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NOISE-INFLUENCED TRANSIENT ENERGY LOCALIZATION IN AN OSCILLATOR ARRAY

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Abstract
The effects of noise on transient energy localization in an array of coupled nonlinear oscillators are examined in this work. The oscillators in the considered arrays are identical to each other. Results obtained through simulations of deterministic systems are compared to those obtained through Euler-Maruyama scheme based simulations with the corresponding stochastic systems. To complement the numerical studies, a Fokker-Planck formalism is also used to analyze the response of the system in the presence of noise. Transient localization phenomena are explored by using time domain and time-frequency analyses, and the insights gained are discussed. Although this type of localization can be detrimental to the performance of a system, the intent of this study is to further our understanding of this behavior and use it for the benefit of a nonlinear system.

1. Introduction

Noise can produce significant changes in the response of nonlinear systems. These effects have traditionally been perceived as undesirable; however noise can lead to desirable benefits [1, 2]. Though the effects of noise have been examined for nonlinear oscillator arrays subjected to harmonic inputs [3-5], effects of noise on transient energy localizations could be of importance as well. In this article, the authors explore phenomena in two different arrays and examine how noise can be used in a beneficial manner. In each case, the system is excited with a sinc pulse. One of the systems is an array of bistable Duffing oscillators, and through a study of this system, it is illustrated that noise can enhance wave-like phenomenon. The second system is an array of nonlinearly coupled monostable oscillators, which is studied to show that noise can suppress a travelling wave, by transferring energy to low-frequency components.

The Euler-Maruyama algorithm [6] is used to numerically simulate the considered systems. The Fokker-Planck equation [7] associated with each system is also studied. Results obtained from the corresponding moment evolution equations will be included in the presentation.

2. Nonlinear oscillator arrays

The equations of motion for the considered oscillator array can be written in the form

$$\begin{cases} \mathrm{m}_1\ddot{x}_1 + \mathrm{c}_1\dot{x}_1 + [\mathrm{k}_{0,1} + \mathrm{k}_{0,2} + \mathrm{k}_{1,1}]x_1 - \mathrm{k}_{0,2}x_2 + \mathrm{k}_{2,1}x_1^3 + \mathrm{k}_{3,1}x_1^3 + \mathrm{k}_{3,2}(x_1 - x_2)^3 \\ = \mathrm{F}sinc + \sigma\dot{W}(t) \\ \vdots \\ \mathrm{m}_i\ddot{x}_i + \mathrm{c}_i\dot{x}_i - \mathrm{k}_{0,i}x_{i-1} + [\mathrm{k}_{0,i} + \mathrm{k}_{0,i+1} + \mathrm{k}_{1,i}]x_i - \mathrm{k}_{0,i+1}x_{i+1} + \mathrm{k}_{2,i}x_i^3 + \mathrm{k}_{3,i}(x_i - x_{i-1})^3 + \mathrm{k}_{3,i+1}(x_i - x_{i+1})^3 \\ = \sigma\dot{W}(t) \\ \vdots \\ \mathrm{m}_n\ddot{x}_n + \mathrm{c}_n\dot{x}_n - \mathrm{k}_{0,n}x_{n-1} + [\mathrm{k}_{0,n} + \mathrm{k}_{0,n+1} + \mathrm{k}_{1,n}]x_n + \mathrm{k}_{2,n}x_n^3 + \mathrm{k}_{3,n}(x_n - x_{n-1})^3 + \mathrm{k}_{3,n+1}x_n^3 \\ = \sigma\dot{W}(t) \end{cases} \quad (1)$$

