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Emerging scales and dynamics from adaptively networked systems

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Abstract—We present a model for the emergence of three important features found in many social and biological systems, and especially in neural networks: modular structures, scale-free distribution of connections strengths, and computational capability. All such features naturally emerge from the interaction of different oscillators, and from the synchronization between their dynamics. Our results are of relevance in enlightening possible biological mechanisms at the basis of the processing and integration of information across distributed neural systems.

1. Introduction

In the last decades, researchers of all fields have recognized the importance of systems composed of elements displaying an oscillatory dynamics, interacting between them on the top of a (complex) network [1, 2]. Many heterogeneous systems can be studied by means of such models: from cells of an organism, to social and technological systems. Most of these systems share some common characteristics: namely, they present a power law (scale-free) scaling in the network connectivity, and elements are organized in modules (community structures) at a mesoscopic scale [3]. Remarkably, such characteristics are not hard-coded by an external entity: instead, they appear (or emerge) in a natural way from the interaction of the elements of the system.

One of the most astonishing examples of such emergence of power-law and modular structure is the brain. It is composed of thousands of millions of neurons, interacting between them by means of intermittent electric currents (known as spike trains). If such electrical activity is modeled by an oscillatory dynamics, another emergent behavior appers: the computational capability of the brain. While no model exists yet able to reproduce at once these three features, their interplay is believed to be the basis of the systems general functioning and performance.

In this contribution, we will show how some simple rules, governing the strengths of the connections between a set of oscillatory elements, is able to create structures with both scale-free and modular topologies [4, 5]. Furthermore, we will show that synchronization between the dynamics of different units can be used to encode information, ultimately yielding a form of computation [6, 7].

2. Emergence of modular and scale-free topologies

The model we introduce to explain the simultaneos appearance of modular and scale-free topologies as a phenomena emerging from synchronization is based on Kuramoto oscillators [8]. This election is motivated by the simplicity of the Kuramoto model, and by the wide array of results (both numerical, analytics, and experimental) available in the Literature, which make it a classical example in synchronization problems. In the Kuramoto model, the phase of the i-th oscillator ϕ_i is defined as

$$\dot{\phi}_i = \omega_i + \lambda \sum_{j \in N_i} w_{ij} \sin(\phi_j - \phi_i) \qquad (1)$$

On the one side, the dynamics of each oscillator is controlled by its own angular velocity ω_i; on the other side, each oscillator also receives external inputs from neighbour nodes (denoted by N_i), trying to driving its dynamics toward a mean global dynamics. While this is standard in the Literature about networking Kuramoto oscillators, we have introduced a time-dependent dynamics in the coefficient governing the strength of the external inputs, i.e. w_{ij}. Specifically, each link connecting two nodes is subject to two forces. First, the strength of links connecting pairs of synchronized nodes is enhanced; this mechanism is known to be relevant in neuronal plasticity, i.e. Hebbian learning. Second, the resources available to each node for connecting to other nodes are limited: therefore, enhancing one connection implies the weakening of another link, so that the sum is maintained. Mathematically, this is defined as:

$$\dot{w}_{ij} = p_{ij} - \left(\sum_{k \in N_i} p_{ik} \right) w_{ij} \qquad (2)$$

w_{ij} denotes the average phase correlation between oscillators i and j over a characteristic memory time T. The first term in the r.h.s. of Eq. 2 therefore accounts for the Hebbian learning: the higher the synchronization, the higher the link weight; the second term limits the total weight associated to each node, thus limiting the available resources.

Once the system is defined, its dynamics is studied by means of two parameter. The first one is the standard time dependent Kuramoto order parameter, quantifying the global synchronization of the nodes in the network:

$$r(t) = \frac{1}{N} \left| \sum_{i=1}^{N} e^{i\phi_i(t)} \right| \qquad (3)$$

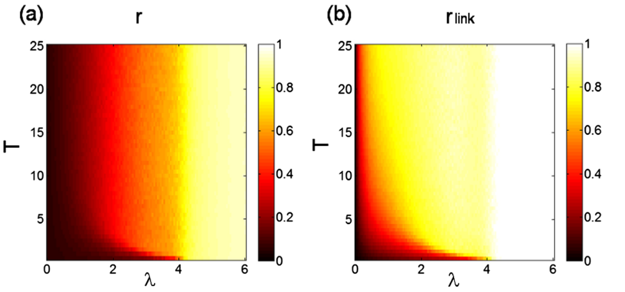


Figure 1: Global and local synchronization indicators, r (left) and r_{link} (right). The color coding is reported in the right columns.

The second one is a local parameter order, quantifying the average synchronization between connected nodes in the network:

$$r_{link} = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \in N_i} w_{ij} r_{ij} \tag{4}$$

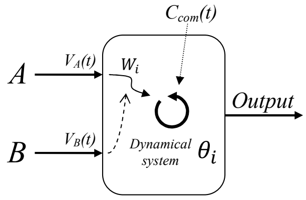
We consider a system with $N = 300$ and $K = 20$ and explore the structural and dynamical features as functions of λ (a global connectivity strength) and T. Figure 1 shows the behavior of r (left) and r_{link} (right) as a function of these two parameters. According to Fig. 1 Left, the adaptive mechanism has the effect of generically enhancing global synchronization in the network to a remarkable extent already for coupling strengths below the critical value. Yet, in Fig. 1 Right it is shown that the growth of r_{link} with λ is, however, much faster than that of r, which delimits (for $\lambda < 1$ and $T > 1$) a wide region with low global synchronization, but strong local synchronization: in other words, in this region there is an emergence of modular (cluster) synchronization.

