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Feedback control for nonholonomic systems using neural oscillator network

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Abstract—In this paper, we propose a novel control approach for some canonical forms of nonholonomic systems using networks of neural oscillators. Key features of a neural oscillator include its ability to generate stable rhythmic signals, and to tune their frequencies and patterns with the environment. Focusing on their versatile entrainment property, we adopt the network as a dynamic feedback controller for nonholonomic systems. We examined effectiveness and robustness of the proposed method by several numerical simulations.

1. Introduction

Robot control using neural oscillators have been extensively studied for the last two decades. Thanks to their entrainment property as nonlinear dynamical system, networks composed of neural oscillators have outstanding ability to generate stable rhythmic signals in response to stimulation from the environment. Thus the robustness of the controller using neural oscillators have been performed in several studies.

A pioneering example of making full use of the entrainment property is the generation of biped walking locomotion proposed by Taga[1]. Moreover, the model generated not only the walking movement but also the running movement by changing the parameters of the networks. As a result, a control method using the entrainment ability was proposed. Another example is an ameba-like modular robot developed by Ishiguro[2], which is composed of several module units that has elastic arms between each other. The stretch/shrink motion of each arm is controlled by the neural oscillators. Even though the module unit could not locomote alone, a coordination of the modules excited by the physical external stimulation enables its locomotion. It can also avoid collision to obstacles by adaptively changing the connection structure among the modules.

Meanwhile, a number of scientists in the field of nonlinear control theory have been intensively working on control problems of *nonholonomic systems* since early 90's[3]. Nonholonomic system is a peculiar class of nonlinear systems with the feature that they can be controllable in nonlinear sense by considering the effect of Lie brackets of the input vector-fields, in spite that they are uncontrollable in linear sense. Such systems often appear in locomotion control problem of mobile robots.

The purpose of this paper, in short, is to combine the two fields; we propose a new approach to feedback control for nonholonomic systems using networks of neural oscillators. Moreover, we examine effectiveness of the method by using the neural network to generate appropriate inputs to make desired state transition for rolling sphere problem. The main purpose of the study is to analyze the robustness of the oscillators based on nonlinear control theory.

2. Neural oscillator networks

In this section, we briefly review a model of neural oscillator network which we use in this paper. A simple neural oscillator is composed of a pair of neural element models inspired by biological nerves systems. A neural element typically exhibits excite-decay response to external stimulation as shown in Figure 1: given a constant input (stimulation), the element generates a corresponding output which decays with respect to time. A neural oscillator consists a pair of elements with mutually *inhibitory* connection between them. Given constant inputs and external stimulation, the oscillator generates synchronized and rhythmic signals depending on the environment[4]. Moreover, given inputs oscillating near natural frequencies of the oscillator, its input-output phase is strongly synchronized. By this entrainment property, the oscillator generates rhythmic stable signals under disturbance condition.

In this paper, we adopt a mathematical model of neural element proposed by Matsuoka[5]. The Matsuoka model is

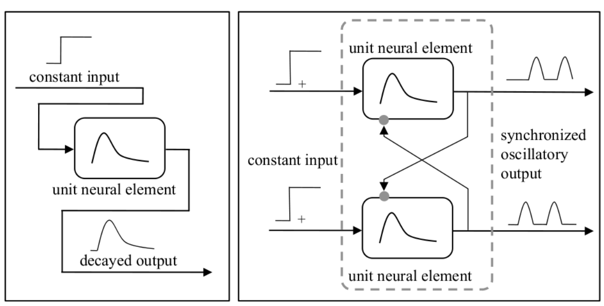


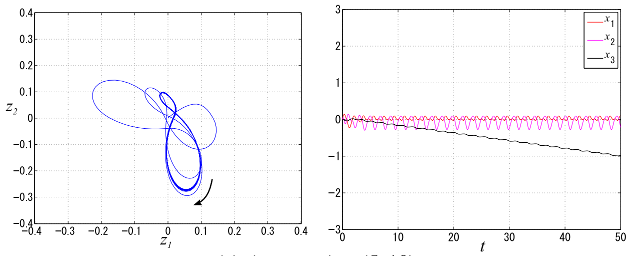
Figure 1: Neural element (left) and their reciprocal connection (right)

formulated as follows:

