Combination of Ultra-Wide Band Characteristic Basis Function Method and Improved Adaptive Model-Based Parameter Estimation in MoM Solution

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***Abstract*-An efficient technique which combines the ultra-wide band characteristic basis function method (UCBFM) and the improved adaptive model-based parameter estimation (IA-MBPE) is introduced for analyzing the wide band electromagnetic scattering problems. The ultra-wide band characteristic basis functions (UCBFs) are generated from characteristic basis functions at the highest frequency in the range of interest. These UCBFs are also available for the entire band. The IA-MBPE is the application of improved adaptive sampling algorithm (IASA) in MBPE. The high sampling efficiency in IASA is realized through extending the regions of searching for the sampling points from one to two simultaneously with the help of parallel computation algorithm. The combination of UCBFM and IA-MBPE results in significant enhancement of computational efficiency reduction in MoM solution. Numerical results from two examples of wide band frequency responses of monostatic RCSs validate the proposed method.**

***Index Term*-ultra-wide band characteristic basis function method; model-based parameter estimation; improved adaptive sampling algorithm; method of moments**

I. Introduction

The computation of the electromagnetic radar cross section (RCS) by the complex objects with large electrical size, such as ships on the sea and tanks on the ground, is important for military applications. Although the parallel technology of the computer is rapidly developing, it is still an arduous task to create a large RCS database of those objects. Meanwhile the computation by using numerical method, such as method of moments (MoM) [1], is very time consuming. The MoM not only places a heavy burden on the CPU time as well as memory requirements but also requires the impedance matrix to be generated and solved for each frequency point. Hence, if the response over a wide frequency band is of interest, the MoM is more computationally intensive.

Several techniques have been proposed to alleviate this problem. In [2-3], the characteristic basis function method (CBFM), is able to reduce the size of the MoM matrix. In the CBFM, the object is divided into a number of blocks, and high-level basis functions called characteristic basis functions (CBFs) are derived for these blocks, which are discretized by using the conventional triangular patch segmentation and Rao-Wilton-Glisson (RWG) basis functions [4]. In[5-7], an interpolation model known as the model-based parameter estimation (MBPE), which takes into account the physics behind the problem, is proposed to minimize the computational cost with a desired accuracy. The MBPE is based on the rational function interpolation. Since it needs solving MoM matrix equation at sampling points directly, the MBPE can hardly deal with wide-band electromagnetic scattering problems from electrically large objects. So in [8], MBPE combined with AMCBFM, is proposed to analyze wide band electromagnetic scattering problems. This method uses the mutual coupling method for generating CBFs, that is time consuming and memory demanding, in CBFM and applies an adaptive sampling algorithm (ASA) [9] for MBPE. In [10], an improved adaptive sampling algorithm (IASA) is presented to obtain the high sampling efficiency. Since the CBFs depend upon the frequency, they need to be generated repeatedly for each frequency. Hence, in [11], the ultra-wide band characteristic basis function method (UCBFM) is developed, without having the generation of CBFs for each frequency repeatedly. The CBFs calculated at the highest, termed UCBFs, entail the electromagnetic behavior at lower frequency range; thus, it follows that they can also be employed at lower frequencies without going through the time consuming step of generating them again. However, in the UCBFM, it is still time consuming for the computation of wideband RCS since it requires repeated solving of the reduced matrix equations at each frequency.

In this paper, the combination of the UCBFM and IA-MBPE is introduced for fast evaluation of wide band scattering problems. In the following sections, the principles of UCBFM and the IA-MBPE are outlined firstly, and then the combination scheme of the UCBFM/IA-MBPE method is developed. Finally, two classical scattering problems are analyzed and the comparisons of the proposed method and traditional methods are provided.

II. Formulation

A. UCBF Method[11]

Let us consider a complex 3-D object illuminated by a plane wave. In a conventional MoM, the whole surface is divided into triangles with size ranging from $\lambda/10$ to $\lambda/20$. Applying

[2.] This work is supported by State Key Laboratory of Millimeter Waves (K201306).

this to the electric field integral equation, one can obtain a dense and complex system of the form

$$Z(k)I(k)=V(k) \tag{1}$$

In (1), Z is the MoM matrix of dimension $N\times N$, I and V are vectors of dimension $N\times 1$, where N is the number of unknown current coefficients and k is the wave number of the free space. For large and complex problem, the matrix filling and matrix equation solving are quite time consuming.