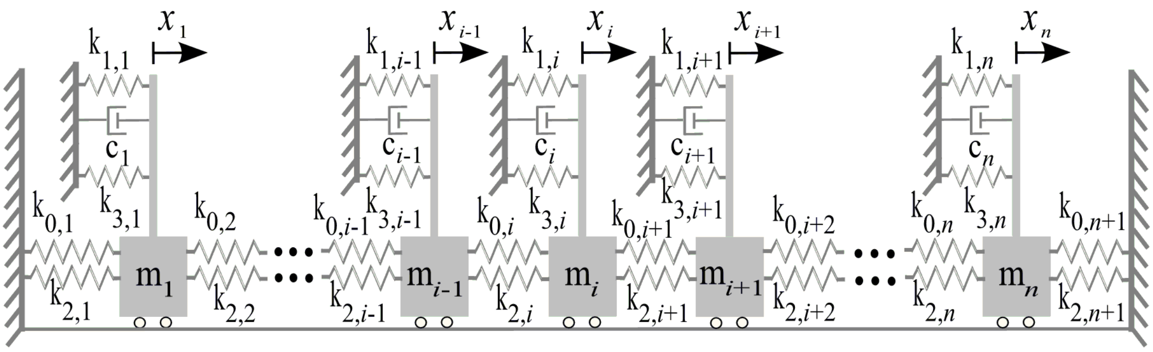


Figure 1. An array of nonlinear coupled oscillators. Each mass is coupled to adjacent masses with linear and nonlinear springs, and linear dampers. In addition, each mass is attached to a fixed local point through linear and nonlinear springs, and linear dampers.

Nomenclature

x_i	position of ith oscillator
$x_{i,1}$	position of ith oscillator in state space
$x_{i,2}$	velocity of ith oscillator in state space
m_i	mass of ith oscillator
$k_{0,i}$	linear coupling spring on left side of ith oscillator
$k_{1,i}$	linear spring constant of ith oscillator
$k_{2,i}$	nonlinear spring constant of ith oscillator
$k_{3,i}$	nonlinear coupling spring on left side of ith oscillator
K_i	$= k_{0,i} + k_{0,i+1} + k_{1,i}$
c_i	damping of ith oscillator
F	forcing amplitude
$\dot{W}(t)$	white noise (derivative of Wiener process)
σ	noise amplitude
sinc	sinc pulse which lasts for one second

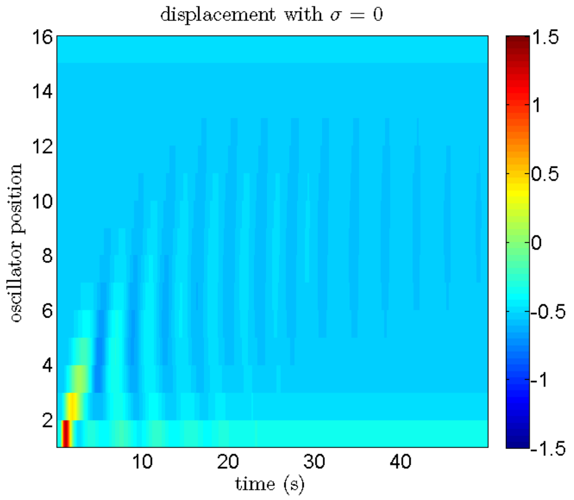
By setting the k_2 (k_3) terms equal to zero, the system becomes an array of monostable oscillators (an array of bistable Duffing oscillators). The Fokker-Planck equation for the ith oscillator of these systems can be written as follows:

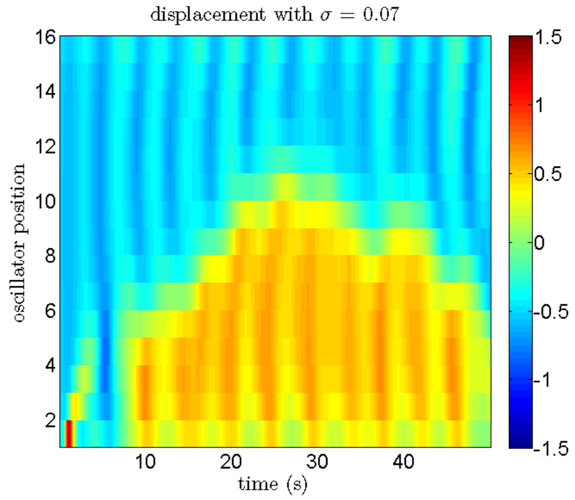
$$\partial_t p = -[\partial_{x_{i,1}} x_{i,2} p + \frac{1}{m_i}\partial_{x_{i,2}} p(-c_i x_{i,2} + k_{0,i} x_{i-1,1} - [k_{0,i} + k_{0,i+1} + k_{1,i}] x_{i,1} + k_{0,i+1} x_{i+1,1} - [k_{2,i} + k_{2,i+1} + k_{3,i}] x_{i,1}^3 + [3k_{2,i} x_{i-1,1} + 3k_{2,i+1} x_{i+1,1}] x_{i,1}^2 - [3k_{2,i} x_{i-1,1}^2 + 3k_{2,i+1} x_{i+1,1}^2] x_{i,1} + [k_{2,1} x_{i-1,1}^3 + k_{2,i+1} x_{i+1,1}^3] + F\mathit{sinc})] + \frac{\sigma^2}{2m_i^2}\frac{\partial^2}{\partial {x_{i,2}}^2} p \tag{2}$$

Further analysis of the Fokker–Planck equation, which is omitted here for brevity, will be included in the presentation.

3. Array of coupled bistable Duffing oscillators

For this system, the k_3 terms are set equal to zero. The oscillators are initialized in their left stable equilibrium position and the first oscillator is excited with a sinc pulse during the first second. The pulse can be seen to have little effect on the other oscillators as it travels through the array, as shown in the upper left portion of Figure 2. By adding Gaussian white noise, a switching, wave-like propagation is observed, where oscillators settle into their right stable equilibrium positions, as shown in the right portion of Figure 2.





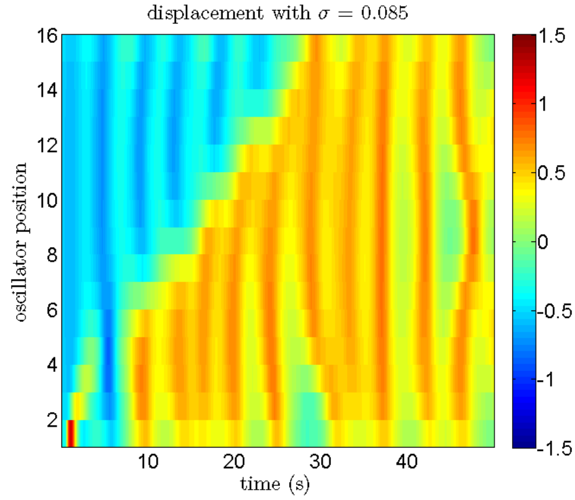


Figure 2. The sinc pulse alone does not cause all of the oscillators to switch from their left well to their right well. By using noise (applied to all oscillators), the switching, wave-like phenomenon is seen to progress through the array. For a moderate amount of noise, the switching behavior can only go through a portion of the array. A higher noise level induces all oscillators to switch wells.

4. Array of nonlinearly coupled monostable oscillators

To realize this system, the k_2 terms are set equal to zero. The oscillators are initialized at their equilibrium position and excited by a sinc function at the first oscillator. For this set of parameter values, the noise acts to attenuate the wave pulse as it travels through the array. This can be seen in the wavelet coefficients of the oscillator displacements. However, low-frequency components are introduced because of the noise as well. If better understood, this noise-influenced phenomenon could be used to inhibit wave propagation.

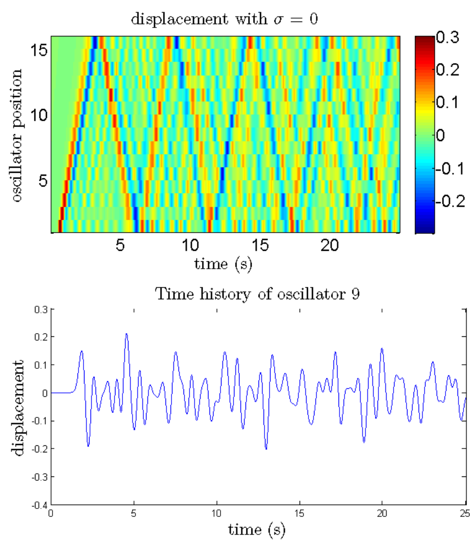


Figure 3. Wave propagation in array, caused by sinc pulse.

