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A numerical study on parametric resonance of intrinsic localized modes in coupled cantilever arrays

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Abstract—In a coupled cantilever array modeled as a coupled ordinary differential equation, symmetric and antisymmetric intrinsic localized modes (ILMs) exist. The symmetric ILM is stable while the other is unstable under the regime in which the ratio in nonlinearities of inter-site and on-site potentials is less than the critical value at which the stability change occurs. This paper shows that a stable ILM loses its stability when the system is parametrically excited. If the amplitude of parametric excitation is large enough, the destabilized ILM wanders in the whole system. The parameter region where the instability occurs are numerically investigated and compared with that in the Mathieu equation. The similarity of the shape of the regions strongly suggests that the instability is caused by the parametric resonance.

1. Introduction

Spatially localized and temporary periodic vibrations often appear in nonlinear coupled oscillators [1]. The energy localized vibration in discrete media which is first discovered by A. J. Sievers and S. Takeno [2] is called intrinsic localized mode(ILM) or discrete breather(DB). Experimental observations of ILM have been reported for a variety of physical system in this decade as well as theoretical and numerical studies. In particular of them, the observation in micro-mechanical cantilever array allow us to expect the realization of applications using ILM in micro/nano-engineering [3], because it was also observed that ILM can move without decaying its energy concentration and can be manipulated by an extraneous stimulus [4].

For the realization of such application, the control scheme for the ILM should be established. The capture and release manipulation using the stability change is proposed as an alternative method to manipulate ILM in a micro-cantilever array [5]. For the manipulation method, it is utilized that a stable ILM begins to move after it loses its stability. Therefore, the manipulation is a way to generate moving ILM. In this paper, we propose another method to generate moving ILM by parametric excitation of the ratio in nonlinearities. We first investigate how moving ILMs behave around a stable ILM. The motion of moving ILM is approximated by a simple equation of pendulum. Then, behaviors of moving ILM created by parametric excitation are shown. Finally, the region where a stable solution loses its stability by parametric excitation is computed for an ILM, and is compared with that of the Mathieu equation.

2. Coupled Cantilever Array

Cantilever array, which is often used as a coupled mechanical resonator in nano/micro-engineering, is one of nonlinear coupled oscillators in which ILM can be observed experimentally [3]. The motion of cantilevers can approximately be described by ordinary differential equation [4–6],

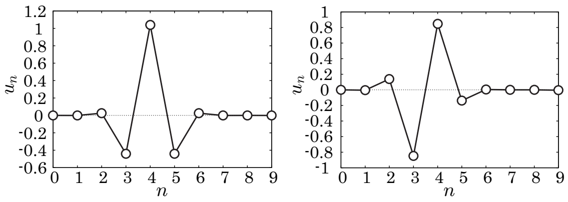
$$\begin{aligned}\ddot{u}_n = &-\alpha_1 u_n - \alpha_2(2u_n - u_{n+1} - u_{n-1})\\ &-\beta_1 u_n^3 - \beta_2(u_n - u_{n+1})^3 - \beta_2(u_n - u_{n-1})^3,\end{aligned} \tag{1}$$

where u_n denotes the displacement of nth cantilever from the equilibrium position. By nondimensionalization of the equation, α_1 and β_2 can be chosen as 1. Coupling coefficients, α_2 and β_2, are parameters of the equation. In these parameters, the nonlinear coupling coefficient β_2 strongly affects the stability of ILM [6]. For this reason, β_2 is chosen as a parameter which is modulated by a time-periodic signal while the linear coupling coefficient is fixed at 0.1.

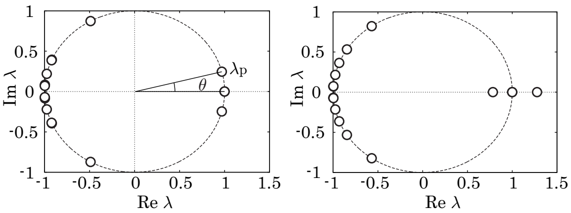
Amplitude distribution of standing ILMs at $\beta_2 = 0.5$ is shown with their Floquet multipliers in Fig.1. The ILMs can be classified into two kinds by their spatial symmetry [7]. One is called Sievers-Takeno mode(ST mode) which stands on a site. The ST mode is stable at $\beta_2 = 0.5$ because all the Floquet multipliers are on unit circle as shown by the left panel of Fig.1(b). The other is Page mode(P mode) centered between sites which is unstable as one of the Floquet multipliers is outside the unit circle.