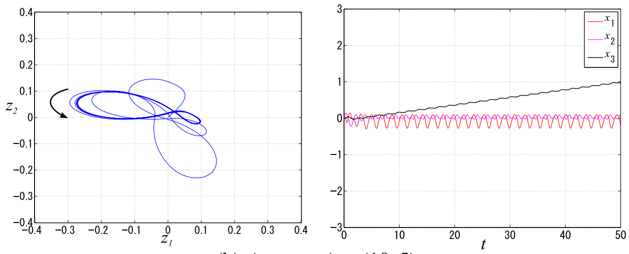
$$\begin{aligned} \dot{z}_i &= -z_i - \sum_{i\neq j}^{n} \omega_{ji} y_j - \beta f_i + u_0 \\ \dot{f}_i &= -f_i + y_i, \\ y_i &= \max(0, z_i) \end{aligned} \quad (1)$$

where $i \in \mathcal{N}, \mathcal{N} := \{1, \cdots, N\}$ denotes index of the neural element, z_i and f_i are its state variables. As for the parameters, β is a coefficient which affects the firing rate in response to external input, u_0 is the external input which is supposed to be constant in this case, and ω_{ij} is the connection intensity from z_j to z_i. We denote $\boldsymbol{\omega} = \{\omega_{ij} | i \neq j, i, j \in \mathcal{N}\}$ the collection of all connection parameters, e.g., $\boldsymbol{\omega} = (\omega_{12}, \omega_{21})^T$ for $N = 2$.

In response to the change of connection structure and the dynamics parameters, the oscillator generates various rythmic signals with different frequencies and patterns[5]. In the rest of paper, we exploit to utilize the versatile rythmic signals as the control input to nonholonomic systems.



(a) $(\omega_{21}, \omega_{12}) = (5, 10)$



(b) $(\omega_{21}, \omega_{12}) = (10, 5)$

Figure 2: Influence of the connection parameters in state responses

3. Feedback control for nonholonomic systems

In this section, we deal with control problem for a typical class of nonholonomic systems, called *Brockett integrator*, as a testbed to examine control ability of the neural oscillator network.

3.1. Brockett integrator

Mechanical systems that undergo non-integrable constraints[6] are called *nonholonomic systems*, which have been widely studied in the field of nonlinear control theory since early '90s[3]. A notable feature of nonholonomic systems is that, although they are *uncontrollable in linear sense*, they still remain *controllable in nonlinear sense*, i.e., the system state can be brought to any desired state from anywhere by appropriate choice of control inputs. This means that a naive control method such as linear state feedback is not valid for nonholonomic systems, however, there left a possibility to control them by making full use of nonlinear dynamics in control. This motivates us to adopt neural oscillator as a feedback controller.

Here we focus on *Brockett integrator* in particular, which is the simplest case of nonholonomic systems. The state equation is shown as follows.

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -x_2 & x_1 \end{bmatrix} \boldsymbol{u}, \quad \boldsymbol{x} \in \mathbb{R}^3, \ \boldsymbol{u} \in \mathbb{R}^2 \quad (2)$$

where $\boldsymbol{x} \in \mathbb{R}^3$ is the system state and $\boldsymbol{u} \in \mathbb{R}^2$ is the control input. The system (2) is controllable in nonlinear sense as mentioned. It is clear that x_1, x_2 are directly controlled by u_1, u_2, while x_3 should be maneuvered indirectly by appropriate coordination of u_1 and u_2; indeed, a key trick is known to assign periodic signals u_1 and u_2 with proper phase gap such as sine–cosine pair of functions[3].

In this paper, we consider a network composed of two Matsuoka-model elements (1), for $\mathcal{N} = \{1, 2\}$, and propose to assign their outputs as the control inputs, i.e.,

$$(u_1, u_2)^{\mathrm{T}} = (\dot{z}_1, \dot{z}_2)^{\mathrm{T}} \quad (3)$$

Here we examine fundamental effect of this method by numerical simulations. Let the parameters in (1) be $\beta = 10$, $u_0 = 1$, and suppose the initial state is given as $(x_1, x_2) = (0.001, 0.002)^{\mathrm{T}}$. For two choices of $\boldsymbol{\omega}$ that $(\omega_{21}, \omega_{12}) = (5, 10), (10, 5)$, the phase trajectories of the oscillator (z_1, z_2) and the time response of all the state variables are shown in Figure 2(a),(b). It shows that the $z_1 - z_2$ trajectory eventually converges to a limit cycle. Different choices of $\boldsymbol{\omega}$ affects the amount of increase (or decrease) of x_3 while x_1 and x_2 keep vibrating in a vicinity of the origin.

The results indicate us that the change of the connection weight $\boldsymbol{\omega}$ would be the crucial factor in the transition of x_3.

3.2. Sensory feedback through ω-parameter

Here we propose a feedback control method which attracts and keeps the system state nearby the origin, by imposing a feedback law for $\boldsymbol{\omega}$ as functions of $\boldsymbol{x}$.