The CBFM begins by dividing the object to be analyzed into blocks. For the best division scheme and the number of blocks M one may refer to [12]. These blocks are characterized through a set of CBFs, constructed by exciting each block with multiple plane waves (MPW), incident from N_{PW} uniformly spaced θ and φ-angles. To calculate the CBFs on the generic ith block , one must solve the following system

$$Z_{ii}(k)J_i^{CBF}=V_i^{MPW} \tag{2}$$

In (2), Z_{ii} is an $N_i\times N_i$ sub-matrix corresponding to the ith block, J_i^{CBF} is a $N_i\times N_{\mathrm{PW}}$ matrix containing original CBFs, and V_i^{MPW} is a $N_i\times N_{\mathrm{PW}}$ matrix containing excitation vectors, where N_i is the number of unknowns relative to ith block. In order to extract Z_{ii} from the original MoM matrix, a matrix segmentation procedure can be used. Next, a new set of orthogonal basis functions, which are linear combinations of the original CBFs, are constructed via the singular value decomposition (SVD) approach. Thus, the redundant information because of the overestimation is eliminated. For simplicity, one can assume that the average number of CBFs after SVD is K. Consequently, the solution to the entire problem is expressed as a linear combination of the $M\times K$ CBFs, as follows

$$I(k)=\sum_{m=1}^{M}\sum_{k=1}^{K}\alpha_m^k(k)J_m^{CBF_k}(k) \tag{3}$$

where $J_m^{CBF_n}$ is the nth CBF of the mth lock. By using the above CBFs, the original large MoM matrix can be reduced, and unknowns are changed to weight coefficient vector α whose order is much smaller than that of the original current coefficient vector I. Finally, after solving the reduced system and substituting solution back to (3), one can obtain the solution of single frequency point.

The ultra-wide band characteristic basis functions (UCBFs) is the CBFs generated at the highest frequency. Since the UCBFs can adequately represent the solution in the entire band of interest, they are used for lower frequencies without going through the time consuming step of generating them again. Fig.1 shows the flowchart of the UCBFM.

B. IA-MBPE Method[10]

The rational function in the form of a fractional polynomial function of the ζ-order numerator and the ν-order denominator employed commonly in MBPE is represented as

$$R(f)=\frac{P_\zeta(f)}{Q_\nu(f)}=\frac{p_0+p_1f+\cdots+p_\zeta f^\zeta}{q_0+q_1f+\cdots+q_\nu f^\nu} \tag{4}$$

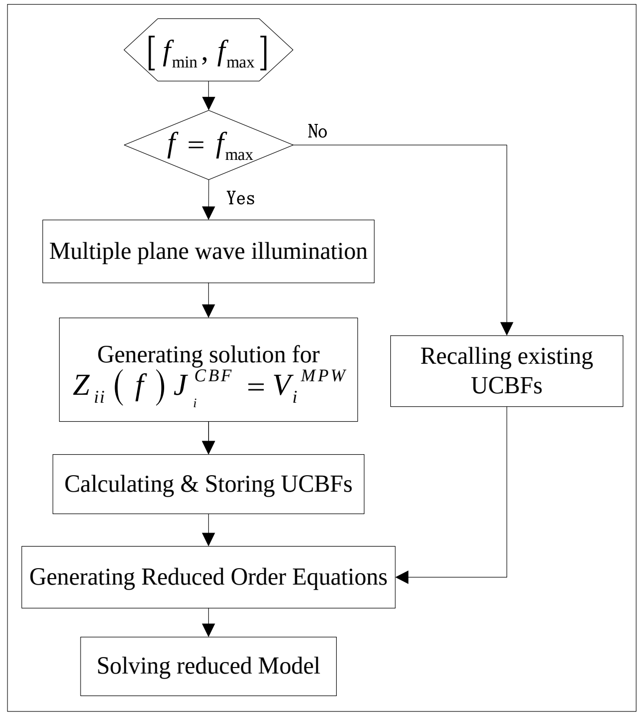


Fig 1. Flowchart of the UCBFM.

where $R(f)$ represents a frequency-domain fitting model and f is the frequency of interest. Since there are $n+d+1$ unknown coefficients (q_0 being arbitrary), a set of $T+1=\zeta+\nu+1$ sample points f_i are required to completely determine $R(f)$. $R(f)$ will then be a curve passing through S_i at f_i for $i=0,1,...,T$. We assume that $R(f)$ exists and has no unattainable frequency points [13] (frequency at which $R(f)$ has a common zero in the numerator and denominator polynomials) for the scattering parameter model that we are trying to attain.

