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Steady-State Analysis using Haar Wavelet Transform in Power Electronics Circuits including Nonlinear Elements

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Abstract– Recently, much attention has been paid to the methods for circuit analysis using wavelet transform. In particular we have proposed some approaches using Haar wavelet, and have showed its effectiveness for the analyses of the circuits including periodic switching of dynamics, such as power electronics circuits. In this paper, we propose the method to deliver the numerical solution of the steady-state periodic solution of the power electronics circuit including nonlinear elements which is driven by a periodic force.

1. Introduction

Recently, much attention has been paid to the methods for circuit analysis using wavelet transform [1-3, 6]. In this paper, we propose the method to derive steady-state periodic solution of the power electronics circuit including the nonlinear element. If we calculate such steady-state waveforms using time-marching methods as Runge-Kutta method, the calculation cost is wasted due to the calculation of the long-term transient response with sufficiently small step size to approximate the discontinuous dynamics caused by switches. To overcome such disadvantage of the time-marching method, in Ref. [6], the Chebyshev polynomials are used as the wavelet basis functions, and the periodic solutions of periodically driven power electronics circuits have been calculated. However it is considered that the calculation should be complicated and the Gibbs-phenomenon-like erros have been seen when the switching is occurred because of the use of the Chebyshev polynomials. In contrast, the proposed method using Haar wavelet can make the calculation simpler and shorten calculation time due to drive steady-state periodic solution without calculating the long-term transient response like time-marching methods. Therefore, it can be said that this method is effective for circuit analysis of the periodic solutions in power electronics circuits. In this paper, we show even if the differential equation becomes nonlinear by nonlinear elements are added to the power electronics circuit, we can solve steady-state periodic solution.

2. Haar Wavelet Matrix

Haar functions are defined on interval [0,1) as follows,

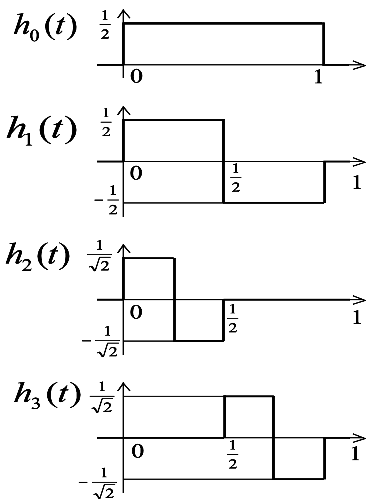

Fig. 1 : Haar wavelet functions for $\alpha = 2$.

$$h_0 = \frac{1}{\sqrt{m}} \tag{1}$$

$$h_i = \frac{1}{\sqrt{m}} \times \begin{cases} 2^{\frac{j}{2}}, & \frac{k-1}{2^j} \le t < \frac{k-\frac{1}{2}}{2^j}, \\ -2^{\frac{j}{2}} & \frac{k-\frac{1}{2}}{2^j} \le t < \frac{k}{2^j}, \\ 0 & otherwise\ in\ [0,1) \end{cases} \tag{2}$$
$$i = 0,1,\cdots,m-1, m = 2^{\alpha}$$

where α is positive integer, and j and k are nonnegative integers which satisfy $i = 2^j + k$, i.e., $k = 0,1,\cdots,2^j - 1 (j = 0,1,2,\cdots.)$. Figure 1 shows the waveforms of the Haar functions for $\alpha = 2$.

$\vec{y}$ is $m \times 1$-dimensional vector whose elements are the discretized expression of $y(t)$ and $\vec{c}$ is $m \times 1$-dimensional coefficient vector. H is $m \times m$-dimensional Haar wavelet matrix defined as

$$H = \begin{bmatrix} \overrightarrow{h_0} \\ \overrightarrow{h_1} \\ \vdots \\ \overrightarrow{h_{m-1}} \end{bmatrix} \triangleq \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & \cdots & h_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mm} \end{bmatrix} \tag{3}$$

where $\overrightarrow{h_i}$ is $1 \times m$ -dimensional Haar wavelet basis vector whose elements are the discretized expression of $h_i(t)$. Note that H is an orthonormal matrix. Using these vectors and matrix, Haar wavelet transform and inverse Haar wavelet transform are described as follows, respectively.

