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IEICE Proceeding Series

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Vol. 2 pp. 475-478
Publication Date: 2014/03/18
Online ISSN: 2188-5079

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2013 International Symposium on Nonlinear Theory and its Applications
NOLTA2013, Santa Fe, USA, September 8-11, 2013

A Method for Reachability Problems of P/T Petri Nets using Algebraic Approach

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Abstract— P/T Petri nets are one of the useful models for discrete event systems. And a firing count vector for transitions is one of the key concepts to describe and evaluate algebraically their behavior. To consider the reachability, from an initial state called an initial marking, M_0 to destination state called a destination marking M_d are the fundamental problems of Petri nets. There are some methods to solve such reachability problems. One method is to use the coverability(reachability) tree, but the method requires a huge amount of calculation in general. On the other hand, the method to use matrix equations and reduction techniques has the advantage, because the method can utilise the algebraic equation properties of Petri nets. In this paper, we proposed an algebraic approach to reachability problems using Fourier-Motzkin method. Not only particular solutions and elementary T-invariants are obtained from the augmented system of state equation by Fourier-Motzkin method, but also the expansion coefficients of the nonnegative integer solution to represent state equations as $Ax = b$ can be obtained by the same algorithm of the Fourier-Motzkin method.

1. Introduction

A Petri net is a particular kind of directed graph, together with an initial state called the initial markings, M_0. The underlying graph of a Petri net is a directed, weighted, bipartite graph consisting of two kinds of notes, called places and transitions, where arcs are either from a place to a transition or from a transition to a place. A state or marking in a Petri net is changed according to the firing rules[1],[2]. Such Petri nets are effectively used for modeling, analyzing, and verifying many discrete event systems[1],[2].

In this paper, we concern structural analysis based on the linear algebra techniques and the state equation $Ax = b := M_d - M_0$, where M_0 and M_d are initial and destination marking vectors, respectively. All generators for T-invariants and all minimal inhomogeneous(i.e., particular) solutions are needed for discussing the feasibility of a group of firing count vectors, x, for the fixed $b := M_d - M_0$[3],[4],[5], where any firing count vector is expanded by means of T-invariant generators and particular solutions[5]. However, it is difficult, in general, to find the nonnegative rational/integer scalar expansion-coefficients. In this paper, we also consider how to find systematically those coefficients through the use of the well-known Fourier-Motzkin method[6],[7].

In section 2, preliminaries are given, and how to find expansion coefficients are described in section 3. In section 4, an example for finding expansion coefficients is described. And section 5 is the conclusion of this paper.

2. Preliminaries

2.1. State Equation

If the destination marking M_d was assumed to be reachable from initial marking M_0 through the firing sequence as $\{t_1, t_2, \cdots, t_d\}$, the state equation can be expressed as

$$M_d = M_0 + A\sum_{k=1}^{d} t_k \tag{1}$$

and eq.(1) can be described like as eq.(2) when $A \in Z^{m\times n}$, $b = M_d - M_0 \in Z^{m\times 1}, x = \sum_{k=1}^{d} t_k \in Z_+^{n\times 1}$

$$Ax = b. \tag{2}$$

Then we can obtain the firing count vector x to solve the solutions of eq.(2), from initial marking $M_0 \in Z_+^{m\times 1}$ to destination marking $M_d \in Z_+^{m\times 1}$.

2.2. Fourier-Motzkin method

The Fourier-Motzkin method is to obtain the set of all elementary vector solutions as the nonnegative integer solutions of $Ax = 0^{m\times 1}$. And the algorithm of the Fourier-Motzkin method is as follows[6],[7].

<Algorithm of Fourier-Motzkin method>

Input: Incidence matrix $A \in Z^{m\times n}$, m, and n.

Output: The set of T-invariants including all minimal support T-invariants.

Initialization: The matrix B is constructed by adjoining the identity matrix $E^{n\times n}$ to the bottom of the incidence matrix $A \in Z^{m\times n}$, with $B = [A^T, E]^T \in Z^{(m+n)\times n}$.