This emergence of modularity is also accompained by the appearance of a weight distribution (i.e., the topology resulting from the competitive adaptation mechanism) following a power-law scaling.

3. Emergence of a computation capability

After demonstrating that synchronization between different dynamical units can be used to create a modular and scale-free structure, we will see how synchronization can be also used to perform simple Boolean computation. While here we will just consider Kuramoto oscillators, the model has been extended to other chaotical oscillators [6] and neuron models [7].

First of all, in order to achieve a computation capability, it is necessary to define a way for representing information: in a Boolean context, this means a way for representing 0 and 1 bits. Here, we code such bits by means of the level of synchronization of each network's unit with two signals, $S(t)$ and $R(t)$. The fundamental element of computation is



$\chi(V_A, V_B)$	Output
≈ 0	$S(t)$
> 0	$V_A(t)$

Figure 2: (Left) Schematic representation of the connectivity of each oscillator. (Right) Expected output as a function of the input signals.

sketched in Fig. 2, and consists of a dynamical system (in this case, a Kuramoto phase oscillator), and two input ports (A and B). The assumption is that almost all networked units are subjected to *a same* external synchronizing signal, in a way that their dynamics (in the absence of any further interaction) would result in a time series synchronous with $S(t)$ (which, from here on, will be taken as the 0-state of the computation). We also assume that a second reference signal $R(t)$, constituting the 1-state, is present in the network, as produced by the evolution, for instance, of *at least one unit* that is not suffering the effect of the common forcing.

As shown in Fig. 2 Left, each Kuramoto oscillator is also forced by an external signal, entering from port A; the strength W of this coupling is the result of the following adaptive dynamics:

$$\frac{dW}{dt} = -W(W - w_1)(W - w_2) + k\,[\Delta(A, B) - thr]\,. \tag{5}$$

$\Delta(A, B)$ represents the phase synchronization error between signals entering ports A and B. When $w_1 = 0.5$ and $w_2 = 1.0$, the first term of the r.h.s. of Eq. 5 creates three equilibrium points, two of them stable (corresponding to $W = 0$ and $W = 1$). The second term forces the system toward one of these two equilibrium points, depending on the synchronization error Δ. Therefore, the dynamics of W is defined in such a way that the coupling strength tends to zero when the synchronization error between the input signals entering ports A and B vanishes, i.e., when both inputs are synchronized, and to a positive value otherwise (Fig. 2 Right).

Once defined the structure of the basic computation unit, we are interested in how this unit can perform simple computations, and how a group of them, interacting above a network, can carry out more complicated tasks. In order to achieve this result, we will focus on the construction of a NAND gate, whose output is zero only when both inputs are ones. This gate is one of the *universal Boolean gates*, in that any other Boolean computation can be performed by using a combination of NAND gates, and therefore any Turing machine can be constructed from them [see [**?**]]. The reference signal R is fed inside port A, and the two input signals are summed (both of them with weight $1/2$) and presented to port B. Only when the two inputs are one, i.e., are they synchronized to the reference signal *R*, *W* tends to zero, and therefore the output follows the *ID* dynamics (i.e., a zero output).

4. Conclusions

In conclusion, we have shown how a network competitive adaptation leads to the emergence of those meso- and macroscale features that are commonly observed in real neural systems. Furthermore, we have shown as a novel computational paradigm may emerge from similar adaptation mechanisms, where the coding and processing of information emerge from adaptive synchronization processes. Our results are of relevance in enlightening possible biological mechanisms at the basis of the processing and integration of information across distributed neural systems, where neural assemblies are known to organize their dynamics in a balance between synchronization and de-synchronization [9, 10, 11], with modifications associated with a number of neurological illnesses, including schizophrenia and Alzheimer disease [**?**].

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References

[1] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, C. S. Zhou, "The synchronization of chaotic systems," *Phys. Rep.*, vol.366, 2002.

[2] L. M. Pecora, T. L. Carroll, "Synchronization in chaotic systems," *Phys. Rev. Lett.*, vol.64, 1990.

[3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, D.-U. Hwang, "Complex networks: Structure and dynamics," *Phys. Rep.*, vol.424, 2006.

[4] R. Gutiérrez, A. Amann, S. Assenza, J. Gómez-Gardeñes, V. Latora, S. Boccaletti, "Emerging Meso- and Macroscales from Synchronization of Adaptive Networks", *Phys. Rev. Lett.*, vol.107, 2011.

[5] S. Assenza, R. Gutiérrez, J. Gómez-Gardeñes, V. Latora, S. Boccaletti, "Emergence of structural patterns out of synchronization in networks with competitive interactions", *Sci. Rep.*, vol.1, 2011.

[6] M. Zanin, D. Papo, I. Sendiña-Nadal, S. Boccaletti, "Computation as an emergent feature of adaptive synchronization," *Phys. Rev. E*, vol.84, 2011.

[7] M. Zanin, F. del Pozo, S. Boccaletti, "Computation Emerges from Adaptive Synchronization of Networking Neurons," *Phys. Rev. E*, vol.6, 2011.

[8] Y. Kuramoto, "Chemical Oscillations, Waves and Turbulence," Springer, New York, 1984.

[9] K. J. Friston, "The labile brain. I. Neuronal transients and nonlinear coupling," *Phil. Trans. Roy. Soc. Lon. B*, vol.355, 2000.

[10] M. Breakspear, L. M. Williams and C. J. Stam, "A novel method for the topographic analysis of neural activity reveals formation and dissolution of 'dynamic cell assemblies'," *J. Comput. Neurosci.*, vol.16, 2004.

[11] J. Benda, A. Longtin and L. Maler, "A synchronization-desynchronization code for natural communication signals," *Neuron*, vol.52, 2006.