$$\vec{c} = H\vec{y}, \qquad (4)$$
$$\vec{y} = H^T\vec{c}(= H^{-1}\vec{c}). \qquad (5)$$

3. Integral and Differential Operator Matrices using Haar Wavelet

The basic idea of the operator matrix has been firstly introduced by using Walsh function [4]. However, in logical way, the matrices introduced by block pulse function are more fundamental. The block pulse function is the set of m rectangular pulses which have $1/m$ width and are shifted $1/m$ each other.

The integral operator matrix of the block pulse function matrix B is defined as the following equation [4, 5].

$$\int_0^i B(\tau)d\tau \equiv Q_B \cdot B(t) \qquad (6)$$

$$Q_{B(m\times m)} = \frac{1}{m}\left[\frac{1}{2}I_{(m\times m)} + \sum_{i=1}^{\infty} P^i_{(m\times m)}\right] \qquad (7)$$

where $B(t)$ is $m \times m$-dimensional matrix whose elements are the discretized expression of the block pulse function $b_i(t),\ i = 0,1,\cdots,m-1$ and

$$P^i_{(m\times m)} = \begin{bmatrix} 0 & I_{(m-i)\times(m-i)} \\ 0_{(i\times i)} & 0 \end{bmatrix}$$

for $i<m$,

$$P^i_{(m\times m)} = 0_{(m\times m)}$$

for $i \geq m$. And the inverse matrix $Q^{-1}_{B(m\times m)}$ is calculated as follows [5].

$$Q^{-1}_{B(m\times m)} = 4m\left[\frac{1}{2}I_{(m\times m)} + \sum_{i=1}^{m-1}(-1)^i P^i_{(m\times m)}\right] \qquad (8)$$

As the Haar wavelet matrix H is the set of the orthogonal functions, the integral matrix of H is given as follows:

$$Q_H = HQ_B^TH^{-1} = HQ_B^TH^T \qquad (9)$$

Similarly, the differential matrix of H can be written as

$$Q_H^{-1} = H(Q_B^T)^{-1}H^{-1} = H(Q_B^T)^{-1}H^T \qquad (10)$$

4. Haar Wavelet Expression of Blanch Characteristics of Nonlinear Time Varying Circuit Elements

Next, we show the Haar wavelet expression of branch characteristics of nonlinear time varying circuit elements for the expression in wavelet domain.

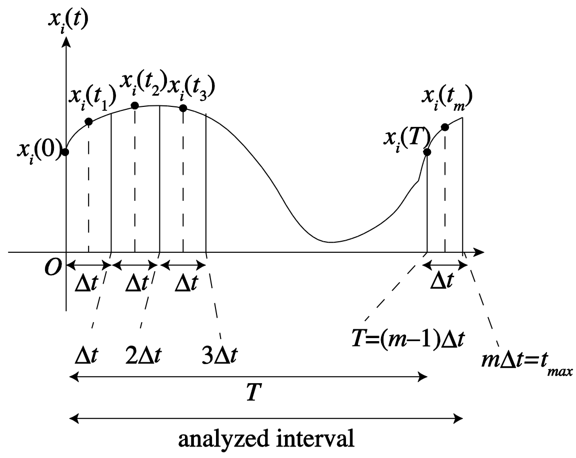

Fig.2: Definition of the analyzed interval and the time step.