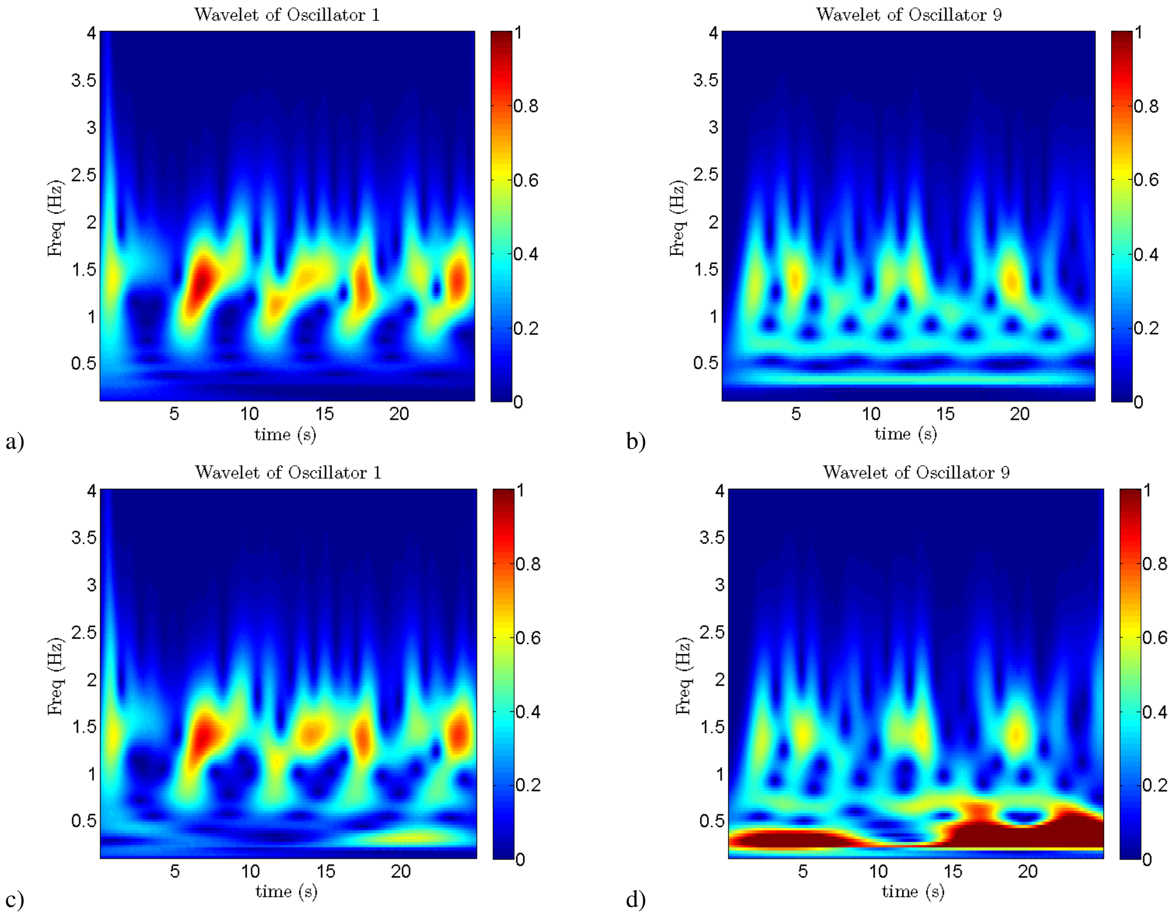


Figure 4. a) and b) Magnitude of wavelet coefficients of oscillator displacements with no noise; c) and d) Magnitude of wavelet coefficients of oscillator displacements with noise ($\sigma = 0.1$). With addition of noise, the wave pulse propagating through the oscillators is attenuated slightly at 1.5 Hz.

Acknowledgements

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