A Hybridization of Multi-Level UV with the Hierarchical Fast Far Field Approximations for 3D Rough Surface Scattering

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***Abstract*-This paper presents a hybrid multilevel UV method with the fast far field algorithm for the random rough surface scattering calculations. In the MLUV, a pre-ranked sampling algorithm is used to further compress the UV matrix pairs. To further improve the efficiency of the algorithm, from the second and third coarsest level, a hierarchical FAFFA is employed to further enhance the efficiency of the algorithm. A rough surface with 32-by-32 wavelengths is simulated with 1.5 million unknowns. The results show the efficiency of the algorithm.**

I. INTRODUCTION

Random Rough surface scattering is a very interesting and important phenomenon in surface science. In real world, there exist a lot of examples that can be explained using the random rough surface scattering theorem, e.g., in macroscopic size world, the electromagnetic wave scattering from soil, lake, river and ocean; and in the microscopic size world, the electromagnetic wave scattering from the rough surface of the silicon substrate surface, and the surface plasma excitation on the silver film with rough surface.

The difficulties in study the random rough surface scattering is that the rough surface is always infinite large and the complex multiple scattering by the roughness of the surface. To model the rough surface scattering, one needs to truncate the surface. However, to ensure the accuracy of the modeling of the surface scattering, the truncated surface size should be electric large. To study the rough surface scattering problem, the MoM based fast algorithms i. e., fast integral equation solutions are always employed because of their accuracy and efficiency.

The Multi-Level UV (MLUV) factorization method is a kernel independent fast integral equation method that factorizes and compresses the low rank off-diagonal submatrices in the MoM matrix using a multi-level scheme [1][2]. It has been used for some scalar scattering problems [3]. In [4], MLUV has been employed for object scattering composited with object and rough surface. In it, the surface is the PEC. In [3][5], UV method is combined with Sparse Matrix Canonical Grid (SMCG) for soil scattering problems, the rough surface is soil interface that can be viewed as the rough interface of a dielectric region.

Instead, in this paper, we employ Multi-Level UV directly. A pre-ranked sampling algorithm is employed to further compress the UV matrix pairs. For the second or the third coarsest levels, the fast far field approximation is also used to further enhance the efficiency of the algorithm. Comparing with other fast methods such as MLFMA, MLUV has a great advantage that it is very easy to be parallelized because 1) the calculation of each UV pairs is independent and 2) in a matrix-vector multiplication, each submatrix-vector multiplication using the UV pairs is also independent. A rough surface of 32-by-32 square wavelengths is simulated with 1.5 million unknowns. The results show the efficiency of the algorithm.

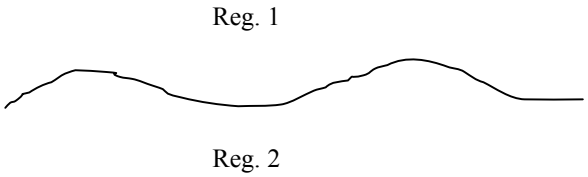

Fig. 1. Two regions rough surface problem.

II. PRE-RANKED MLUV+FAFFA

A. Integral equations

For two regions rough surface problem as shown in Fig. 1, the PMCHWT is used and listed below.

$$\boldsymbol{E}^{inc}(\boldsymbol{r})|_{\tan} = (L_1 + L_2)\boldsymbol{J}'(\boldsymbol{r})|_{\tan} - (K_1 + K_2)\boldsymbol{M}(\boldsymbol{r})|_{\tan}, \quad (1)$$

$$\eta_0 \boldsymbol{H}^{inc}(\boldsymbol{r})|_{\tan} = (K_1 + K_2)\boldsymbol{J}'(\boldsymbol{r})|_{\tan} + (\varepsilon_{r1}L_1 + \varepsilon_{r2}L_2)\boldsymbol{M}(\boldsymbol{r})|_{\tan}. \quad (2)$$

$$L_i = \frac{-i\omega\mu_i}{4\pi\eta_0}\int_S ds'(\bar{\bar{I}} - \frac{1}{k_i^2}\nabla\nabla')G_i(\boldsymbol{r},\boldsymbol{r}')\cdot, \quad (3)$$

$$K_i = -\frac{1}{4\pi}\int_S ds'\nabla G_i(\boldsymbol{r},\boldsymbol{r}')\times, \quad (4)$$

where $\boldsymbol{J}'(\boldsymbol{r}') = \eta_0 \boldsymbol{J}(\boldsymbol{r}')$ and η_0 is the free space wave impedance, and subscript $i = 1, 2$ is the indices of the regions.