The following operations a), b) are repeated from $i = 1$ to $m = |P|$, where $|P|$ means the cardinality of the place set P.

a) Add to the matrix B all the columns which are linear combinations of pairs of columns of B and which annul the i-th row of B.

b) Eliminate from B the columns in which the i-th element is nonzero. ∎

When this algorithm has finished, each column of the submatrix $C \in Z_+^{n\times r}$ which is obtained by deleting the rows from the first to the m-th from the final outputted matrix $B \in Z_+^{(m+n)\times r}$ is a T-invariant. However, in general, this submatrix C includes also non-minimal-support T-invariants[7]. Therefore if the following operation c) is added and applied to C, only minimal support T-invariants are obtained.

c) Each column vector $u_i \in Z^{n\times 1}$ which satisfies the rank condition $q(u_i)$ =rank$A'(u_i)) + 2$ is removed from the submatrix $C = [u_i] \in Z_+^{n\times r}$. Here, $q(u_i)$ is the number of nonzero elements of $u_i \in Z_+^{n\times 1}$ for $Au_i = 0^{m\times 1}$ and $A'(u_i)$ is composed of the columns of A, of which columns are corresponding to nonzero elements of $u_i \in Z_+^{n\times 1}$.

But, this method can be applied to $Ax = 0$, and this means that obtained solutions are T-invariants. So, to obtain the particular solutions(firing count vectors), we need to make such changes to the eq.(2) considering the augmented incidence matrix as follows:

$$\widetilde{A} = [\ A \quad -b\ ] \in Z^{m\times(n+1)}. \tag{3}$$

then eq.(2) would be expressed by eq.(3) and augmented $\widetilde{x} \in Z^{n+1}$,

$$\widetilde{A}\widetilde{x} = 0. \tag{4}$$

Here, eq.(4) can be applied to the algorithm of §2.2.

3. Finding Expansion Coefficients for a Firing Count Vector by T-Invariants and Particular Solutions

§2.2 expressed how to obtain nonnegative solutions x of $Ax = b$ using the algorithm of the Fourier-Motzkin method. Finding expansion coefficients are useful for analyzing behavior verification of P/T Petri nets efficiently[8].

3.1. An Arbitrary Firing Count Vector by Means of T-Invariants and Particular Solutions

A firing count vector $x \in Z_+^{n\times 1} (x \in X)$ is expressed by using $u_i^{(4)} \in U_4$ = "the set of minimal support T-invariants" and $v_j^{(4)} \in V_4$ = "the set of fundamental particular solutions" as follows[5]:

$$x = \textstyle\sum_{i=1}^{l_4} \alpha_i^{(4)} u_i^{(4)} + \sum_{j=1}^{k_4} \beta_j^{(4)} v_j^{(4)}, \quad \sum_{j=1}^{k_4} \beta_j^{(4)} = 1, \tag{5}$$

where $l_4 = |U_4|$, $k_4 = |V_4|$, and $\alpha_i^{(4)}, \beta_j^{(4)} \in Q_+^{1\times 1}$.

We call eq.(5) as the level 4 expression in this paper. Moreover we have another expression for $x \in Z_+^{n\times 1}$ if we use $U_5 = \{U_4, U_5\backslash U_4\}$ = "the set of minimal T-invariants" and $V_5 = \{V_4, V_5\backslash V_4\}$ = "the set of minimal particular solutions" as follows[5]:

$$x = \textstyle\sum_{i=1}^{l_5} \alpha_i^{(5)} u_i^{(5)} + \sum_{j=1}^{k_5} \beta_j^{(5)} v_j^{(5)}, \tag{6}$$

where $\sum_{j=1}^{k_5} \beta_j^{(5)} = 1$, $l_5 = |U_5|$, $k_5 = |V_5|$, and $\alpha_i^{(5)}, \beta_j^{(5)} \in Z_+^{1\times 1}$. Then eq.(6) is rewritten as follows

