Full-Wave Analysis of Multiport Microstrip Circuits by Efficient Evaluation of Multilayered Green's Functions in Spatial Domain

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***Abstract* — In this paper, a full wave analysis of multiport microstrip circuits is presented, which is based on the efficient computation of spatial domain dyadic Green's functions and accurate modeling of method of moments (MoM) with perfectly matched ports. With fast and accurate extracting all the surface wave and adequate leaky wave modes, the spatial domain dyadic Green's functions of layered structures can be evaluated by combining the discrete complex image method (DCIM) and the all modes method in near and non-near region, respectively. The Delta-Gap model is adopted to model the exciting port, while the others are perfectly matched by pre-determining the current exponentially decaying. The matrix pencil method (MPM) are adopted to extract the scattering parameters of a 4-port microstrip branch-line hybrid. Very good agreement between simulation and measurement results have been found from the provided example.**

I. Introduction

The fast development in microwave and millimeter wave integrated circuit design has been playing an important role in the modern communication technologies. Therefore, more and more attention has been paid to the rigorous, accurate and fast modeling and simulation methods of multilayered circuits [1]. The multilayered dyadic Green's functions in spatial domain and the mixed potential integral equation (MPIE) have been proved to be one of the most efficient method in full wave analysis of complicated planar multilayered structures with the MoM [2]-[5]. As is well-known, the multilayered Green's functions can be expressed in closed form in spectral domain, and then inversed to spatial domain by calculating the Sommerfeld integrals (SI) [2], [3]. In our recent work [6], [7], [10], the spatial domain Green's functions have been fast and accurately evaluated by means of the combination of the DCIM [8] and the all modes method [9] in near and non-near region, respectively, where the surface and leaky wave modes are extracted by means of a modified dichotomy method and the consecutive perturbation algorithm [10]. With the obtained dyadic Green's functions, the RWG-based MoM can be well constructed with adopting the Delta-Gap voltage excitation model [12].

In this paper, a full wave analysis of multiport microstrip circuits is presented, which is based on the efficient computation of spatial domain dyadic Green's functions and accurate modeling of MoM with perfectly matched ports. The multilayered Green's functions involved in this paper are calculated by means of the combination of the DCIM and the all modes method that have been developed recently. To validate the correctness and efficiency of the presented method, a 4-port branch-line hybrid coupled line filter is simulated, fabricated and measured. Very good agreement between simulation and measurement results have been found from the provided example.

II. Evaluation of Green's Functions

With equivalent expression of the transmission line Green's functions (TLGF) in fractional forms, all the components of the dyadic Green's functions can be rewritten in a convenient way in spectral domain, as is shown in [6]. By using the polynomial denominator of the Green's functions, all the surface wave modes for lossless case can be accurately extracted by a modified dichotomy method [6]. Furthermore, for the layered microstrip structures with moderate thickness and loss, not only all the surface wave but also adequate leaky wave modes have to be extracted to ensure the smoothness of the kernel in the SI. By applying the consecutive perturbation algorithm proposed in [6] and [10], all the necessary modes can be traced and their corresponding residues can be analytically calculated simultaneously. For convenience, the computation method of the spatial domain Green's functions of layered medium is summarized as follows [10]:

a) In near field region, $\rho \le 0.05\lambda$ DCIM [8], [14]:

$$G(\rho) = \frac{A}{4\pi}\left(G_{Q-S} + G_{SWP} + G_{CIM}\right) \tag{1}$$

$$G_{Q-S} = G\big|_{\rho\to\infty} \cdot \frac{e^{-jk_0\rho}}{\rho} \tag{2}$$

$$G_{SWP} = -2\pi j \sum_{i}^{N_{SWP}} \mathrm{Res}_i H_0^{(2)}\left(k_{\rho i}\rho\right) k_{\rho i} \tag{3}$$

$$G_{CIM} = \sum_{i=1}^{N_c} a_i \frac{e^{-jk_0 r_i}}{r_i} \tag{4}$$

where ρ is the projection of the distance between the source and field point on *x-o-y* plane, *A* is a coefficient depending on the medium, G_{Q-S}, G_{SWP} and G_{CIM} stand for the quasi-static term, the surface wave term and the complex image term, respectively. Res_i and $H_0^{(2)}(\bullet)$ stand for the residue of the i-*th* surface wave mode and the zeroth-order Hankel function of the second kind. a_i and r_i are the amplitudes and complex distances of the image terms.

b) In non-near field region, $\rho > 0.05\lambda$ All modes method:

$$\int_{SIP}(\bullet)\,dk_\rho = \int_\Gamma(\bullet)\,dk_\rho - 2\pi j\cdot(\sum_i^{N_{SWP}} R_{k_{\rho i}} + \sum_j^{N_{LWP}} R_{k_{\rho j}}) \tag{5}$$

$$\int_\Gamma(\bullet)\,dk_\rho = \int_\Gamma[\tilde{G}^+(k_\rho)+\tilde{G}^-(k_\rho)]\cdot H_n^{(2)}(k_\rho\rho)k_\rho dk_\rho \tag{6}$$

where Γ stands for the deformed branch cut as depicted in [5]. $\tilde{G}^+$ and $\tilde{G}^-$ stand for the spectral-domain Green's functions on the top and bottom Riemann sheet, respectively. N_{SWP} and N_{LWP} are the number of surface and leaky wave modes, respectively. All the extracted modes involved in (5) can be fast and accurately extracted by the consecutive perturbation algorithm as is proposed in [6] and [10].

