# A PLANAR NEAR-FIELD TO FAR-FIELD TRANSFORMATION USING AN EQUIVALENT MAGNETIC CURRENT APPROACH 

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#### Abstract

An alternate method is presented for computing far-field antenna patterns from planar near-field measurements. The method utilizes near-field data to determine equivalent magnetic current sources over a fictitious planar surface which encompasses the antenna, and these currents are used to ascertain the farfields. An electric field integral equation (EFIE) is developed to relate the near-fields to the equivalent magnetic currents. Method of moments (MOM) procedure is used to transform the integral equation into a matrix one. The matrix equation is solved with the conjugate gradient method (CGM), and in the case of a rectangular matrix, a least squares solution for the currents is found without explicitly computing the normal form of the equation. Near-field to far-field transformation for planar scanning may be efficiently performed under certain conditions by exploiting the block Toeplitz structure of the matrix and using CGM and Fast Fourier Transform (CGFFT) thereby drastically reducing computation and storage requirements. Numerical results are presented by extrapolating the far-fields using experimental near-field data.


## I INTRODUCTION

The earliest works based on the modal expansion method, in which the fields radiated by the test antenna are expanded in terms of planar, cylindrical or spherical wave functions [1-2]. The primary drawback of the mode expansion technique is that when a Fourier transform is used, the fields outside the measurement region are assumed to be zero, particulary for the planar case. The equivalent current approach which represents an alternate method of computing far-fields from measured near-fields has been recently explored [3-4]. In this approach the integral equation pertaining to measured near-fields and equivalent magnetic currents is a decoupled one with respect to the coordinate axes for the planar scanning case. A method of moments procedure with point-matching is used in which the equivalent currents are expanded in terms of two-dimensional pulse basis functions with unknown coefficients. The matrix may be rectangular or square depending on the number of data points and number of unknown currents chosen. If the spacing of the field points and currents patches are chosen the same, the resulting matrix is block Toeplitz. The structure of the matrix can be exploited by noting that a two dimensional Fourier transform may be utilized to evaluate some terms in CGM. This results in a tremendous decrease in storage and computation. The numerical integration in the process of creating the moment matrix elements can be avoided by taking the limiting case of the integral equation. For this special case, instead of using equivalent magnetic currents as equivalent sources, an equivalent magnetic dipole array is used to replace the aperture of the antenna.

## II THEORY

Let us consider an arbitrary shaped antenna radiating into free-space and let the aperture of the antenna be in a plane surface. Using the equivalence principle and image theory the aperture of the antenna $S_{0}$ is replaced by equivalent magnetic current $M_{\text {, which res res into free-space. We are focusing our }}$ attention to find the far-fields from the measured electric near-fields only via the equivalent magnetic current approach,

$$
\begin{equation*}
\bar{E}_{\text {meas }}=\bar{E}(\bar{M}) \tag{1}
\end{equation*}
$$

where $\mathrm{E}_{\text {mee }}$ is the measured electric near-field. The electric field at any point P can be found from,

$$
\begin{equation*}
\bar{E}(\bar{r})=-\int_{S_{0}}\left[\bar{M}\left(\bar{r}^{\prime}\right) \times \nabla \mathrm{g}\left(\bar{r}, \bar{r}^{\prime}\right)\right] \mathrm{d} s^{\prime} \tag{2}
\end{equation*}
$$

where $\nabla^{\prime}$ is the gradient operator according to the primed variables and $g\left(r, r^{\prime}\right)$ is the three-dimensional Green's function. For the planar scanning case the near-field measurement is performed over a planar surface which is assumed to be parallel with the source plane. The aperture of the antenna $\left(\mathrm{S}_{0}\right)$ is assumed to be a rectangular plate in the $x$ - $y$ plane. The distance between the source plane $\left(S_{0}\right)$ and measuring plane is $d$. For planar scanning the $x$ and $y$ components of the electric near-fields are usually measured. Taking only the $x$ and $y$ components of the measured electric near-fields into account in equation (2), the obtained integral equation is a decoupled one with respect to the two components of the magnetic currents. After formulating the E-field integral equations, a MOM procedure is used to transform them into matrix equations. Utilizing point matching the following two decoupled matrix equations are obtained,

$$
\begin{align*}
E_{\text {meas }, x} & =-G M_{y}  \tag{3a}\\
E_{\text {maas }, y} & =G M_{x} \tag{3b}
\end{align*}
$$

where $\underline{E}_{\text {mees } x}$ and $\underline{E}_{\text {meas, }}$ are the vectors which contain the x and y components of the measured electric nearfields, respectively, $\underline{\underline{M}}$ and $\underline{M}$, are the vectors which contain the unknown coefficients and $\underline{\underline{G}}$ is the moment matrix for the planar scanning case. The explicit expression for matrix $\underline{\mathrm{G}}$ is given by,

$$
\begin{equation*}
\underline{G}_{k, 1}=\int_{\Omega_{1}} \int_{e^{-j k_{0} R}}^{4 \pi R^{2}}\left(z_{k}-z^{\prime}\right) \quad\left[j k_{o}+\frac{1}{R}\right] d s^{\prime} \tag{4}
\end{equation*}
$$

where $\Omega_{\text {, }}$ is the area of the $\ell^{\text {th }}$ patch and $R$ is the distance between the $k^{\text {th }}$ field point $\left(r_{k}\right)$ and source point $\left(\mathbf{r}^{\prime}\right)$. Exploiting the block Toeplitz structure of the matrix some terms in the iterative process can be computed using a two-dimensional FFT. Instead of using equivalent magnetic currents as sources in the general magnetic current approach, equivalent magnetic dipole array can also be used to replace the aperture of the test antenna. This equivalent magnetic dipole array approximation can be treated as a limit of the integral equation approach. For this limiting case it can be assumed that the numerical integrations in the process of creating the moment matrix elements in equation (4) are executed using a one-point approximation. Using this one-point approximation in equations (4) an equivalent magnetic dipole array approximation is developed. The advantage of this approximation is that the numerical integration in the process of creating the moment matrix elements can be avoided.