$$x = \textstyle\sum_{i=1}^{l_5} \alpha_i^{(5)} u_i^{(5)} + v_j^{(5)}, \tag{7}$$

where $\beta_j^{(5)} = 1, v_j^{(5)} \in V_5$, and $\alpha_i^{(5)} \in Z_+^{1\times 1}$. We call eq.(6) or (7) as the level 5 expression in this paper. After that we discuss about the level 5 here , and also eq.(7) is rewritten as follows:

$$x = \sum_{i=1}^{l} \alpha_i u_i + v_j \tag{8}$$

where $\alpha_i \in Z_+^{1\times 1}$, $u_i \in U := \{\ u_i \in Z_+^{n\times 1}\ ;\ Ax = b$ T-invariants, and $i = 1, 2, \cdots, l\ \}$, $v_j \in V := \{\ v_j \in Z_+^{n\times 1}\ ;\ Ax = b$ particular solutions, $j = 1, 2, \cdots, k\ \}$, after here.

3.2. How to Find Expansion Coefficients

Eq.(8) means any nonnegative solutions (firing count vectors) of state equation eq.(2) can be obtained by the linear combinations of T-invariants and a particular solution.

When U, V, and $x \in Z_+^{n\times 1}$ are given, eq.(8) can be rewritten as follows:

$$\sum_{i=1}^{l} \alpha_i u_i = x - v_j \tag{9}$$

by transposition of v_j. And eq.(9) expresses

$$[\ u_1, u_2, \cdots, u_l\ ]\ \alpha = [\ x - v_j\ ]. \tag{10}$$

And on eq.(10),

$$[\ u_1, u_2, \cdots, u_l\ ] \rightarrow A', \quad \alpha \rightarrow x', \quad [\ x - v_j\ ] \rightarrow b'$$

are transposed, eq.(10) can be expressed as follows:

$$A'x' = b'. \tag{11}$$

This means that eq.(11) is the same type of equation as eq.(2). Then the same algorithm of the Fourier-Motzkin method expressed in §2.2 can be also applied to such problems as finding expansion coefficients for any reachable firing count vectors by T-invariants and a particular solution.

4. Example

Let's consider a Petri net shown in Fig.1, where a black dot on place p_1 is an initial marking, and small white circles on places are destination markings.

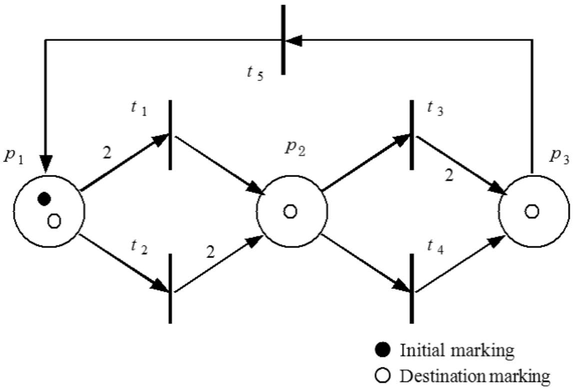

Fig.1 Example of Petri nets.

4.1. T-invariants and particular solutions

On this case, the incidence matrix of $A \in Z^{m\times n}$ is

$$A = \begin{bmatrix} -2 & -1 & 0 & 0 & 1 \\ 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 2 & 1 & -1 \end{bmatrix} \in Z^{3\times 5},$$

and the difference of marking $b \in Z^{m\times 1}$ from $M_0 \in Z_+^{m\times 1}$ to $M_d \in Z_+^{m\times 1}$ is

$$b = M_d - M_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \in Z^{3\times 1}.$$

