Analysis of Electromagnetic Scattering from Complicated Objects Using Nonconformal IE-DDM

X. Wei, J. Hu, M. Tian, R. Zhao, M. Jiang, Z. P. Nie
Dept. of Microwave Engineering,
University of Electronic Science and Technology of China
Cheng du, 610054, China
hujun@uestc.edu.cn

***Abstract*-This paper extends the integral equation domain decomposition method (IE-DDM) with Robin Transmission Condition(Robin TC) to the electromagnetic analysis of general objects of arbitrary shape, even for composite conducting-dielectric structures. Meanwhile, nonconformal mesh processing is formulated for the IE-DDM. The method is based on the CFIE-GCFIE integral equation formulation. First, we present the IE-DDM formulations for composite conducting-dielectric objects. Then, the nonconformal mesh processing is introduced into the IE-DDM. And last, the numerical results are also given to prove the validity of the present method.**

I. INTRODUCTION

The integral equation (IE) method is a popular tool which has many important applications e.g., in the radar technology, antenna design and microwave engineering. An efficient and stable integral equation framework for solving scattering problem of complex and multi-scale platform is a great need, especially for composite conducting-dielectric structures. And lots of studies have been conducted by many researchers [1-5]. Several methods based on IE have been proposed, including surface coupled integral equations method [6] , hybrid volume-surface integral equations method [7] . IE method can analyze conducting-dielectric structures of arbitrary shape by using the combination of electric-field integral equation(EFIE) for PEC part and other kind SIEs for dielectric part. The popular methods such as EFIE-PMCHWT [8-9] or EFIE-JMCFIE still remain challenges on the treatment of junctions between conducting part and dielectric part. The complex combinations of integral equations and conformal mesh processing are uneasy for sophisticated models, even though EFIE-CFIE-PMCHWT [10-11] formulation was present for solving special junction parts. Surface integral equation domain decomposition method (IE-DDM) [12-13] for field problem of composite object with homogeneous material properties performs well while a new generalized combined field integral equation is also employed for IE-DDM, which leads to well-conditioned matrix equations.

In this paper we analyze conducting object and composite conducting-dielectric structures using IE-DDM that makes treatment of junctions uncomplicatedly. Moreover, nonconformal interpolation method [14] is applied in it to make mesh processing more flexible.

The rest of this paper is organized as follows. In Section 2 the formulations about IE-DDM and general combined field integral equation (GCFIE) are given. Nonconformal interpolation method is described in Section 3. In Section 4 numerical results are shown to prove the validity of this method, followed by the conclusions.

II. IE-DDM FORMULATIONS

In this section, firstly we present the basic theorem of integral equation domain decomposition method. Then G-CFIE will be discussed in IE-DDM framework when analyzing the scattering field of a composite conducting-dielectric structure.

A. *Framework of IE-DDM*

We consider that the whole computational domain Ω is divided into some non-overlapping subdomains Ω_i ($\mathrm{i}=1,\cdots,\mathrm{N}$) (Fig.1), $\Omega=\cup_{i=1}^{N}\Omega_i$.

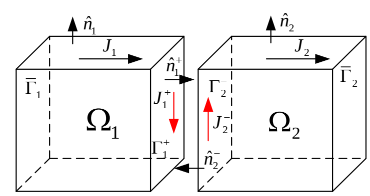

Fig.1. Non-overlapping IE-DDM Scheme

$\overline{\Gamma}_{\mathrm{i}}$ and $\overline{\Gamma}_{\mathrm{ij}}$ are the original as well as new interface respectively, that is to say

$$\begin{aligned}\overline{\Gamma_i}&=\partial\Omega_i\cap\partial\Omega\\ \overline{\Gamma_{ij}}&=\partial\Omega_i\cap\partial\Omega_j\end{aligned}\tag{1}$$

While applying the idea above to electromagnetic scattering model, we can get the basic formulations for each subdomain Γ_{i}, (2.1) for $\overline{\Gamma_i}$ and (2.2) for $\overline{\Gamma_{ij}}$.