3. Moving ILMs around a Stable ILM

For the stable ILM, the eigenvector of a Floquet multiplier near +1 corresponds to the pinning mode [8, 9]. If the stable ILM is perturbed to the direction of the pinning mode, the locus of ILM oscillatory moves. In order to show



(a) Amplitude distribution



(b) Floquet multipliers

Figure 1: Sievers-Takeno mode and Page mode.

the motion of the locus of ILM, we define a projection $\mathcal{G}:\mathbb{R}^{2N}\to\mathbb{C}$ which is defined as follows [10]:

$$
\begin{aligned}
h = \mathcal{G}(\boldsymbol{u},\dot{\boldsymbol{u}}) = \sum_{n=1}^{N}\Bigg\{&\left(\frac{1}{2}\dot{u}_n^2 + U_{\mathrm{O}n}(u_n)\right)e^{i\frac{2\pi}{N}n} \\
&+U_{\mathrm{I}n}(u_n-u_{n-1})e^{i\frac{2\pi}{N}(n+\frac{1}{2})}\Bigg\},
\end{aligned} \tag{2}
$$

where N is equal to 8. The projection can extract the locus of ILM by $X = \dfrac{\arg h}{2\pi}\left(N+\dfrac{1}{2}\right)$. The velocity of the locus is obtained by taking the central difference.

The oscillatory motions of the perturbed ILMs are shown in Fig.2. The initial conditions $\boldsymbol{u}_m$ is created as follows:

$$
\begin{aligned}
\tilde{\boldsymbol{u}}_m &= \boldsymbol{u}_{\mathrm{ST}} + (\boldsymbol{u}_{\mathrm{P}} - \boldsymbol{u}_{\mathrm{ST}})\frac{m}{20}, \\
\boldsymbol{u}_m &= k\tilde{\boldsymbol{u}}_m, \quad (m = 1,\ 2,\ \cdots,\ 19),
\end{aligned} \tag{3}
$$

where k is a scaling factor determined such that the initial conditions are on the same energy surface on which the two standing ILMs exist. In this paper, the total energy of Eq.(1) is fixed at 2.5. The vectors $\boldsymbol{u}_{\mathrm{ST}}$ and $\boldsymbol{u}_{\mathrm{P}}$ correspond to an ST mode and a P mode, respectively. Note that $\dot{\boldsymbol{u}}_m$ is set to be zero because $\dot{\boldsymbol{u}}_{\mathrm{ST}} = \dot{\boldsymbol{u}}_{\mathrm{P}} = 0$. As shown in Fig.2, the period of the oscillatory motion increases with distance from the stable ILM to the initial position. In addition, the shape of the motion is apart from the sinusoidal motion when the initial position is close to the unstable ILM. These two facts imply that the motion of the moving ILM can be approximated by the equation of motion of pendulum.

Here we consider a pendulum whose equation of motion is described by $\ddot{\theta} = -\omega_0^2\sin\theta$. By transforming variables to the locus of ILM, we have

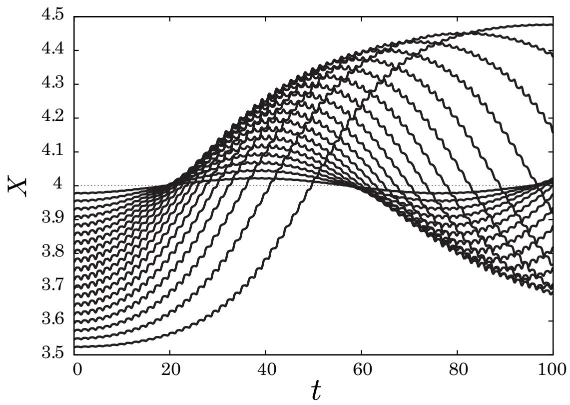


Figure 2: Time evolution of the locus of ILMs. The small fluctuations are caused by oscillations of each oscillator.

$$
\ddot{X} = -\frac{1}{2\pi}\Omega_0^2\sin(2\pi X), \tag{4}
$$

where $\Omega_0 = \dfrac{2\pi}{T_0}$ is the period of oscillatory motion when the perturbation to the direction of the pinning mode is small. Therefore, T_0 is determined by the angle of the Floquet multiplier λ_{p} shown in Fig.1(b), namely, $T_0 = T_{\mathrm{b}}\dfrac{2\pi}{\arg\lambda_{\mathrm{p}}}$, where T_{b} is the period of the stable ILM. Since λ_{p} moves along unit circle when the parameters of Eq.(1) is varied, Ω_0 is a function of the coefficients and the total energy of Eq.(1). Fig.3 shows how Ω depends on β_2. Because of the stability change at $\beta_2 \simeq 0.545358$ [6], both curves discontinue there.

