTION BY A COATED WEDGE

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1. Introduction

We study the problem of diffraction of a normally incident wave by a wedge coated with thin dielectric layers. The effects of a thin material coating are simulated by the appropriate Generalized Impedance Boundary Conditions (GIBCs of the third order in our case) and Contact Conditions (CCs) at the edge of the wedge. This problem was extensively studied during the last few years [1-8]. This interest is due to importance of such a model in the simulation of scatterers constructed with the aid of thin multylayered coatings. Some of the known results for this problem were incomplete or incorrect, the others were too restrictive for applications. Recently the problem of diffraction by a coated wedge has been solved for almost normal incidence of a plane wave [3,8] by a convergent perturbation method. In particular, the closed form solution for normal incidence has been obtained for new CCs and GIBCs of the third order [7,8]. We exploit this solution for constructing the far field asymptotics which is uniform with respect to the angles of incidence φ_0 and obseravation φ. In this approach we neglect the nonhomogeneous plane waves with the complex phase which are deduced from the exact solution. These waves vanish exponentially for large distances from the edge. The uniform asymptotics is derived from the nonuniform one.

2. The closed form solution

Let the plane wave

$$E_z^i(\rho,\varphi) = D_1 \exp(-i\,k\,r\cos(\varphi-\varphi_0))$$

be incident on the wedge of the opening $2\Phi > \pi$ coated by thin dielectric layers. The thin layers on a perfectly conducting substrate are simulated by the GIBCs (Fig.1)

$$(\alpha_j + \beta_j\partial_{x^2})\left(\frac{\partial E_z}{\partial y}\right) + (\gamma_j + \delta_j\partial_{x^2}^2)\,E_z\Big|_{S_j} = 0 \ , \ j = 1,2,$$

where the constants α, β, γ, δ are defined in [5,7,8]. We add two contact conditions at the edge

$$\left[\frac{\beta}{\delta}\left(\frac{\partial E_z}{\partial y}\right)\right]_{-\Phi}^{\Phi} = 0$$

$$\left[\frac{\beta\delta}{\Delta}\frac{\partial}{\partial x}\left(\frac{\partial E_z}{\partial y}\right) + \frac{|\delta|^2}{\Delta}\frac{\partial E_z}{\partial x}\right]_{-\Phi}^{\Phi} = 0 \ ,$$

where $\Delta_j = \alpha_j\bar{\delta}_j - \beta_j\bar{\gamma}_j$, $j = 1, 2$, $\alpha, \beta, \gamma, \delta$ depend on the parameters of the coating [4]

$$[f.]_{-\Phi}^{\Phi} = \lim_{r\to 0}(f_1(r,+\Phi) - f_2(r,-\Phi)) \ .$$

The wave field E_z (E-polarization) also satisfies the Helmholtz equations, the condition at infinity and Meixner's condition.

The solution is presented in the form of Sommerfeld integral

$$E_z(r,\varphi) = \frac{1}{2\pi i}\int_\gamma g(\alpha+\varphi)\, e^{-ikr\cos\alpha}\, d\alpha \quad ,$$

where $g(\alpha)$ is the solution of the Maliuzinets' equations. The spectral function $g(\alpha)$ can be determined in a closed form

$$g(\alpha) = \Psi_g(\alpha)\,\sigma_{\varphi_0}(\alpha)\,U(\alpha) \quad .$$

$$\Psi_g(\alpha) = \frac{\Psi(\alpha,\chi_1^1))\,\Psi(\alpha,\chi_3^1)\,\Psi(\alpha-2\Phi,\chi_1^2)\,\Psi(\alpha-2\Phi,\chi_3^2)}{\Psi(\alpha,\chi_2^1)\,\Psi(\alpha-2\Phi,\chi_2^2)} \,,$$