The interpolation function $R(f)$ can be calculated with the recursion formulas in (6)-(7) initialized with (5) and using the inverse differences defined in (8)-(9).

$$\begin{cases}P_0=S(f_0)\\Q_0=1\\P_1=\varphi_1(f_1,f_0)P_0+(f-f_0)\\Q_1=\varphi_1(f_1,f_0)\end{cases} \tag{5}$$

$$\begin{cases}P_t=\varphi_1(f_t,f_{t-1},\dots,f_0)P_{t-1}+(f-f_{t-1})P_{t-2}\\Q_t=\varphi_1(f_t,f_{t-1},\dots,f_0)Q_{t-1}+(f-f_{t-1})Q_{t-2}\\t=2,3,...,T\end{cases} \tag{6}$$

$$R_t(f)=\frac{P_t(f)}{Q_t(f)},\quad t=0,1,...,T \tag{7}$$

$$\varphi_1(f_i,f_0)=\frac{f_i-f_0}{S(f_i)-S(f_i)},\quad i=1,2,...,T \tag{8}$$

$$\begin{aligned}&\varphi_t\left(f_i, f_{t-1}, \ldots, f_0\right)\\&=\frac{f_i-f_{t-1}}{\varphi_{t-1}\left(f_i, f_{t-2}, \ldots, f_0\right)-\varphi_t\left(f_{t-1}, f_{t-2}, \ldots, f_0\right)}\\&i=t, t+1, \ldots, T;\ t=2,3, \ldots, T\end{aligned} \tag{9}$$

The inverse differences in(8)-(9), determined recursively from the sample points, are essentially the polynomial coefficients defining $R(f)$. The rational expressions $R_t(f)$ are partial fractions of (4). Every new sample point increases the order of the rational function by one, until $R(f)= R_T(f)$.

The IASA is defined to work in the interval $[f_0, f_1]$. Define the residual error as

$$E_t(f)=\left|R_t(f)-R_{t-1}(f)\right| \tag{10}$$

Suppose the accuracy is δ. As a first step, an arbitrary third frequency point f_2 is selected which lies in the interval $[f_0, f_1]$. The interpolation function $R_1(f)$ is generated from the samples (f_0, S_0) and (f_2, S_2), while $R_2(f)$ is recursively updated using (6)-(9) and the sample (f_1, S_1). S_t is determined from CEM analyzes at f_t. The residual $E_2(f)$ is determined in the interval $[f_0, f_2]$ by evaluating it at a large number of equi-spaced frequency points over that frequency band. At the maximum of the residual, a new sample point (f_3, S_3) is selected.

For iteration t, the algorithm is used to calculate φ_t, P_t and Q_t from (6)-(9) in order to recursively determine $R_t(f)$. Assuming that the sample (f_t, S_t) was selected in the interval $[f_i, f_j]$, the residual $E_t(f)$ is determined in the intervals $[f_0, f_i]$ and $[f_j, f_1]$. Two new sample points (f_{t+1}, S_{t+1}) and (f_{t+2}, S_{t+2}) are chosen at the maximum of the relative residual in the intervals $[f_0, f_i]$ and $[f_j, f_1]$, respectively. The process is repeated until the relative residual becomes less than δ. The IASA automatically selects and minimizes the number of sample points.

C. *UCBFM/IA-MBPE Method*

The frequency point $f_{\max}$ is calculated first by the UCBFM for storing the UCBFs. The flowchart of UCBFM/IA-MBPE is shown in Fig.2. The frequency point $f_{\max}$, $f_{\min}$ and f_2 are selected firstly and then f_t (t>2) is selected. S_t represents for the RCS, which taking the UCBFM analyzing at f_t. $\psi_t=\{f_0, f_1, \ldots, f_t\}$.