The period of the locus of moving ILM is derived by

$$
T = 4\frac{2\pi}{T_0}K(k), \tag{5}
$$

where $k = \sin(\pi(4-X))$, and K is the complete elliptic integral of the first kind. The solid curve in Fig.4 shows Eq.(5) with $T_0 = 76.215$. The solid circles indicate the peaks of Fourier spectrum of each motion shown in Fig.2. The peaks are well consistent with the theoretical curve. Consequently, the motion of the moving ILM around standing ILMs can approximately be described by Eq.(4) unless the energy concentration of the moving ILM decays.

4. Parametric Resonance

It is well known that the pendulum can be excited by a parametric forcing. Thus, an oscillatory motion can be induced by a parametric excitation for ILM as well as the pendulum. Here we substitute β_2 in Eq.(1) by $\beta_2 + \epsilon\sin\nu t$. The motion of the destabilized ILM at $\epsilon = 0.4$ and $\nu = \dfrac{2\pi}{T_0/2}$ is shown in Fig.5. The fluctuation of the amplitude

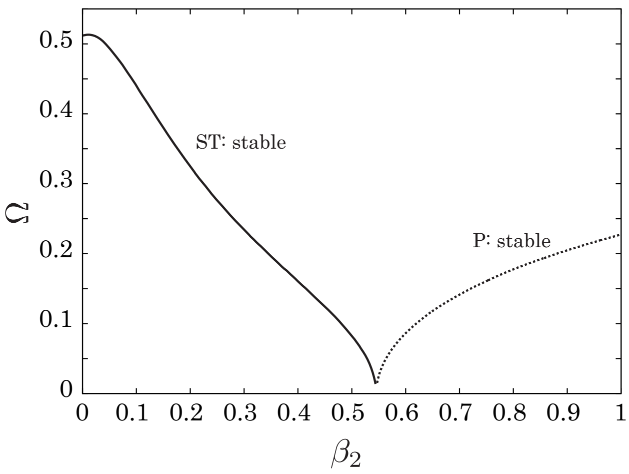


Figure 3: Dependency of Ω with respect to β_2. The solid and dotted curves correspond to the period around ST mode and P mode, respectively.

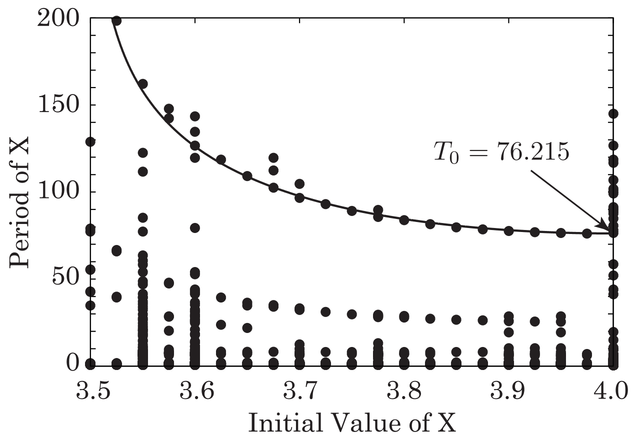


Figure 4: Period of the reciprocal motion around the stable ILM. Solid circles indicate the peaks of Fourier spectrum. The solid curve is drawn by Eq.(5) with $T_0 = 76.215$.

of oscillator can be seen around $t = 1900$. This implies that the ILM loses its stability by the parametric excitation. After $t > 2200$, the locus of the ILM wanders in the whole system.

For other amplitudes of parametric excitation, the trajectories of the locus are shown in Fig.6. The dotted curve corresponds to the case shown in Fig.5. It starts to move in one direction with an almost constant velocity for $t > 2200$ as mentioned above. On the other hand, the ILM cannot overcome the unstable P mode for $\epsilon = 0.2$ although the ST mode becomes unstable. The oscillation around the ST mode grows until $t < 2800$, but its amplitude goes to zero again. At $\epsilon = 0.5$, the ILM escapes from the initial position faster than the case of $\epsilon = 0.4$. The moving ILM shows a random motion for $1500 < t < 3000$.