Capacitor:

$$v(t) = v(0_-) + \frac{1}{C}\int_0^t i(\tau)d\tau, \quad v_0 \coloneqq v(0_-)$$
$$V = V_0 + C_w^{-1}Q_HI$$
$$\text{or} \quad I = C_wQ_H^{-1}[V - V_0] \qquad (11)$$
$$C_w = Hdiag[C(i_0,t_0), C(i_1,t_1), \cdots, C(i_{m-1},t_{m-1})]H^T$$

Inductor:

$$i(t) = i(0_-) + \frac{1}{L}\int_0^t v(\tau)d\tau, \quad i_0 \coloneqq i(0_-)$$
$$I = I_0 + L_w^{-1}Q_HV$$
$$\text{or} \quad V = Q_H^{-1}L_w[I - I_0] \qquad (12)$$
$$L_w = Hdiag[L(i_0,t_0), L(i_1,t_1), \cdots, L(i_{m-1},t_{m-1})]H^T$$

Resistor:

$$v(t) = Ri(t)$$
$$V = R_wI,\ R_w = diag[R] \qquad (13)$$
$$R_w = Hdiag[R(i_0,t_0), R(i_1,t_1), \cdots, R(i_{m-1},t_{m-1})]H^T$$

5. Method to Find Steady-State Periodic Solutions

Consider the following ordinary differential equation,

$$\dot{x} = f(x,t) \triangleq A(x,t)x + u(t) \qquad (14)$$

where $x(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T \in R^{n\times 1}$ is an unknown state variable vector, $A(x,t) \in R^{n\times n}$ is a nonlinear time varying parameter matrix, and $u(t) = [u_1(t)\ u_2(t)\ \cdots\ u_n(t)]^T \in R^{n\times 1}$ is an external force vector. The system is driven by the periodic external force or parameter with period T. Assume that we can find the periodic solution $x_p(t)$ with period T, i.e., $x_p(t) = x_p(t+T)$ for all t. In order to find the steady-state periodic solution, we should find the solution for the interval [0,1] under the appropriate boundary conditions. For the wavelet expression of the differential equations, we define the discretized expression of $x(t)$ and $u(t)$ as $\overrightarrow{x_i} = [x_i(t_1)\ x_i(t_2)\ \cdots\ x_i(t_m)]^T \in R^{m\times 1}$ and $\overrightarrow{u_i} =$

$[u_i(t_1)\ u_i(t_2)\ \ \cdots\ \ u_i(t_m)]^T \in R^{m\times 1}$ for $i = 1,2,\cdots,m$ respectively.

The wavelet transformed expression of Eq. (14) can be derived as

$$Q_m^{-1}[X - X_0] = A_H X + U \tag{15}$$

where $X = [(H\overrightarrow{x_1})^T\ \ (H\overrightarrow{x_2})^T\ \ \cdots\ \ (H\overrightarrow{x_n})^T]^T \triangleq [X_1^T\ \ X_2^T\ \ \cdots\ \ X_n^T]^T \in R^{mn\times 1}$ is an unknown wavelet coefficients vector, $X_0 = [(H\overrightarrow{x_{10}})^T\ \ (H\overrightarrow{x_{20}})^T\ \ \cdots\ \ (H\overrightarrow{x_{n0}})^T]^T \triangleq [X_{10}^T\ \ X_{20}^T\ \ \cdots\ \ X_{n0}^T]^T \in R^{mn\times 1}$ and $U = [(H\overrightarrow{u_1})^T\ \ (H\overrightarrow{u_2})^T\ \ \cdots\ \ (H\overrightarrow{u_n})^T]^T \in R^{mn\times 1}$.

Note that $\vec{x}_{i0}$ is also unknown for this case. Moreover,

$$Q_m^{-1} = \begin{bmatrix} Q_H^{-1} & 0 & \cdots & 0 \\ 0 & Q_H^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_H^{-1} \end{bmatrix} \in R^{mn\times mn} \tag{16}$$

and, $A_H \in R^{mn\times mn}$ is the wavelet region expression of A derived from Sect. 4. At this moment, as both X and X_0 are unknown, we cannot solve this algebraic equations.