## III. NUMERICAL RESULTS

Consider a microstrip array consisting of $32 \times 32$ uniformly distributed patches on a $1.5 \mathrm{~m} \times 1.5 \mathrm{~m}$ surface. The operating frequency is 3.3 GHz . The array is considered to be in the $x-y$ plane. For the planar
modal expansion approach, the near-fields are measured on a plane $3.24 \mathrm{~m} \times 3.24 \mathrm{~m}$ at a distance of 35 cm from the array. There are $81 \times 81$ points measured 4 cm apart. Measurements are performed using a WR 284 waveguide. For the first equivalent magnetic dipole array approximation, a $1.56 \mathrm{~m} \times 1.56 \mathrm{~m}$ surface with $39 \times 39$ uniformly distributed magnetic dipoles are used for the source plane and a $3.24 \mathrm{~m} \times 3.24 \mathrm{~m}$ surface with $81 \times 81$ measured near-field points are used for the field plane. So the solution obtained here is a least squares one with exploiting the block Toeplitz structure of the matrix and utilizing CGFFT. Fig. 1 shows the normalized absolute value of the electric far-field component $\left|\mathrm{E}_{\theta}\right|$ versus $\Theta$ in dB for $\Phi=90$ using the equivalent magnetic dipole array approximation and planar modal expansion. This is the co-polarization pattern. The result obtained using the equivalent magnetic dipole array approximation and the result obtained using the planar modal expansion show excellent agreement up to $\pm 60^{\circ}$ of $\Theta=0^{\circ}$ and show acceptable agreement in the rest of the elevation range. For Figure 1 it is known that the conventional planar modal expansion would provide acceptable results up to $\tan ^{-1}\left(\frac{87}{35}\right)=68^{\circ}$. Perhaps that is the reason the planar modal expansion results tend to deviate from our results beyond $\pm 60^{\circ}$. For the second equivalent magnetic dipole array approximation, a $1.48 \mathrm{~m} \times 1.48 \mathrm{~m}$ surfice with $37 \times 37$ uniformly distributed magnetic dipoles are used for the source plane and a $1.48 \mathrm{~m} \times 1.48 \mathrm{~m}$ surface with $37 \times 37$ measured near-field points are used for the field plane to enable use of CGFFT. It is important to note that for this case the conventional planar expansion method cannot be used because the area of the measured plane is equal to the area of the actual aperture of the test antenna. So the maximum angle for accurate far-field $\left(\Theta_{s}\right)$ when the planar modal expansion method is used is equal to zero. Fig. 2 shows the normalized absolute value of the electric far-field component $\left|E_{\mathrm{e}}\right|$ versus $\Theta$ in dB for $\Phi=90$ using both equivalent magnetic dipole array approximations. This is the co-polarization pattern. The result obtained using the first approximation and the result obtained using the second one show good agreement up to $\pm 40^{\circ}$ of $\Theta=0^{\circ}$ and show acceptable agreement in the rest of the elevation range. Summarizing our experimental results it can be concluded that the equivalent magnetic dipole array approximation gives accurate far-field patterns at angles beyond which the conventional planar modal expansion technique. For example, for Figure 2, the conventional planar modal expansion will not work. The accuracy of this approximation depends mainly upon the accuracy of the measured near-field data and does not depend on the measurement configuration. When a least squares solution is obtained for the equivalent sources the calculated far-fields are not as sensitive to the errors in the data as for the approximation when a square matrix solution is done. A simple method is presented for computing far-field antenna patterns from near-field measurements.

## REFERENCES

[1] A. D. Yaghjian: "An overview of Near-Field Antenna Measurements," IEEE Trans. Antennas and Propảgat., Vol. AP-34, pp. 30-45, January 1986.
[2] J. J. H. Wang: An Examination of the Theory and Practice of Planar Near-Field Measurement," IEEE Trans. Antennas and Propagat., Vol. AP-36, pp.746-753, June 1988.
[3] S. Ponnapolli: "Near-Field to Far-Field Transformation Utilizing the Conjugate Gradient Method," in Application of Conjugate Gradient Method in Electromagnetics and Signal Processing, Vol. 5, in PIER, T. K. Sarkar, Ed. New York: VNU Science Press, Ch. 11, Dec. 1990.
[4] S. Ponnapolli, T. K. Sarkar, P. Petre: "Near-Field to Far-Field Transformation Using an Equivalent Current Approach," Proc. of Int. Conf. on Antennas and Propagation (AP-S 91), London, Ontario, June 1991.


Co-polarization characteristic for $\Phi=90$ cut for a $32 \times 32$ patch microstrip array using planar modal expansion and equivalent magnetic dipole array approximation.


Co-polarization characteristic for $\Phi=90$ cut for a $32 \times 32$ patch mictrostrip array using least squares and square matrix solutions.

