

COMPLEX RAY ANALYSIS OF TROUGH SCATTERING

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1. Introduction

The electromagnetic back scattering from a rectangular trough on the ground is a typical example of scattering prediction of open-ended cavities which have recently been paid attention and many investigations by means of GTD, SBR or ray-mode methods have been reported<sup>[1-3]</sup>.

In this paper, the incident plane wave is first expanded into a set of complex source point(CSP) beams<sup>[4]</sup>, and the complex ray paraxial approximation(CRPA)<sup>[5]</sup> is used to evaluate the beam scattering from a trough. The total scattering is obtained by the phasor summation of individual beam scattering from the trough, as well as the GTD contribution from the edges. The numerical examples for monostatic and bistatic trough scattering are given.

2. Analysis

Rectangular trough scattering is considered as a two-dimensional problem (Fig.1). The radar echo width (REW) is defined as

$$\sigma = \lim_{\rho \rightarrow \infty} 2\pi\rho \frac{|E_s|^2}{|E_i|^2} \tag{1}$$

where  $E_i$  is the incident plane wave amplitude with the assumption  $E_i=1$ , and  $E_s$  denotes the scattering wave amplitude at a very long distance  $\rho \rightarrow \infty$ .

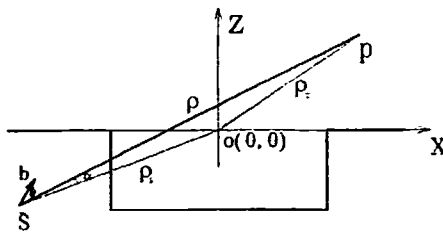


Fig.1 Costructure geometry

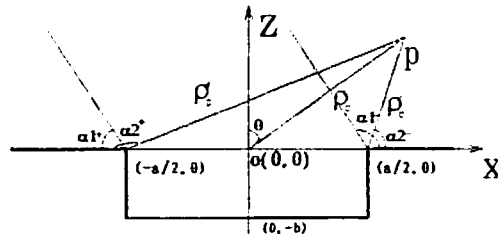


Fig.2 Trough edge diffraction

The incident unit plane wave can be expressed in terms of 2-D CSP field as a summation of Gaussian beams

$$\tilde{u}_i = \sum \frac{dx}{\sqrt{jb\tilde{\rho}}} \exp(-kb) \exp(jk\tilde{\rho}) \quad (2)$$

where  $dx$  is the CSP spacing,  $b$  is the CSP beamwidth parameter, and  $\tilde{\rho} = \sqrt{(x - \tilde{x}_s)^2 + (z - \tilde{z}_s)^2}$  denotes the distance from complex source point  $\tilde{S}(\tilde{x}_s, \tilde{z}_s)$  to field point  $P(x, z)$ .

Suppose a CSP beam undergoes  $n$  times of internal reflection in the trough, then the beam axis trajectory can be easily traced by SBR procedure. Because of the planar structure of trough, no additional divergence coefficient is introduced at reflections, and therefore the latest reflection beam field is obtained by<sup>[6]</sup>

$$\tilde{u}_{i,n} = \tilde{u}_{i,1} \prod_{m=1}^n R_m \frac{\sqrt{l_1 - jb}}{\sqrt{\sum_{m=1}^n l_m - jb}} \prod_{m=2}^n \exp(jkl_m) \quad (3)$$

where  $\tilde{u}_{i,1}$  is the incident field at the first reflection point,  $R_m$  is the Fresnel reflection coefficient,  $\tilde{\rho} = \sum_{m=1}^n l_m - jb$  denotes the distance from the equivalent CSP of latest reflection to the far-zone observer with  $l_m$  denoting the segment length before the  $m$ -th reflection. For the coordinates shown in Fig.1, the complex distance is expressed by

$$\tilde{\rho} = \rho_p + \rho_s \cos \alpha - jb \cos \theta \quad (4)$$

and then the scattering field of a CSP beam from the trough is

$$\tilde{u}_p = \frac{c}{\sqrt{jk\rho_p}} \exp[jk(\rho_s \cos \alpha - jb \cos \theta)] \exp(jk\rho_p) \quad (5)$$

$$c = \frac{dx}{\sqrt{j}} \exp(-kb) \sqrt{k} \prod_{m=1}^n R_m \quad (6)$$

The total reflection field of CSP beams from the trough is given as

$$\tilde{u}_e = \frac{\exp(jk\rho_p)}{\sqrt{k\rho_p}} \sum_i c_i \exp[jk(\rho_s \cos \alpha_i - jb \cos \theta_i)] \quad (7)$$

and the REW of CSP beam reflection from trough is obtained by Eq.(1) as

$$\frac{\sigma}{\lambda} = \left| \sum c_i \exp(jk\rho_s \cos \alpha_i) \exp(kb \cos \theta_i) \right|^2 \quad (8)$$

From GTD, the diffracted field of plane wave at edges and its REW contribution are given as (Fig.2)

$$u_d^\pm = D_{e,m}^\pm \frac{1}{\sqrt{k\rho_p}} \exp(\pm jka \cos \alpha_2^\pm) \exp(jk\rho_p) \quad (9)$$

$$D_{e,m}^\pm = \frac{\exp(j\pi/4) \exp(\pi/n)}{n\sqrt{2\pi}} \left[ \frac{1}{\cos \frac{\pi}{n} - \cos \frac{\alpha_2^\pm - \alpha_1^\pm}{n}} \mp \frac{1}{\cos \frac{\pi}{n} - \cos \frac{\alpha_2^\pm + \alpha_1^\pm}{n}} \right] \quad (10)$$

$$\frac{\sigma_d}{\lambda} = \left| D_{e,m}^+ \exp(jka \cos \alpha_2^+) + D_{e,m}^- \exp(jka \cos \alpha_2^-) \right|^2 \quad (11)$$

Making the phasor summation of Eqs.(3) and (9), one obtains the total scattering field from the trough and the corresponding REW contribution.