To determine the boundary condition, we set the analyzed interval as shown in Fig. 3. The time step $\Delta t = \frac{T}{m-1}$ and $t_{max} = T + \Delta t$. Because of the feature of the matrix $Q_{B(m\times m)}$, time t_i is calculated as $t_i = \frac{\Delta t}{2} + (j-1)\Delta t\ \ (j = 1,2,\cdots,m)$. Due to the periodicity, the relationship $x_i(t_1) = x_i(t_m)$ for all $i = 1,2,\cdots,n$ is derived. From Eq. (5), this relationship is rewritten as follows.

$$[h_{11}\ h_{21}\ \ \cdots\ \ h_{m1}]X_i = [h_{1m}\ h_{2m}\ \ \cdots\ \ h_{mm}]X_i \tag{17}$$

then,

$$[h_{11} - h_{1m}\ \ h_{21} - h_{2m}\ \ \cdots\ \ h_{m1} - h_{mm}]X_i = 0 \tag{18}$$

Setting $[h_{11} - h_{1m}\ \ h_{21} - h_{2m}\ \ \cdots\ \ h_{m1} - h_{mm}] \triangleq h_b \in R^{1\times m}$ and $\mathrm{diag}(h_b) \triangleq H_b \in R^{n\times mn}$, the relationship

$$H_b X = 0 \tag{19}$$

is derived.

To derive the unknown vector $\vec{x}_0$, we consider the relationship between X and X_0. From Eq. (15), we see the matrix $Q_H^{-1}X_{i0}$ from $Q_m^{-1}X_0$. From the relationship $X_{i0} = H\vec{x}_{i0}$,

$$Q_H^{-1}X_{i0} = Q_H^{-1}H\vec{x}_{i0} \tag{20}$$

If we set $Q_H^{-1}H \triangleq [q_{ij}] \in R^{m\times m}$,

$$Q_H^{-1}H\vec{x}_{i0} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{bmatrix} \begin{bmatrix} x_i(0) \\ x_i(0) \\ \vdots \\ x_i(0) \end{bmatrix}$$

$$= \begin{bmatrix} q_{11} + q_{12} + \cdots + q_{1m} \\ q_{21} + q_{22} + \cdots + q_{2m} \\ \vdots \\ q_{m1} + q_{m2} + \cdots + q_{mm} \end{bmatrix} x_i(0) \triangleq q_0 x_i(0) \tag{21}$$

Then we define $Q_0 = \mathrm{diag}(q_0) \in R^{mn\times n}$, Eq.(15) is rewritten as

$$(Q_m^{-1} - A_H)X - Q_0\vec{x}_0 = U \tag{22}$$

From Eqs.(22) and (19), we can derive $n(m-1)$-dimensional algebraic equations as follows,

$$\begin{bmatrix} Q_m^{-1} - A_H & -Q_0 \\ H_b & 0 \end{bmatrix} \begin{bmatrix} X \\ \overrightarrow{x_0} \end{bmatrix} = \begin{bmatrix} U \\ 0 \end{bmatrix} \tag{23}$$

In this equation, the number of the unknown of the variables coincides with the dimension of the equation. Therefore, we can derive all the unknown variables to solve it. Finally, we derive the approximated solution of Eq. (14) from Eq. (5).

5. Example

In this section, we show the simple example to confirm the effectiveness of the proposed method. The simple boost converter circuit show in Fig. 3 is analyzed in this example. In this circuit, resistance R gives nonlinear characteristic as follows,

$$i(t) = (g_1 + g_3 {v_C}^2)v_C \tag{24}$$

The circuit parameter is shown in Table 1 and we set $g_1 = 0.08$ and $g_3 = 0.0016$. The circuit equations written as follows,

$$\begin{bmatrix} i_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} -\dfrac{R_s(1 - s(t)) + R_s s(t)}{L} & -\dfrac{s(t)}{L} \\ \dfrac{s(t)}{C} & -\dfrac{g_1 + g_3 v_C^2}{C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{E - s(t)V_f}{L} \\ 0 \end{bmatrix} \tag{25}$$

where

$$s(t) = \begin{cases} 0, & for\ 0 \le t \le T_D \\ 1, & for\ T_D \le t \le T \\ s(t-T), & for\ all\ t > T \end{cases} \tag{26}$$