$$\left(\overline{E}_i^s\Big|_{\overline{\Gamma_i}}+\overline{E}_i^{inc}\Big|_{\overline{\Gamma_i}}+\sum_{j=1,\neq i}^{N}\overline{E}_j^s\Big|_{\overline{\Gamma_i}}\right)\times n_i=0$$
$$n_i\times\left(\overline{H}_i^s\Big|_{\overline{\Gamma_i}}+\overline{H}_i^{inc}\Big|_{\overline{\Gamma_i}}+\sum_{j=1,\neq i}^{N}\overline{H}_j^s\Big|_{\overline{\Gamma_i}}\right)=\overline{J}_i\Big|_{\overline{\Gamma_i}} \tag{2.1}$$

$$\left(\overline{E}_i^s\Big|_{\overline{\Gamma_{ij}}}+\sum_{j=1,\neq i}^{N}\overline{E}_j^s\Big|_{\overline{\Gamma_{ij}}}\right)\times n_i=0$$
$$n_i\times\left(\overline{H}_i^s\Big|_{\overline{\Gamma_{ij}}}+\sum_{j=1,\neq i}^{N}\overline{H}_j^s\Big|_{\overline{\Gamma_{ij}}}\right)=\overline{J}_i\Big|_{\overline{\Gamma_{ij}}} \tag{2.2}$$

Scattering fields in (2.1), $\overline{E}_m^s$ and $\overline{H}_m^s$ $(m=i\ ,\ j)$, arise from the sources on $\overline{\Gamma_m}$, however fields in (2.2) are only the product of sources on interfaces, $\overline{\Gamma_{ij}}$ and $\overline{\Gamma_{ji}}$.

B. *Formulation for Composite Object in G-CFIE*

Quite a few SIE formulations have been proposed for solving EM scattering from homogeneous region. However not all of them perform well when they encounter field problem of composite conducting-dielectric structure. G-CFIE[13] as a stable SIE method will be discussed in IE-DDM framework when analyzes these problems.

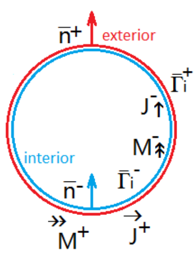

Fig.2. Homogeneous dielectric: G-CFIE Scheme

Unlike other SIEs, the homogeneous region is considered two subdomains, exterior and interior part, Fig.2., each of them has its own SIE formulations (3.1) and (3.2).

$$\left(\overline{E}_i^s\Big|_{\overline{\Gamma_i^+}}+\overline{E}_i^{inc}\Big|_{\overline{\Gamma_i^+}}\right)\times n_i^+=\overline{M}_i^+\Big|_{\overline{\Gamma_i^+}}$$
$$n_i^+\times\left(\overline{H}_i^s\Big|_{\overline{\Gamma_i^+}}+\overline{H}_i^{inc}\Big|_{\overline{\Gamma_i^+}}\right)=\overline{J}_i^+\Big|_{\overline{\Gamma_i^+}} \tag{3.1}$$

$$\left(\overline{E}_i^s\Big|_{\overline{\Gamma_i^-}}\right)\times n_i^-=\overline{M}_i^-\Big|_{\overline{\Gamma_i^-}}$$
$$n_i^-\times\left(\overline{H}_i^s\Big|_{\overline{\Gamma_i^-}}\right)=\overline{J}_i^-\Big|_{\overline{\Gamma_i^-}} \tag{3.2}$$

Now we apply G-CFIE into IE-DDM framework then derive the matrix equation of IE-DDM for composite object as (4), assuming Γ_1 is a PEC sub-domain and Γ_2 is homogeneous dielectric region.

