

# Stretching Method Using Chebyshev Polynomial for Linear Sparse Array Antenna Design

Efri Sandi, Fitri Yuli Zulkifli, Basari and Eko Tjipto Rahardjo  
 Antenna Propagation and Microwave Research Group (AMRG)  
 Department of Electrical Engineering, Faculty of Engineering, Universitas Indonesia  
 Kampus Baru UI Depok,  
 Depok 16424, INDONESIA

**Abstract** — In this paper, a novel non-uniform stretching method of linear sparse array antenna design is proposed. This method is developed by applying new approach of inter-element spacing stretching strategy based on Chebyshev polynomial technique. The performance of proposed method is compared to full array antenna configuration with half-wavelength spacing. The simulation result shown the stretching-Chebyshev method has improved sidelobes level (SLL) compared to full array configuration with significant reduced number of array elements.

**Index Terms** — Stretching Method; Sparse Array Design; Non-uniform elements spacing; Chebyshev Polynomial.

## 1. Introduction

The purpose of development sparse array antenna is to achieve a desired radiation pattern with the minimum number of elements [1][2]. The sparseness characteristic is useful where weight and size of antenna are limited, such as in radar phased array antenna, radio astronomy communication, satellite communications and multiple-input-multiple-output (MIMO) in Long Term Evolution (LTE)-advanced cellular network [3][4].

In the phased array application such as a radar system, a large spatial dimension of aperture and a large number of antenna elements are required to obtain high resolution radar. Moreover, in phased array, each antenna element requires many subsystems therefore increase the production cost of the system [5].

In this paper, a method of linear sparse array antenna design is proposed. It is configured by inter-element spacing stretching strategy namely stretching coefficient with uniform excitation for each element (*isophoric* array). Therefore the stretching coefficient is developed based on Chebyshev polynomial function. This approach is different with Chebyshev array was proposed by Dolph [6] more than 50 years ago. The Dolph-Chebyshev approach is used to determine coefficient amplitude tapering in the uniform spacing configuration.

## 2. Proposed Stretching Method

The proposed method is developed from classical array with uniform elements spacing  $\lambda/2$  to sparse array

configuration by stretching strategy [7]. The element spacing will be stretched by using a coefficient based on mathematical formulation Chebyshev polynomial. It is provided by appropriate array factor and desired pattern with SLL target.

By expanding the array factor using Chebyshev polynomial and determining SLL target equal to recursion Chebyshev formula, the amplitude coefficient  $a_n$  for each element can be obtained. By referring correlation between amplitude excitation and inter-element spacing to the array factor, the element spacing is inverse of amplitude tapering, then the stretching coefficient ( $d_n$ ) can be expressed as follow:

$$d_n = \frac{1}{a_n} \quad (1)$$

The set of stretching coefficient is normalized by traditional element spacing  $\lambda/2$ . By using normalized coefficients, the sparse array configuration is obtained. The configuration will reduce number of elements required to achieve desired radiation pattern without grating lobe and low SLL.

## 3. Result and Discussion

To proof the proposed method, we use microstrip array antenna to perform as a linear full array compare with sparse array antenna configurations. The linear microstrip antenna array is designed at frequency 5 GHz, uses the substrate material of Taconic TLY-5 with dielectric permittivity  $\epsilon_r = 2.2$  and thickness = 1.58 in.

The sparse array configuration with proposed stretching method is developed from 9 elements as shown in Fig. 1a. The elements spacing is stretched by applying stretching coefficients based on Chebyshev polynomial as shown in Table 1. The spatial dimension after stretching is equal to 16 elements full array configuration with elements spacing  $\lambda/2$  as shown in Fig. 1b.



(a)



(b)

Fig. 1, Sparse array stretching method: (a) Sparse array 9 elements with spatial dimension  $17.8\lambda$ ; (b) Full array 16 elements with spatial dimension  $17.8\lambda$ .

TABLE I.

Stretching Coefficient For 9 Elements

Spacing Number ( $n$ )	Chebyshev Coefficients ( $a_n$ )	Stretching Coefficients ( $d_n$ )	Elements Spacing ( $\lambda$ )
1	0.26	3.85	1.92
2	0.52	1.92	0.96
3	0.81	1.23	0.62
4	1.00	1.00	0.50
5	1.00	1.00	0.50
6	0.81	1.23	0.62
7	0.52	1.92	0.96
8	0.26	3.85	1.92

The simulation result of the  $S_{11}$  and 2D far field plot linear full array configuration is showed in Fig.2. The radiation characteristics of sparse array 9 elements simulation showed that the antenna gain is 19.0 dB, the 3dB beamwidth is  $4.5^\circ$  and the SLL is -20.5dB. It shows significant improvement compared to 16 elements full array.

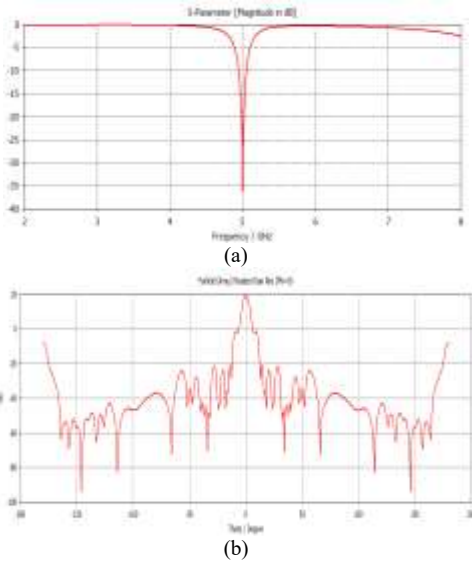


Fig. 2, Radiation pattern of sparse array configuration: (a)  $S_{11}$ ; (b) 2D far field plot 9 elements sparse array.

Furthermore, the proposed stretching method is applied to others sample of linier sparse array design to proof usefulness of the proposed method. The stretching method is applied to configure 17 elements and 33 elements sparse array design and compared to full array configuration with equal spatial

dimension. The simulation result of radiation pattern is then tabulated as shown in Table II.

TABLE II.

Overall Performances Comparison

No	Array Configuration	Number of Element	Aperture Length ( $\lambda$ )	Radiation Performances		
				Gain (dB)	3dB Beam width ( $^\circ$ )	SLL (dB)
1	Full Array ( $\lambda/2$ )	16	17.8	18.8	4.5	-13.6
	Sparse Array	9	17.8	19.0	4.5	-20.5
2	Full Array ( $\lambda/2$ )	33	31.9	21.5	2.3	-13.3
	Sparse Array	17	31.9	21.3	2.6	-20.9
3	Full Array ( $\lambda/2$ )	58	60.2	24.4	0.7	-20.9
	Sparse Array	33	60.2	26.1	0.4	-26.6

The overall comparison showed that sparse array design using stretching method has better characteristics for larger number of element (massive array) with the same ratio of element efficiency and has slightly improvement of gain and 3dB beamwidth.

#### 4. Conclusion

A novel stretching method of linear sparse array antenna design based on Chebyshev polynomial has been proposed and simulated. The simulation result showed the performances of proposed method is better than full array configuration with uniform spacing  $\lambda/2$ .

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