Expanding the currents with the Rao-Wilton_Gillison Basis functions and using the Galerkin method, equations (1) and (2) can be discretized into a MoM matrix equation.

$$\begin{bmatrix} \bar{\bar{Z}}^{L_1} + \bar{\bar{Z}}^{L_2} & -(\bar{\bar{Z}}^{K_1} + \bar{\bar{Z}}^{K_2}) \\ \bar{\bar{Z}}^{K_1} + \bar{\bar{Z}}^{K_2} & \varepsilon_{r1}\bar{\bar{Z}}^{L_1} + \varepsilon_{r2}\bar{\bar{Z}}^{L_2} \end{bmatrix} \begin{bmatrix} \bar{I}^E \\ \bar{I}^M \end{bmatrix} = \begin{bmatrix} \bar{V}^E \\ \bar{V}^M \end{bmatrix}. \quad (5)$$

where

$$Z_{p,q}^{L_i} = \frac{-i\omega\mu_i}{4\pi\eta_0}\int_S ds\boldsymbol{f}_p(\boldsymbol{r})\cdot \int_S ds'[\bar{\bar{I}} - \frac{1}{k_i^2}\nabla\nabla']G_i(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{f}_q(\boldsymbol{r}'), \quad (6)$$

$$Z_{p,q}^{K_i} = -\frac{1}{4\pi}\int_{S_p} ds\boldsymbol{f}_p(\boldsymbol{r})\cdot\int_{S_q} ds'\nabla G_i(\boldsymbol{r},\boldsymbol{r}')\times\boldsymbol{f}_q(\boldsymbol{r}'), \quad (7)$$

(5) can be solved iteratively using method such as GMRES, BICG-stab etc. In this paper we use a hybrid pre-ranked MLUV+FAFFA to accelerate the matrix-vector multiplications.

B. Pre-ranked MLUV

In the original MLUV, the coarse-coarse sampling algorithm is used to sample the subscatterers from the field and source groups. This sampling scheme always gives over sampled subscatterers sets and thus ensures the accuracy of MLUV. However, because both rows and columns are over sampled, the computational load for UV factorizing the submatrices will be unnecessary large. In this paper, we use a pre-ranked approach. In this approach the coarse-coarse sampling is applied first to obtain a submatrix that is greatly smaller than the original submatrix. Subsequently the rank-prevealed QR factorization is employed to obtain the indices sets of the significant rows and columns. According to these two indices sets, we can sample the rows and columns from the original submatrix and then construct the UV pairs following the original UV procedure.

C. Hierarchical Fast Far Field Approximations

When the electric size of the problem becomes very large, the sub-matrix describing the interactions between a node at finer level and its interaction list node can be very large although the sub-matrix is low rank. The complexity of the MLUV thus becomes also very large for solving this kind of problem. In [6], a FAFFA method is employed to solve for the electric large problem. In the FAFFA method, the Green's function of any two points respectively in two well separated groups can be approximated using the following equation [7]

$$e^{ikR}/R \simeq \frac{\eta_0}{-i4\pi\omega\mu_i} e^{i\boldsymbol{k}_{m,n}\cdot(\boldsymbol{r}-\boldsymbol{r}_m)}\alpha_{i,m,n}^{far} e^{-i\boldsymbol{k}_{m,n}\cdot(\boldsymbol{r}'-\boldsymbol{r}_n)}, \quad (8)$$

where $\alpha_{i,m,n}^{far} = \frac{-i\omega\mu_i}{4\pi\eta_0} e^{ik_i R_{m,n}} / R_{m,n}$.

However, to ensure the accuracy of the far field approximation, we should use the weak criterion [7]

$$ratio = R_{m,n} / L_{gp} \geq \beta, \quad \text{i=1,2} \quad (9)$$

where β is a real constant and k is the wave number. In [8], a multilevel FAFFA is implemented for solving scattering problems. However, in the coarsest level the distances between each far interaction group is different which will definitely affect the efficiency.

In order to further enhance the efficiency, in this paper, an adaptive hierarchical FAFFA (HFAFFA) algorithm is devised. In this method, the FAFFA starts from the second and third coarsest levels. For any two well separate groups at these levels, if the *ratio* satisfies the weak criterion (9), the FAFFA is performed to approximate the interactions between these two groups. Otherwise, divide each group into four sub-groups and perform the same procedure to these sub-groups recursively. Thus an HFAFFA is constructed. The HFAFFA has the advantage that the *ratio* of any two well separate groups between which the interactions are approximated using FAFFA will not change greatly and hence the efficiency enhanced.

III. NUMERICAL RESULTS

All results in this paper are calculated using a server with two 2.60 GHz Intel Xeon CPUs with 16 cores and 32 threads. The shared memory is 128 GB. The program is parallelized employing OpenMP technique with all the 32 threads.