$$\begin{bmatrix} A_{\Gamma_1\Gamma_1}^{J^+J^+} & & & & \\ & A_{\Gamma_2^+\Gamma_2^+}^{J^+J^+} & A_{\Gamma_2^+\Gamma_2^+}^{J^+M^+} & & \\ & A_{\Gamma_2^+\Gamma_2^+}^{M^+J^+} & A_{\Gamma_2^+\Gamma_2^+}^{M^+M^+} & & \\ & & & A_{\Gamma_2^-\Gamma_2^-}^{J^-J^-} & A_{\Gamma_2^-\Gamma_2^-}^{J^-M^-} \\ & & & A_{\Gamma_2^-\Gamma_2^-}^{M^-J^-} & A_{\Gamma_2^-\Gamma_2^-}^{M^-M^-} \end{bmatrix}\begin{bmatrix} \overline{J^+}_{\Gamma_1} \\ \overline{J^+}_{\Gamma_2^+} \\ \overline{M^+}_{\Gamma_2^+} \\ \overline{J^-}_{\Gamma_2^-} \\ \overline{M^-}_{\Gamma_2^-} \end{bmatrix}$$
$$=\begin{bmatrix} b_{\Gamma_1}^J \\ b_{\Gamma_2^-}^J \\ b_{\Gamma_2^-}^M \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} D_{\overline{\Gamma_1}\overline{\Gamma_{12}}}^{J^+J^+} & B_{\overline{\Gamma_1}\overline{\Gamma_2}}^{J^+J^+} & B_{\overline{\Gamma_1}\overline{\Gamma_2}}^{J^+M^+} & & \\ B_{\overline{\Gamma_2}\overline{\Gamma_1}}^{J^+J^+} & & & C_{\Gamma_2^+\Gamma_2^-}^{J^+J^-} & C_{\Gamma_2^+\Gamma_2^-}^{J^+M^-} \\ B_{\overline{\Gamma_2}\overline{\Gamma_1}}^{M^+J^+} & & & C_{\Gamma_2^+\Gamma_2^-}^{M^+J^-} & C_{\Gamma_2^+\Gamma_2^-}^{M^+M^-} \\ & C_{\Gamma_2^-\Gamma_2^+}^{J^-J^+} & C_{\Gamma_2^-\Gamma_2^+}^{J^-J^+} & & \\ & C_{\Gamma_2^-\Gamma_2^+}^{J^-J^+} & C_{\Gamma_2^-\Gamma_2^+}^{J^-J^+} & & \end{bmatrix}\begin{bmatrix} \overline{J^+}_{\Gamma_1} \\ \overline{J^+}_{\Gamma_2^+} \\ \overline{M^+}_{\Gamma_2^+} \\ \overline{J^-}_{\Gamma_2^-} \\ \overline{M^-}_{\Gamma_2^-} \end{bmatrix} \tag{4}$$

Where, the matrix blocks $A_{\Gamma_i\Gamma_i}^{J^\pm J^\pm}$ are self-coupling and $B_{\overline{\Gamma_i}\overline{\Gamma_j}}^{J^\pm J^\pm}$ stands for mutual coupling in (6). Blocks $C_{\overline{\Gamma_2^+}\overline{\Gamma_2^+}}^{J^+J^+}$ or $B_{\overline{\Gamma_{ij}}\overline{\Gamma_{ji}}}^{J^\pm J^\pm}$ is sparse motar matrices derived from Robin or Neumann transmission conditions, respectively.

III. Computation of Mortar Matrix on Non-Conformal Touching Face

In non-overlapping IE-DDM, to enforce field continuity on nonconformal touching face is very important for the accuracy of DDM. Here, a cement technique [14] is used to allow nonconformal interpolation to overcome this difficulty.

Firstly, we assume that all subdomains are have non-conforming meshes with each other. Meanwhile, we could introduce the discrete spaces: each Ω_i is a space provided with its own grid T_i,1<i<N, such that $\Omega_i=\cup_{\mathrm{T}_i^m\in\mathrm{T}_i}w_i^m$ and for grid $w_i^m\in\mathrm{T}_m$, $1<m<\#\mathrm{T}_i$. We consider that the w_i^m sets belonging to the meshes of simple type (triangles or tetrahedras).

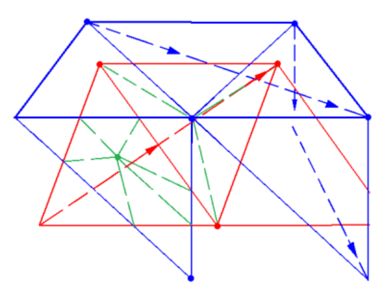

Fig.3. Nonconformal meshes interpolation

Then, we define over each subdomain two non-conforming spaces X_k and Y_k by the space of traces over each Γ_{ij} of elements as follows:

$$X_i = \left\{ v_m \middle| v_m \in \mathrm{T}_i, v_i \in \Gamma_{ij}, 1 < m < \#\left(\mathrm{T}_i \cap \Gamma_{ij}\right) \right\}$$
$$Y_j = \left\{ u_m \middle| u_m \in \mathrm{T}_j, u_m \in \Gamma_{ji}, 1 < m < \#\left(\mathrm{T}_j \cap \Gamma_{ji}\right) \right\} \quad (5)$$

Then , we can get a set of elements P_{ij} by projecting space X_i and Y_j with each other , that $P_{ij} = \left\{ p_m \middle| p_m \in (X_i \cap Y_j) \right\}$ and $v_p = \sum_{i=1}^{m} p_i$, $v_p \in X_i$.