Now let us consider the following *parameter feedback law*:

$$\begin{cases} K_i &= 1 \\ \omega_{ij}(\boldsymbol{x}) &= 10 + (-1)^j 5 \,\mathrm{erf}(x_3) \end{cases}, \quad i, j \in \mathcal{N} \quad (4)$$

where K_i is the proportional gains and

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (5)$$

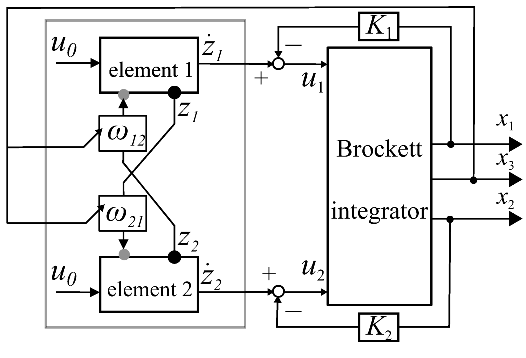


Figure 3: Structure of the proposed feedback controller

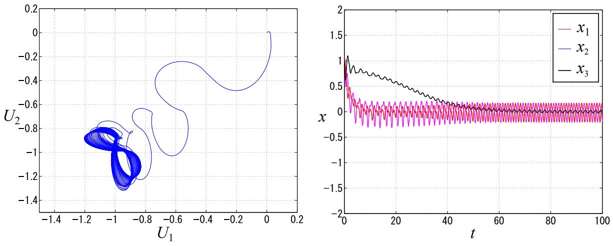


Figure 4: Control of Brockett Integrator via parameter feedback through $\boldsymbol{\omega}$

is the Gauss error function, which is adopted in purpose to limit the magnitude of $\omega_{ij}(\boldsymbol{x})$ up to a constant value as $|x_3|$ increases. The overall structure of the feedback system is shown in Fig. 3

Let us see the simulation results. This time, the initial state is $\boldsymbol{x} = [1, 1, 1]^{\mathrm{T}}$. Figure 4 shows the phase trajectory of the input integrations (U_1, U_2) and the time response of all the state values $\boldsymbol{x}$. The state trajectory successfully converged near the origin with the aid of proportional and parameter feedback.

The Lissajous plot changed with the state transition from the initial state and the state trajectory converged to oscillating near the origin. These results represent effectiveness of the proposed method.

4. Application to rolling sphere problem

In this section, we apply the proposed approach to control of a spherical mobile robot system, which is a typical problem in nonholonomic control and more difficult to control than Brockett integrator. To transit the state of the spherical robot arbitrarily, second-order Lie brackets should be taken into consideration while first-order Lie brackets in Brockett integrator. That is, frequencies and patterns of inputs become more important.

4.1. State equation of the spherical mobile robot

Let us define the posture angles and positions of the sphere as shown in Figure 5, where (x, y, z) is absolute coordinate system, (x', y', z') is sphere-fixed coordinate system, an intersection point of the surface with negative z'-axis is initial grounding point O'. Then, we define the intersection point of positive x-axis with the surface as the *north pole*, θ as the *latitude*, and φ as the *longitude*.

We assume that the sphere is in contact with the ground without any spin nor slip sideways. We also suppose that we can apply the latitudinal and longitudinal angular velocities, $\dot{\theta}, \dot{\varphi}$ as the control inputs, respectively. Then, we obtain the following state equation.

$$\begin{bmatrix} \dot{\theta} \\ \dot{\varphi} \\ \dot{\psi} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \sin\varphi & 0 \\ -\rho\cos\varphi\sin\psi & -\rho\cos\varphi \\ \rho\cos\varphi\cos\psi & -\rho\sin\varphi \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (6)$$

The state equation (6) is also known to be controllable in nonlinear control theory. In other words, appropriate periodic inputs $\dot{\theta}, \dot{\varphi}$ would generate states transition arbitrarily.

4.2. Control strategy

Now we propose a control strategy for spherical mobile robot by using the network of the neural oscillators. Let us suppose again a pair of Matsuoka-model elements (1), for $\mathcal{N} = \{1, 2\}$, and assign the control inputs for the spherical robot system as

$$(u_1, u_2)^T = (\dot{z}_1, \dot{z}_2)^T \quad (7)$$

$$\begin{cases} \dot{z}_1 = -\theta - \omega_{21} y_2 - \beta f_1 + u_0 \\ \dot{z}_2 = -\varphi - \omega_{12} y_1 - \beta f_2 + u_0 \\ \dot{f}_1 = -f_1 + y_1 \\ \dot{f}_2 = -f_2 + y_2 \end{cases} \quad (8)$$
$$\begin{cases} y_1 = \max(0, z_1) \\ y_2 = \max(0, z_2) \end{cases}$$

where connection weight $\boldsymbol{\omega}$ is the parameter to change the periodic outputs generated by the neural network. Here we intend to produce primitive motion of the sphere by tuning the connection parameters $\boldsymbol{\omega}$.