Then the augmented matrix of A can be described as follows:

$$\widetilde{A} = \begin{bmatrix} -2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 2 & -1 & -1 & 0 & -1 \\ 0 & 0 & 2 & 1 & -1 & -1 \end{bmatrix} \in Z^{3\times 6}$$

by eq.(3). And by the algorithm in §2.2, we can express the matrix B of Fig.1 using $B = [\ \widetilde{A} \ \ E\ ] \in Z^{(m+n+1)\times(n+1)}$, as follows:

$$B = \left[\begin{array}{cccccc} -2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 2 & -1 & -1 & 0 & -1 \\ 0 & 0 & 2 & 1 & -1 & -1 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right].$$

From matrix B, T-invariants and particular solutions are obtained by using the algorithm of the Fourier-Motzkin method as follows:

$$\begin{array}{ll} u_1 = (\ 1\ \ 0\ \ 1\ \ 0\ \ 2\ )^T, & v_1 = (\ 0\ \ 1\ \ 1\ \ 0\ \ 1\ )^T, \\ u_2 = (\ 1\ \ 1\ \ 0\ \ 3\ \ 3\ )^T, & v_2 = (\ 0\ \ 2\ \ 0\ \ 3\ \ 2\ )^T, \end{array} \tag{12}$$

where $u_i \in U := \{u_i \in Z_+^{n\times 1}\}$; $Ax = b$ T-invariants and $v_j \in V := \{v_j \in Z_+^{n\times 1}\}$; $Ax = b$ particular solutions.

4.2. Expansion coefficients

Here, let's consider about one firing count vector

$$x = (\ 3\ \ 2\ \ 3\ \ 3\ \ 8\ )^T, \tag{13}$$

and try to obtain the expansion coefficients α with one of the particular solutions $v_1 = (\ 0\ \ 1\ \ 1\ \ 0\ \ 1\ )^T$ using the same algorithm as the Fourier-Motzkin method used above.

On this case, b' on eq.(11) is

$$\begin{aligned} b' &= [\ x - v_1\ ] \\ &= (\ 3\ \ 2\ \ 3\ \ 3\ \ 8\ )^T - (\ 0\ \ 1\ \ 1\ \ 0\ \ 1\ )^T \\ &= (\ 3\ \ 1\ \ 2\ \ 3\ \ 7\ )^T, \end{aligned}$$

and A' on eq.(11) is

$$\begin{aligned} A' &= [\ u_1,\ u_2\ ] \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 3 \\ 2 & 3 \end{bmatrix}. \end{aligned}$$

Then in this case, matrix B of eq.(11) can be described as follows:

$$B = \left[\begin{array}{ccc} 1 & 1 & -3 \\ 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 3 & -3 \\ 2 & 3 & -7 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

because of eq.(3) and $B = [\ \widetilde{A} \ \ E\ ]$. Add to the matrix B all the columns which are linear combinations of pairs of columns which annul the 1st row of B. In this case there are 2 pairs. The 1st pair is in the 1st and 3rd columns of above B. and 2nd pair is in the 2nd and 3rd columns of above B. Add to the matrix B the 2 columns which annul the 1st row of B expressed as follows:

$$B = \left[\begin{array}{ccc|cc} 1 & 1 & -3 & 0 & 0 \\ 0 & 1 & -1 & -1 & 2 \\ 1 & 0 & -2 & 1 & -2 \\ 0 & 3 & -3 & -3 & 6 \\ 2 & 3 & -7 & -1 & 2 \\ \hline 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right].$$

Eliminate from B the columns which the 1st element is nonzero, and add the column which is the linear combination of a pair of columns which annul the 2nd row of B.

$$B = \left[\begin{array}{rr|r} 0 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & -2 & 0 \\ -3 & 6 & 0 \\ -1 & 2 & 0 \\ \hline 3 & 0 & 6 \\ 0 & 3 & 3 \\ 1 & 1 & 3 \end{array}\right]$$

And devide all elements of the 3rd column by the base number 3.

$$B = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 2 \\ 1 \\ 1 \end{array}\right]$$