The projection set P_{ij} between nonconforming space makes the mortar element method easily. The Robin TC formulation in Galerkin testing is as :

$$\alpha\left(\int_{\Gamma_{ij}} \vec{w}_i^k(\vec{r}) \cdot \vec{E}_i^s\left(\vec{v}_i^l(\vec{r}')\right)\Big|_{\Gamma_{ij}} d\vec{r} - \sum_{j=1, j\neq i}^{N} \int_{\Gamma_i^+} \vec{w}_i^k(\vec{r}) \cdot \vec{E}_j^s\left(\vec{p}_{ij}(\vec{r}')\right)\Big|_{\Gamma_{ij}} d\vec{r} \right) \quad (6)$$
$$+(1-\alpha)\left(\int_{\Gamma_i^+} \vec{w}_i^k(\vec{r}) \cdot \left(\overset{+}{n_i} \times \vec{H}_i^s\left(\vec{v}_i^l(\vec{r}')\right)\Big|_{\Gamma_{ij}} \right) d\vec{r} - \int_{\Gamma_i^+} \vec{w}_i^k(\vec{r}) \cdot \vec{v}_i^l(\vec{r}')\Big|_{\Gamma_{ij}} d\vec{r} \right)$$
$$+(1-\alpha)\left(\sum_{j=1, j\neq i}^{N} \int_{\Gamma_i^+} \vec{w}_i^k(\vec{r}) \cdot \left(\overset{+}{n_i} \times \vec{H}_j^s\left(\vec{p}_{ij}(\vec{r}')\right)\Big|_{\Gamma_{ij}} \right) d\vec{r} \right) = 0$$

Where, $\vec{w}_i^k$ is testing function and $\vec{v}_i^l$ is original basic function in $\overline{\Gamma}_{ij}$, $\vec{p}_{ij}$ is basic function which belongs to P_{ij} , projecting space. In (6), double integral terms in scattering fields can be done in original basic functions and surface current terms , single integral term , should be expanded in projecting space.

IV. NUMERICAL RESULTS

In this section the present method is verified and compared with other available data. A coated PEC sphere and two aircrafts models are investigated.

A. Coated PEC sphere model

Consider the case of a perfectly conducting sphere with two different materials, which are permittivity 6.0 and permittivity 4.0 respectively. A combination of three parts that two are dielectric and another is PEC is set for model, λ_0 is wavelength in free space. The bistatic RCS is calculated with 38,835 unknowns and result is correspond to MIE result, in Fig.4.

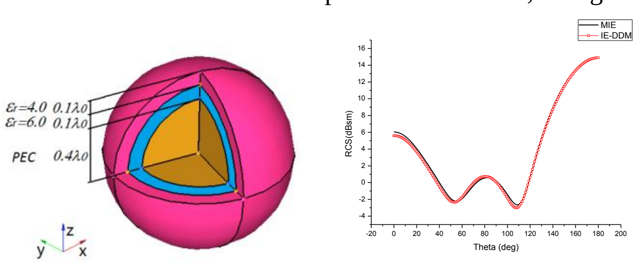

Fig.4.Domain decomposition for coated PEC sphere model

B. EM scattering from simplified aircraft model

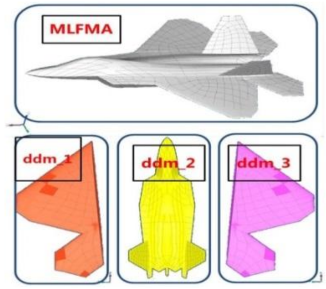

Fig.5.Domain decomposition for simplified aircraft model

In order to show the efficiency and accuracy of nonconformal IE-DDM, the bistatic RCS of a simplified PEC aircraft model is solved. An incident plane wave at 1.0 GHz frequency illuminates from $\theta = 90°$, $\phi = 0°$, the whole model is divided into 3 parts showed in Fig.4. The number of unknowns by the MLFMA and IE-DDM is about 1.6 million and 1.38 million respectively. A good agreement between MLFMA and IE-DDM is also achieved in Fig.5. The SAI preconditioner is used to accelerate the iteration of matrix equation.