### 3. Numerical Examples and Discussions

Based on the above formulations, numerical calculations have been finished on microcomputer by use of C++ language. Fig.3 and Fig.4 show the monostatic REW distribution for wide and narrow trough,

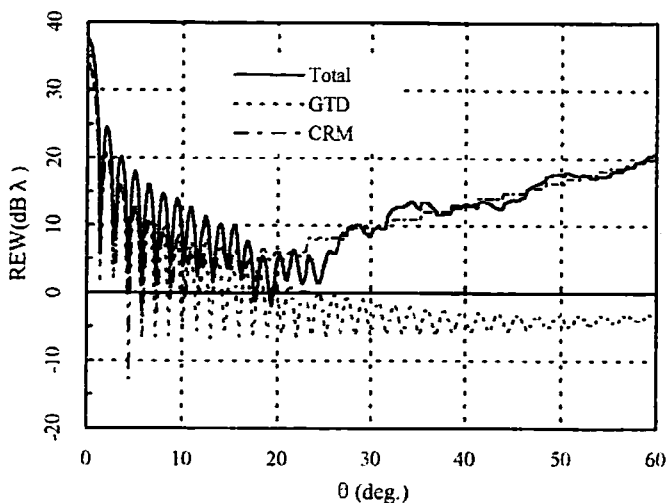


Fig.3 Wide trough backscattering (H-polarization,  $b=2.8\lambda$ ,  $a=20\lambda$ .)

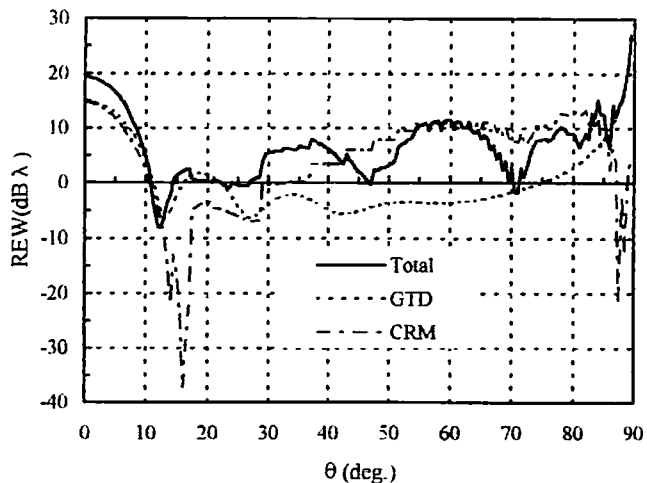


Fig.4 Narrow trough backscattering (H-polarization,  $b=1.6\lambda$ ,  $a=2.2\lambda$ .)

respectively. From Fig.3 (trough depth  $b=2.8\lambda$ , trough width  $a=20\lambda$ ), one finds that the CPS beam reflection is the dominant contribution to trough REW, and the edge GTD contribution is only a secondary one except the area near the reflection boundary. In Fig.4 ( $b=1.6\lambda$ ,  $a=2.2\lambda$ ), the GTD contribution is more important than that in Fig.3.

Fig.5 shows a bistatic REW distribution of wide trough ( $b=2.8\lambda$ ,  $a=20\lambda$ ) where the incidence angle is

+30°, and the maximum bistatic REW is located at -30°.

Comparing Fig.3 and Fig.4 with corresponding results by ray-mode method<sup>[2,3]</sup>, one can find they agree to each other, i.e. both methods have almost the same accuracy. However, complex ray analysis is more simple, which needs only a few seconds of CPU time on 486 microcomputer to finish the all calculations for Fig.3 or Fig.4.

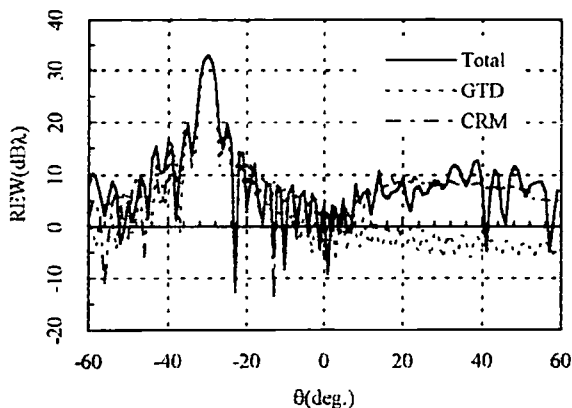


Fig.5 Wide trough bistatic scattering (H-polarization,  $b=2.8\lambda$ ,  
 $a=20\lambda$ , incidence angle +30°)

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