Then we can obtain the coefficients, and try to check the coefficients using eq.(8)

$$x = \sum_{i=1}^{l} \alpha_i u_i + v_j$$

with the obtained coefficients, T-invariants and one particular solution from eq.(12) as follows:

$$\begin{aligned} &\alpha_1 = 2, \ \ \alpha_2 = 1, \\ &u_1 = (\ 1\ \ 0\ \ 1\ \ 0\ \ 2\ )^T, \quad u_2 = (\ 1\ \ 1\ \ 0\ \ 3\ \ 3\ )^T, \\ &v_1 = (\ 0\ \ 1\ \ 1\ \ 0\ \ 1\ )^T. \end{aligned}$$

Then, the firing count vector can be calculated using the obtained coefficients just the same as eq.(13) as follows:

$$\begin{aligned} x &= 2\,(\,1\,0\,1\,0\,2\,)^T + (\,1\,1\,0\,3\,3\,)^T + (\,0\,1\,1\,0\,1\,)^T \\ &= (\,3\,2\,3\,3\,8\,)^T. \end{aligned}$$

5. Conclusions

For any firing count vector $x \in Z_+^{n\times1}$ expressed by means of minimal support or minimal T-invariants and fundamental or minimal particular solutions for state equation in P/T Petri nets, how to find such T-invariants and particular solutions has been shown through the Fourier-Motzkin method. Finding expansion coefficients are useful for analyzing behaviour verification of P/T Petri nets efficiently. In this paper, we described that the same algorithm of the Fourier-Motzkin method as the algorithm for obtaining T-invariants and particular solutions can be also applied to obtain the expansion coefficients. These results are useful for the reachability analysis and the scheduling for the fixed initial and destination markings.

In future studies, we'd like to improve the algorithm for derivation of unobtainable solutions in P/T Petri nets using Fourier-Motzkin Method.

Acknowledgments

This research was supported in part by a Grant-in Aid for Scientific Researches(C) Grant Numbers 24501221, 23560543 Japan Society for the Promotion of Science(JSPS) and by FUT Research Promotion Fund(Cluster D).

References

[1] T. Murata, "Petri nets: Properties, analysis and applications," Proc. IEEE, vol.77, no.4, pp.451–580, 1989.

[2] C. Giraut and R. Valk, Petri Nets for Systems Engineering, A Guide to Modeling, Verification, and Applications, pp.53–72 and pp.278–316, Springer-Verlag, Berlin, Heidelberg, 2003.

[3] A. E. Kotsin and S. A. Tchoudaikina, "Yet another reachability algorithm for Petri nets," SIGACT News, vol.29, no.4, pp.98–110, 1998.

[4] A. Hiramitsu, M. Takata, K. Inaba, T. Yoshida, T. Matsumoto, and S. Moro, "A new algorithm of executability of a solution of state equation for P/T Petri net," IEICE Tech. Rep., CAS2001-64 or CST2001-17, Nov. 2001.

[5] A. Muraya, T. Matsumoto, S. Moro, and H. Hasegawa, "All fundamental particular solutions are needed to express an arbitrary firing count vector in Petri nets," IEICE Trans. Fundamentals, Vol.E88–A, No.1, pp.399–404, Jan. 2005.

[6] J. Martinez and M. Silva, "A simple and fast algorithm to obtain all invariants of a generalized Petri net," Procs. of Second European Workshop on Application and Theory of Petri Nets, Informatik Fachberichte 52, pp.301–310, Springer Publishing Company, Berlin, 1982.

[7] M. Takata, T. Matsumoto, and S. Moro, "A new algorithm to derive generators for both invariants and particular solutions of state equation for a P/T Petri net —Extended Fourier-Motzkin method," IEICE Tech. Rep., CAS2001-65 or CST2001-18, Nov. 2001.

[8] T. Matsumoto, M. Osogami, and S. Moro, "How to obtain coefficients for a firing count vector expanded by T-invariants and particular solutions in P/T Petri nets," Proc. of NOLTA'06, Bologna, Italy, pp.407–410, Sept.2006.