III. Numerical Results

To demonstrate the efficiency and accuracy of the UCBFM/IA-MBPE technique, two numerical examples are investigated. The objects of the numerical simulations are illuminated by a normally incident theta-polarized plane wave from $\theta_{in}=180^{\circ}$, $\varphi_{in}=0^{\circ}$, and the frequency range starts from 0.1GHz and terminates at 0.3GHz. We set $\delta= 0.01$ for the IASA and chose N_{PW} = 800 for UCBFM. All the simulations were run on a notebook equipped with 2 Dual Core at 2.3GHz (only one core was used) and 8GB of RAM.

The first example is the monostatic RCS by a PEC ellipsoid shown in Fig.3. The geometry is automatically divided into four blocks and the discretisation in triangular patches involves

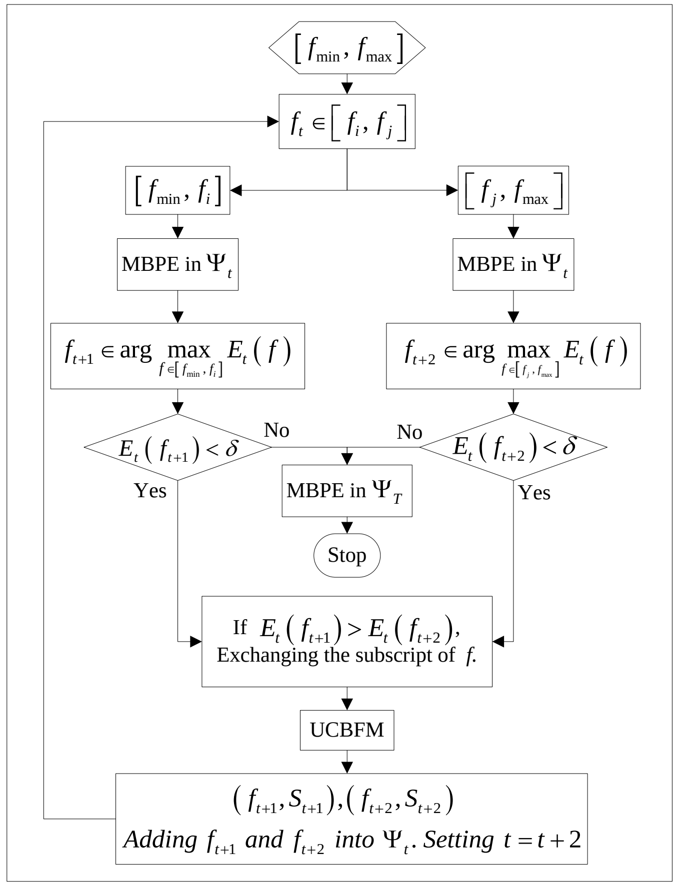
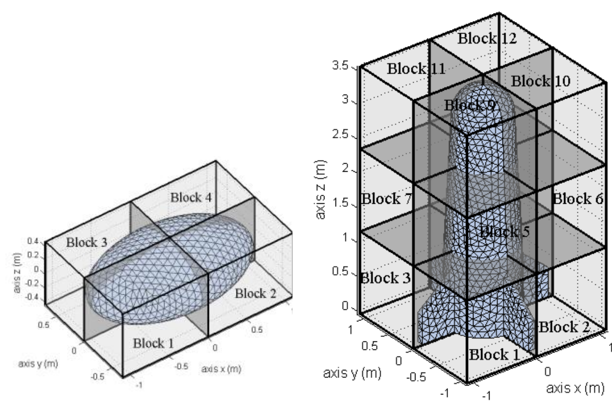


Fig 2. Flowchart of UCBFM/IA-MBPE technique.



(a) ellipsoid (b) missile

Fig 3. Geometries of two examples.

almost 2115 unknowns. After SVD procedure we totally obtain 283 UCBFs. The results are compared with those derived by using UCBFM and direct MoM, as shown in Fig.4. The monostatic RCS calculated by UCBFM/IA-MBPE coincide very well with direct MoM. Hence, the presented method is accurate in wide band electromagnetic scattering analysis.