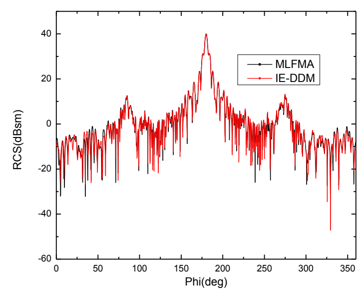

Fig.6.Bistatic RCS for simplified aircraft model

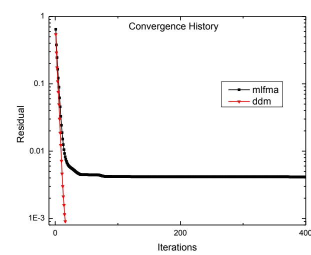

Fig.7. The convergence of IE-DDM and MLFMA for simplified aircraft model

	Memory/GB	CPU Time/h:m
MLFMA	21.6	22:10
IE-DDM	18	15:40

Table 1: Computational Statics of IE-DDM and MLFMA

C. Simplified helicopter model

As shown in Fig. 7, the entire model is divided into 6 closed regions. Each region is meshed independently according to geometry complexities and available computational resources. Due to the non-conformal feature of the proposed IE-DDM, each of the sub-regions can be meshed independently.

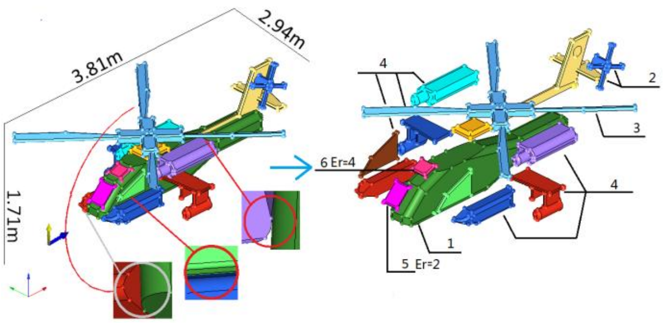

Fig.8.Geometry Model of a simplified helicopter
(Incident wave from θ=90°,ϕ=0° in 1.5GHz frequency.)

Part 5 and 6 are dielectric structures, their permittivities are 2.0 and 4.0 respectively, while others are all PEC structures. Then the results of scattering fields by different parts of this model are given in Fig.8.

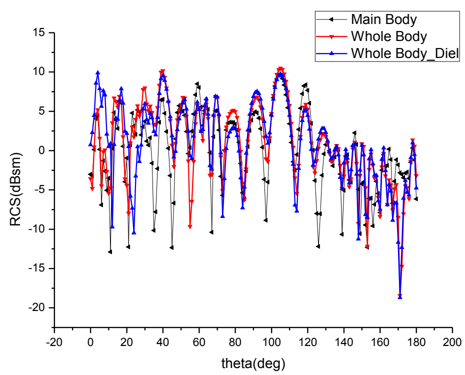

Fig.9.Results comparison of different combined model
(Main and Whole Body are all PEC Parts combination, difference in part 4, while Whole Body_Diel has dielectric structure substituted for PEC, part 5 εr=2.0 and part 6 εr=4.0)

Part	1	2	3	4	5	6	Total
Unknowns	61,389	8,148	28,335	63,510	3,684	5,520	170,586

Table 2: Unknowns of each Part

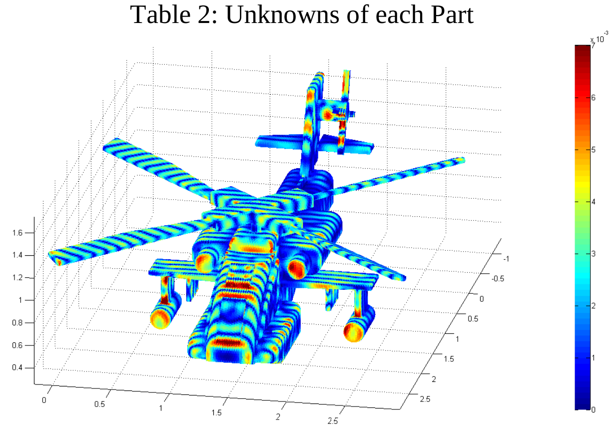

Fig. 10.The current distribution of helicopter model

V. CONCLUSION

In this paper, nonconformal IE-DDM is applied for analysis of electromagnetic scattering from complicated conductor and composite objects. The great benefit of this framework is that it divides the computational domain into subdomains independently, simplifies model processing and reduces the heavy burden of mesh generation. Moreover, it improves greatly the property of matrix, can achieve stable and accurate solution of complicated multi-scale objects in real world.

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