By assuming a small oscillation $X = X_0 + \xi$ ($\xi \ll 1$), a small excitation $\epsilon \ll 1$, and by transforming the variable of time $t \to \dfrac{t}{\nu}$, we have,

$$\begin{aligned}\ddot{\xi} &= -\left(\frac{\Omega}{\nu}\right)^2 \left(1 + \frac{1}{\Omega^2}\frac{\partial^2\Omega}{\partial\beta_2^2}\epsilon\sin t\right)\xi, \\ &= -\omega'^2(1+\epsilon'\sin t)\xi.\end{aligned} \tag{6}$$

This equation is the same form as the Mathieu equation $\ddot{x} = -\omega^2(1+\epsilon\sin t)x$. The comparison between the unstable regions for Eq.(6) and for the Mathieu equation is shown in Fig.7. The solid squares indicate the combination $(\omega',\ \epsilon')$ at which $|\arg h(t) - \arg h(0)| > 10^{-10}$ is realized within $1000T_b$. Thus, the unstable region of ILM shown in the figure is much smaller than the true one. The solid curves are the boundaries of the unstable regions of the Mathieu equation which is numerically obtained. As clearly shown in the figure, the unstable regions of ILM are well coincide with that of the Mathieu equation except the region found in $\omega' < 0.2$.

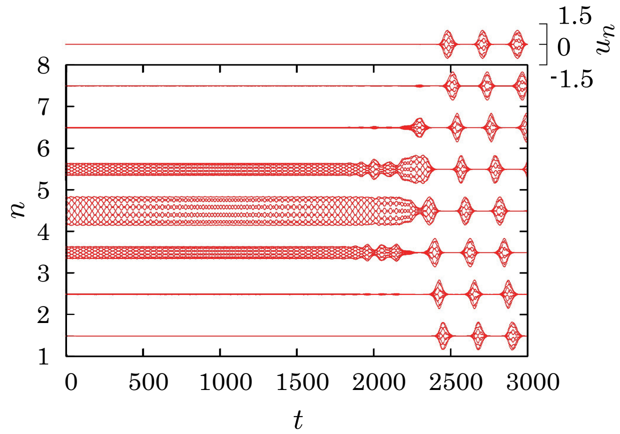


Figure 5: Destabilized ILM at $\epsilon = 0.4, \nu = \dfrac{2\pi}{35}$. The ST mode shown in Fig.1(a) is used as the initial condition. The parametric excitation is started at $t = 0$.

5. Conclusion

It has been shown that the motion of moving ILM can be approximated by the equation of pendulum. In addition, the destabilization of the stable ILM was observed when the parameter was modulated sinusoidally as well as the pendulum. On the basis of the fact that the unstable regions for ILM are well coincide with those of the Mathieu equation, the mechanism of losing stability is the parametric resonance. Therefore, the ILM can be destabilized by parametric excitation at least β_2 is close to 0.545358 at which the stability change occurs.

For the future work, the region where the wandering ILM is caused will be investigated. If the region is relatively wide, the parametric excitation will be a useful method to generate moving ILM in future applications.

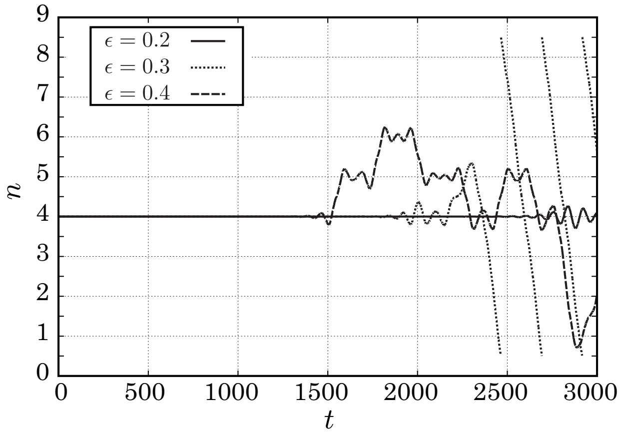


Figure 6: Time evolution of the loci of destabilized ILMs at $\nu = \dfrac{2\pi}{35}$.

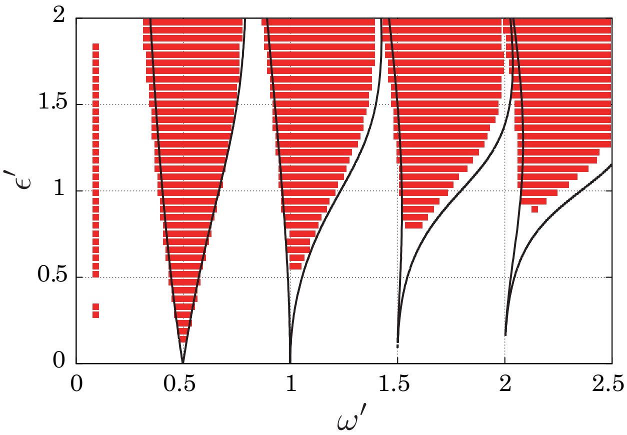


Figure 7: Unstable region where the stable ILM loses its stability within $t < 3000$. The solid curves are boundaries of the unstable regions of $\ddot{x} = -\omega^2(1 + \epsilon \sin t)x$.

Acknowledgments

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