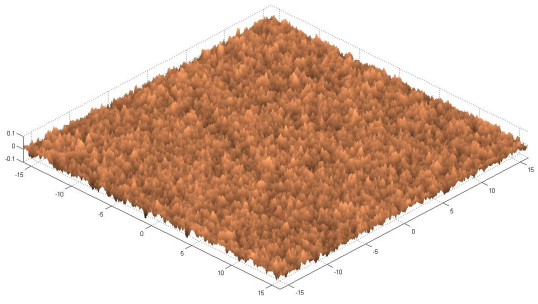

Fig. 2, The profile of Gaussian random rough surface with exponential correlation function. RMSh=0.021; Corre. Length=0.21; Epsr=15.14+1.27i.

The rough surface simulated is ploted in Fig. 2. The root mean square height (RMSh) is 0.021 wavelength. The Coorelation Length is 0.21 wavelength. The dielectric constant of region 2 is 15.14+1.27i. The infinite rough surface is truncated into a 32-by-32 square wavelengths finite surface. If one wavelength is meshed into 16 grids and RWG basis functions are used, the total number of unknowns including both the electric current unknowns and the magnetic current unknowns is 1,572,864.

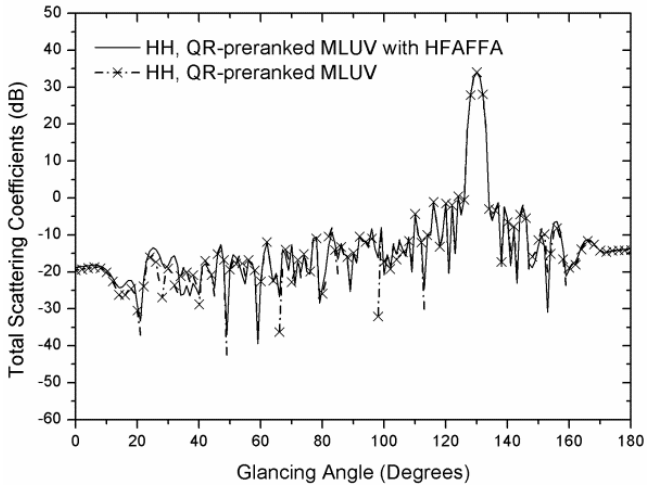

Fig. 3 HH polarized scattering coefficients of one realization.

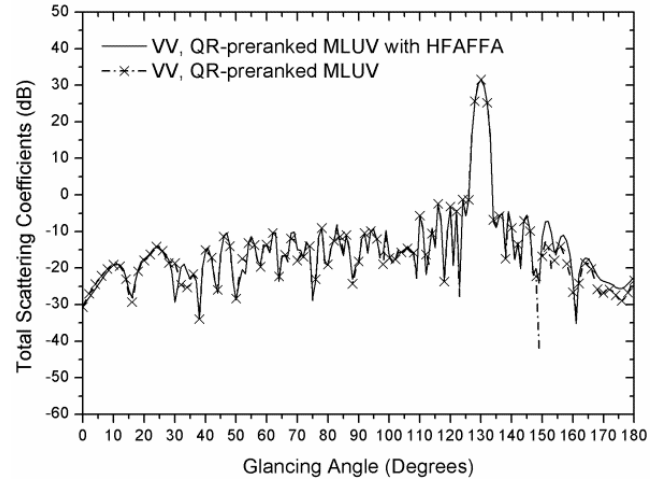

Fig. 3 VV polarized scattering coefficients of one realization

Fig. 3 and Fig. 4 are the calculated HH and VV polarized total scattering coefficients of one realization. We see that the results obtained with using HFAFFA and without using HFAFFA agree very well. Note the incident angle is 40 degree which is just 50 degrees for the glancing angle. Tapered plane wave is used for the rough surface is finite in size.

TABLE I
PERFORMANCE COMPARISON

Methods	Preranked MLUV	Preranked MLUV with HFAFFA
Memory	123 GB	108 GB
Total CPU time	5 hours	2 hours

Table I is a brief comparison of these two methods. We see that with using HFAFFA the CPU time consumption is less than half of that without using HFAFFA, while the memory consumption is also reduced.

IV. CONCLUSION

This paper presents a hybrid multilevel UV method with the fast far field algorithm for the random rough surface scattering calculations. In the MLUV, a pre-ranked sampling algorithm is used to further compress the UV matrix pairs. To further improve the efficiency of the algorithm, from the second and third coarsest level, the FAFFA is employed to further enhance the efficiency of the algorithm. A rough surface with 32-by-32 wavelengths is simulated with 1,572,864 unknowns. The results demonstrate the accuracy and efficiency of the proposed algorithm.

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