With a frequency increment of 1MHz, we obtain 201 sample points for direct MoM, while only 22 sample points are needed for UCBFM/IA-MBPE. To show the efficiency of the presented method, the total simulation time which include matrix fill-

TABLE I
COMPUTATIONAL TIMES FOR THE DIFFERENT METHODS OF AN ELLIPSOID

	MoM	UCBFM	MoM/IA-MBPE	UCBFM/IA-MBPE
Total time	8.008 h	7.764 h	1.137 h	0.846 h
Saving	—	3.05%	85.80%	89.44%

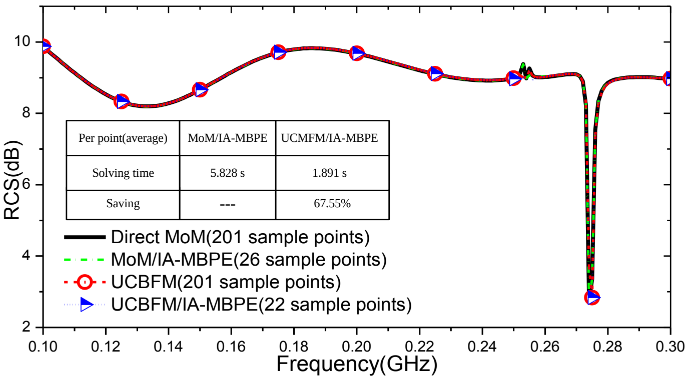


Fig. 4 The ellipsoid monostatic RCS estimated by different methods.

TABLE II
COMPUTATIONAL TIMES FOR THE DIFFERENT METHODS OF A MISSILE

	MoM	UCBFM	MoM/IA-MBPE	UCBFM/IA-MBPE
Total time	35.079 h	27.986 h	11.689 h	8.306 h
Saving	—	20.22%	66.68%	76.33%

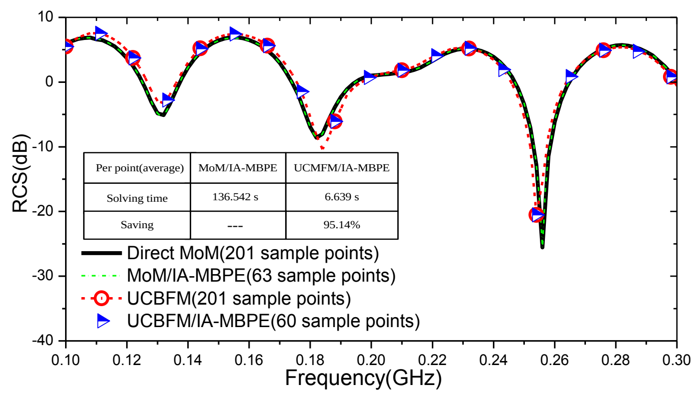


Fig. 5 The missile monostatic RCS estimated by different methods.

ing time and solving time is shown in Table I. The direct MoM requires about 8.008 hours to obtain the solution, While only 0.846 hours are needed for UCBFM/IA-MBPE.

The second example is a PEC missile shown in Fig.3. The geometry is automatically divided into twelve blocks, and the discretisation in triangular patches leads to total unknowns of 4263. After SVD procedure we obtain 765 UCBFs. Fig.5 shows the monostatic RCS by the conductive missile. It can be seen from this figure that the calculated results from UCBFM/IA-MBPE has a good agreement with those of the point to point calculation by direct MoM.

Table II shows the efficiency of the UCBFM/IA-MBPE method. Over the whole frequency band, the total number of the sample points for UCBFM/IA-MBPE is 60, while the direct MoM need 201 sample points. The computational time is reduced from 35.079 hours to 8.306 hours.

The comparison between MoM/IA-MBPE and UCBFM/IA-MBPE shows that the superiority of UCBFM/IA-MBPE over IAMBPE reveals with the increase of the size of the scatterer indicating that UCBFM/IA-MBPE is most suitable for the solving of wide band scattering problems with electrically large objects.

IV. CONCLUSION

In this paper, an efficient approach that combines the UCBFM with IA-MBPE is successfully implemented to fast and efficiently analyze wide band scattering problems. Numerical results demonstrate the high accuracy and efficiency of the proposed hybrid method. The scattering problem of electrically very large and complex objects can be handled, since UCBFM is able to reduce the size of the MoM matrix for fast solving. The IA-MBPE was used in order to further speed up the wide band analysis. Hence, the hybrid method takes both the advantages of UCBFM and IA-MBPE and therefore can reduce considerable total computational time and memory requirements in wide band and electrically large size problems.

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