Then, the Haar wavelet expression of branch characteristics of nonlinear load resistance can be derived from Eq. (24) as

$$H\vec{\imath} = H\mathrm{diag}[g_1 + g_3 {v_{C1}}^2, g_1 + g_3 {v_{C2}}^2, \cdots, g_1 + g_3 {v_{Cm}}^2]\vec{v}_C \tag{27}$$

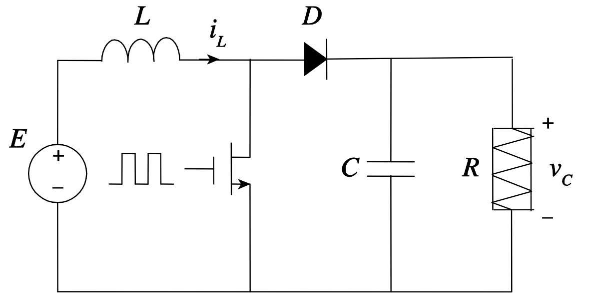

Fig. 3: A simple boost converter with nonlinear load R.

Table 1: Parameter values for boost converter.

Parameter	value
Inductance L	0.2mH
Capacitance C	0.2mF
Input voltage E	16V
Diode forward drop V_f	0.8V
Switching period T	100μs
On-time T_d	45μs
Switch on-resistance R_s	0.001 Ω
Diode on-resistance R_d	0.001Ω

where $\vec{\imath}$ and $\vec{v}_c$ are the discretized expression of the current through and the voltage across the load, respectively. As the matrix H is orthonormal, Eq. (27) can be rewritten as follows,

$$H\vec{\imath} = H\text{diag}[g_1 + g_3{v_{C1}}^2, g_1 + g_3{v_{C2}}^2, \cdots, g_1 + g_3{v_{Cm}}^2]H^T H\vec{v}_C \quad (28)$$

If we set $I = H\vec{\imath}$ and $V_C = H\vec{v}_C$, and define the matrix G_w as

$$G_w = H\text{diag}[g_1 + g_3 v_{C1}^2, g_1 + g_3 v_{C2}^2, \dots, g_1 + g_3 v_{Cm}^2]H^T,$$

we can derive the wavelet transformed form of Eq. (24) as

$$I = G_w V_C \quad (29)$$

as shown in Eq. (13). Using this relationship, the wavelet expression of Eq. (25) becomes a nonlinear algebraic equation. This equation can be solved by the method such as Newton method.

Figure 4 shows the calculation results for the proposed method. The most precise approximation is the case for α=8 in the example. In Fig.4, we can see that the approximation approaches to the result of spice as the value of α become larger and larger. About the stability of the obtained steady state solution and elucidation of the difference of both ends of the interval seems to be the future works.

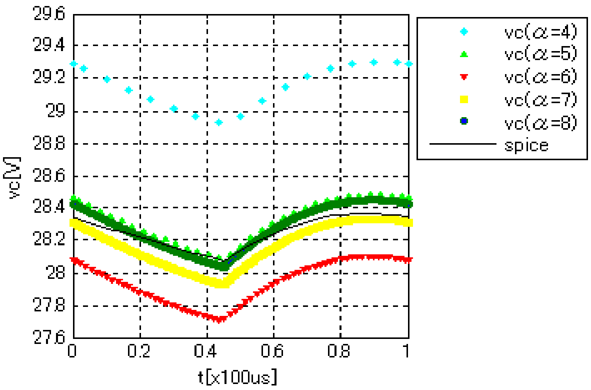

Fig. 4: Calculation results for the proposed method for $\alpha = 4, 5, 6, 7, 8$.

6. Conclusion

In this paper, we have proposed the method to derive steady-state periodic solution of the power electronics circuit including the nonlinear element, and confirmed its performance using the simple power electronics circuit example. The substantiate of the proposed method to the more stiff nonlinear power electronics circuits and proposing the method can be applied a realistic diode seem to be the future works.

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