Now we show some simulation results to relate the connection weight $\boldsymbol{\omega}$ to the motion of the sphere. Parameters for the Matsuoka-models are set to $\rho = 0.085$, $\beta =$

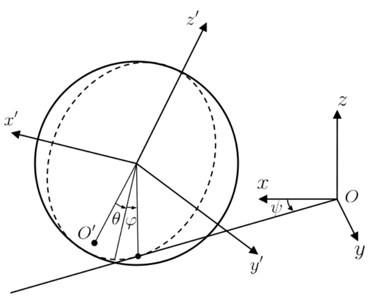


Figure 5: Coordinate setting of the sphere-floor system

10, $u_0 = 1$, while the initial state is $[\theta, \varphi, \psi, x, y]^{\mathrm{T}} = [0.001, 0.002, 0, 0, 0]^{\mathrm{T}}$, Lissajous plots of the neural oscillator, as well as the x-y trajectories of the sphere's position, are shown in Figure 6, 7, 8.

In the case where ω_{21} and ω_{12} are close to each other, the Lissajous plot of the neural oscillator (z_1, z_2) draws a figure-8 like curve, namely a pair of two closed curves with *opposite directions* alternatively; it implies that first-order effect of Lie bracket motion are canceled out to each other and results in a higher-order Lie bracket motions. On the other hand, in case that a relative difference between them turns bigger, the figure describes some clockwise or anti-clockwise circles which result in first-order Lie bracket motions. Indeed, the ball-plate problem has a so-called second-order controllability structure[7], in the sense that the system state is maneuvered by the effect of both the first and the second-order Lie brackets. The proposed controller is capable of producing all types of effects by tuning the connection parameters.

In summary, we observe that it is possible to realize forward locomotion, clockwise rotation and anti-clockwise rotation of the sphere by appropriate choices of $\boldsymbol{\omega} = (\omega_{21}, \omega_{12})$. Combining these primitive motions (forward, right and left turn) enables us to maneuver the sphere to any desired position.

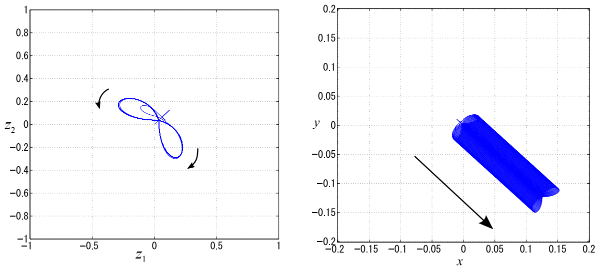


Figure 6: Forward locomotion: $(\omega_{21}, \omega_{12}) = (5, 5)$

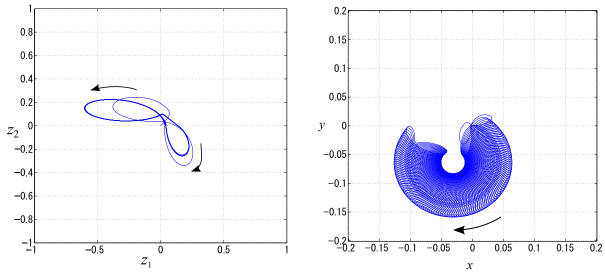


Figure 7: Clockwise rotation: $(\omega_{21}, \omega_{12}) = (8, 5)$

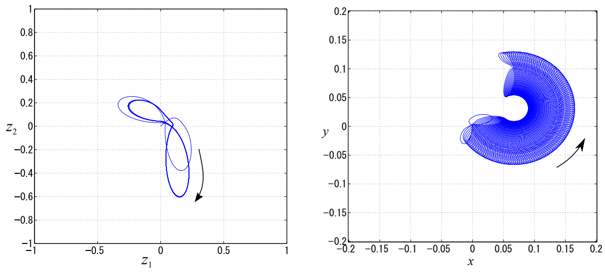


Figure 8: Anti-clockwise rotation: $(\omega_{21}, \omega_{12}) = (5, 8)$

5. Conclusion

In this paper, we proposed a control method for nonholonomic systems by using the network of neural oscillators, and examined its effectiveness and robustness by several simulations. We showed the target system were robustly controlled with the proposed *parameter feedback* through the connection weights $\boldsymbol{\omega}$. We also showed that some fundamental locomotions for the spherical mobile robot can be achieved by the proposed neural oscillator controller.

We are currently working on implementing a complete feedback control for the spherical robot by using neural oscillators. Robustness of the proposed method in real(physical) situation, against model uncertainties and severe disturbances from environment, should also be examined.

Acknowledgments

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