$$\Psi(u,v) = \Psi_\Phi(u+\Phi+\pi/2-v)\,\Psi_\Phi(u+\Phi-\pi/2+v)\,,$$

Ψ_Φ is Maliuzhinets' function, $\sigma_{\varphi_0}(\alpha) = \mu\cos\mu\varphi_0/(\sin\mu\alpha - \sin\mu\varphi_0)$, $\mu = \pi/(2\Phi)$. and $U(\alpha)$ is the solution of the functional equations [3,8]

$$U(\alpha-(-1)^j\Phi) - U(-\alpha-(-1)^j\Phi) = h_0^j(\alpha) \quad ; \quad j = 1,2\,,$$

$$h_0^j(\alpha) = \frac{\sin\alpha\left(D_0^j + E_0^j\cos\alpha\right)}{\Psi_g(\alpha-(-1)^j\Phi)\,\sigma(-\alpha-(-1)^j\Phi)\,P_j(\alpha,\chi)} \quad ,$$

where

$$P_j(\alpha,\chi) = (-1)^{j+1}\Big(\sin^3\alpha + (-1)^j\, i\,\delta_j/(k\,\beta_j)\sin^2\alpha + (\alpha_j - \beta_j\,k^2)/(k\,\beta_j)\sin\alpha +$$

$$+(-1)^j\, i\,(\gamma_j - \delta_j\,k^2)/(k^3\,\beta_j)\Big) = \prod_{\nu=1}^{3}\left((-1)^{j+1}\sin\alpha - (-1)^\nu\sin\chi_\nu^j\right) \quad ,$$

and $\sin\chi_\nu^j$ are determined via the roots of the polinomials $P_j(\alpha,\chi)$ of $\sin\alpha$. The expressions for the unknown constants E_0^j, D_0^j can be obtained with the aid of the CCs and condition at infinity. Therefore, the solution $U(\alpha)$ is deduced from the functional equations in a closed form [1,6,7,8]

$$U(\alpha) = \sigma^{(0)}(z,h) - \sigma^{(0)}(\varphi_0,h) - G_g \quad ,$$

$$\sigma^{(0)}(z) = \frac{i}{8\Phi}\int_{iR} d\tau \sum_{j=1}^{2}(-1)^{j+1}\, h_0^j(\tau)\,\sigma_j(\tau,z) \quad ,$$

$$\sigma_j(t,z) = \sin\mu t/(\cos\mu t + (-1)^j\sin\mu z) \;,\; G_g = -D_1/\Psi_g(\varphi_0) \;.$$

The exact solution in the form of Sommerfeld integral is used for deduction of the nonuniform far field asymptotics ($kr \to \infty$). We exploit the nonuniform with respect to φ, φ_0 asymptotics to determine the uniform one.

3. Uniform asymptotics

The incident wave generates a reflected wave from one face, when only one side of the wedge is illuminated ($\Phi < \varphi_0 < \pi - \Phi$). Two reflected waves spread from the wedge's faces when both sides are illuminated ($|\varphi_0| < \pi - \Phi$) by the incident wave. To describe

the transition region near the termination line between light and shadow for the incident (or reflected) wave we should use the expression for the cylindrical waves spreading from the edge and the transition function, that is Fresnel integral

$$F(x) = \frac{1}{\sqrt{i\pi}} \int_{-\infty}^{x} \exp(it^2)\, dt \ .$$

If we neglect the nonuniform plane waves which exponentially vanish at infinity, the uniform expression (with respect to φ_0 and φ) takes the form

$$E_z(r,\varphi) = D_1 \exp\left(-ikr\cos[\varphi-\varphi_0]\right) F\left(\sqrt{2kr}\cos\left(\frac{\varphi-\varphi_0}{2}\right)\right) +$$

$$+ R_1 \exp\left(-ikr\cos[2\Phi-\varphi-\varphi_0]\right) F\left(\sqrt{2kr}\cos\left(\frac{2\Phi-\varphi-\varphi_0}{2}\right)\right) +$$

$$+ R_2 \exp\left(-ikr\cos[2\Phi+\varphi+\varphi_0]\right) F\left(\sqrt{2kr}\cos\left(\frac{2\Phi+\varphi+\varphi_0}{2}\right)\right) +$$

$$+ \frac{e^{ikr+i\pi/4}}{\sqrt{2\pi kr}} \left(g(\varphi-\pi) - g(\varphi+\pi) + \frac{D_1}{2\cos\left[\frac{\varphi-\varphi_0}{2}\right]} + \frac{R_1}{2\cos\left[\frac{2\Phi-\varphi-\varphi_0}{2}\right]} + \right.$$

$$\left. + \frac{R_2}{2\cos\left[\frac{2\Phi+\varphi+\varphi_0}{2}\right]} \right) \left(1 + O\left((kr)^{-3/2}\right)\right) .$$

where $R_j = -D_1 \frac{P_j(\varphi_0+(-1)^j\Phi,x)}{P_j(-\varphi_0-(-1)^j\Phi,x)}$, $j = 1, 2$ are the reflection coefficient for the wedge's faces. The directions $\varphi = \pi - 2\Phi - \varphi_0$, $\varphi = \pi - 2\Phi + \varphi_0$ correspond to the rays of termination for the reflected waves, and $|\varphi - \varphi_0| = \pi$ corresponds to the light-shadow boundary for the incident wave.

4. Numerical simulation and discussion

The uniform asymptotics can be easily used for numerical simulation of the far field. The only difficulty is in accurate computation of the spectral function $g(\alpha)$. The singularities in the last three terms of the asymptotics on the light-shadow boundaries are compensated by the singularities of $g(\varphi-\pi)$ or $g(\varphi+\pi)$. The results of computations were obtained for different dielectric coatings of the wedge's faces. Among the covering materials we considered the dielectrics which are the commercially available radar absorbers. The Figure 2 demonstrates the results of calculations of intensity $|E_z|^2$ for the thin material coating with $kh_j = 0.3 + i0.1$, $(\epsilon_j/\mu_j)^{1/2} = 0.5 - i0.01$, $j = 1,2$, $k\rho = 10$, $\varphi_o = \pi/2$, $\Phi = 3\pi/4$. The computations were performed for different coatings as well as for different wedge's openings. In the report we shall demonstrate the influence of different types of coatings. We also compare the numerics for the coated wedge with those ones obtained for the perfectly conducting wedge. From this comparison useful recommendations for design of the coated scatterers with wedges can be concluded.

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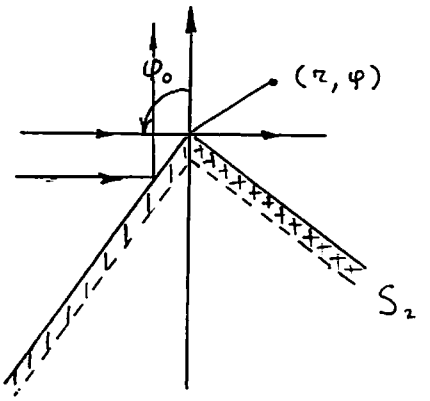

Fig. 1

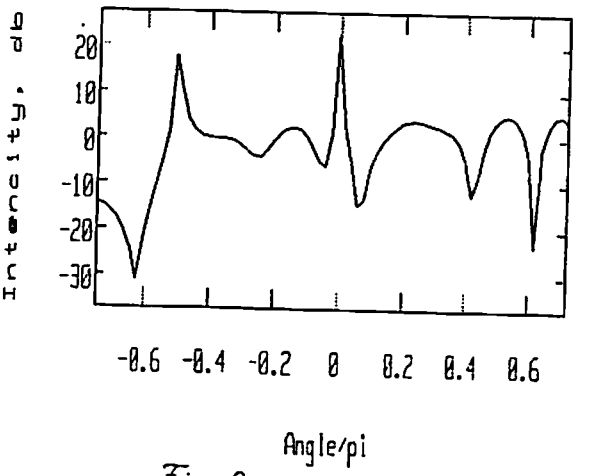

Fig. 2

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