III. MPIE Formulation and MoM

The EFIE governing the total current density can be established by enforcing the boundary condition that revokes the vanishing of the total tangential electric field on the conductor surface [4]. However, to avoid the two-dimensional infinite Sommerfeld integrals with highly oscillating, slowly decaying and hyper-singular kernel involved in the EFIE, the MPIE has been preferred and widely used in layered structures, which is composed of vector and scalar potentials with weakly singular kernels [13], [17]. The MPIE can be formulated as below:

$$\vec{E}_t^{imp} = \begin{bmatrix} j\omega\mu_0\left\langle \bar{\bar{G}}^A(\bar{r}\,|\,\bar{r}');\bar{J}(\bar{r}')\right\rangle \\ -\dfrac{1}{j\omega\varepsilon_0}\nabla\left(\left\langle G^\Phi(\bar{r}\,|\,\bar{r}'),\nabla'_S\cdot\bar{J}(\bar{r}')\right\rangle\right)\end{bmatrix}_t \tag{7}$$

where $\vec{E}^{imp} = V_p\delta(\bar{r}-\bar{r}_p)$ stands for the impressed electric field, $\bar{r}_p$ is the location of the port. $\bar{\bar{G}}^A(\bullet)$ and $G^\Phi(\bullet)$ are the dyadic and scalar Green's functions of the vector and scalar potential, respectively.

After obtaining the Green's functions in spatial domain, the MoM can convert the MPIE into a matrix equation. The triangular patches and RWG basis functions are adopted in this paper for the sake of modeling the arbitrarily shaped geometries. With the Galerkin's procedure applying to (7), the integral equation can be derived as [17]:

$$\begin{aligned} &-j\omega\varepsilon_0\int_{T_m}\vec{E}_t^{imp}(\bar{r})\cdot\bar{f}_m(\bar{r})ds \\ &= k_0^2\sum_{n=1}^N I_n\int_{T_m}\int_{T_n}\bar{f}_n(\bar{r}')\cdot\bar{\bar{G}}^A(\bar{r}\,|\,\bar{r}')\cdot\bar{f}_m(\bar{r})ds'ds \\ &+\sum_{n=1}^N I_n\cdot\int_{T_m}\nabla\left(\int_{T_n}G^\Phi(\bar{r}\,|\,\bar{r}')\left(\nabla'_S\cdot\bar{f}_n(\bar{r}')\right)ds'\right)\cdot\bar{f}_m(\bar{r})ds \end{aligned} \tag{8}$$

where $\bar{f}_n$ and $\bar{f}_m$ stand for the RWG basis and weighting functions, respectively. T_n and T_m are the triangular pairs containing the source ($\bar{r}'$) and field ($\bar{r}$) point, respectively. By using the Green's identity and the numerical Gaussian integral over triangular meshes, the integral equation (8) can be deduced as a matrix equation. As is proposed in [12], the delta-gap voltage model is adopted to excite each physical port, which introduces half-RWG subsections to approximate the induced current from the mathematical point of view. Therefore, the MoM matrix equation can be expressed as [12]:

$$\begin{bmatrix} Z^{ff} & Z^{fh} \\ Z^{hf} & Z^{hh}\end{bmatrix}\cdot\begin{bmatrix} I^f \\ I^h\end{bmatrix} = \begin{bmatrix} 0 \\ V^{inc}\end{bmatrix} \tag{9}$$

where the superscripts f and h refer to full- and half-RWG subsections, respectively. In detail, the matrix element involved in (9) can be calculated as [17]:

$$\begin{aligned} Z_{mn} &= k_0^2\int_{T_m}\int_{T_n}\bar{f}_n(\bar{r}')\cdot\underline{\underline{G}}^A(\bar{r}\,|\,\bar{r}')\cdot\bar{f}_m(\bar{r})ds'ds \\ &-\int_{T_m}\int_{T_n}\left(\nabla'_S\cdot\bar{f}_n(\bar{r}')\right)G^\Phi(\bar{r}\,|\,\bar{r}')\left(\nabla_S\cdot\bar{f}_m(\bar{r})\right)ds'ds \end{aligned} \tag{10}$$

Meanwhile, the right hand vector in (9) can be calculated by integration of the impressed excitation electric field over the half-RWG meshes, as is shown below:

$$\begin{aligned} V^{inc} &= -j\omega\varepsilon_0\int_{T_p^-}V_p\delta(\bar{r}-\bar{r}_p)\cdot\hat{n}_p\cdot\frac{l_p}{2A_p^-}\bar{\rho}_p^- ds \\ &= j\omega\varepsilon_0\cdot V_p \end{aligned} \tag{11}$$

where l_p is the length of the triangular edge on the port, A_p^- is area of triangle T_p^-, $\bar{\rho}_p^-$ is a position vector towards the free vertex of T_p^-.

For multiport circuits, all the non-exciting ports should be perfectly matched by enforcing the surface current decaying from the reference plane to the port end exponentially. In this paper, a modified method based on [18] is used to realize that perfectly matching. In detail, the propagation constant on microstrip feed line is firstly determined from the surface current distribution, and then, the reference planes can be selected at least 1 "wavelength" away from the port ends. In this section of the feed line, the normalized current can be pre-determined by artificially enforced as an exponential decaying form. It is not difficult to solve the modified MoM matrix equation when these current coefficients on the matching feedline are known in advance.

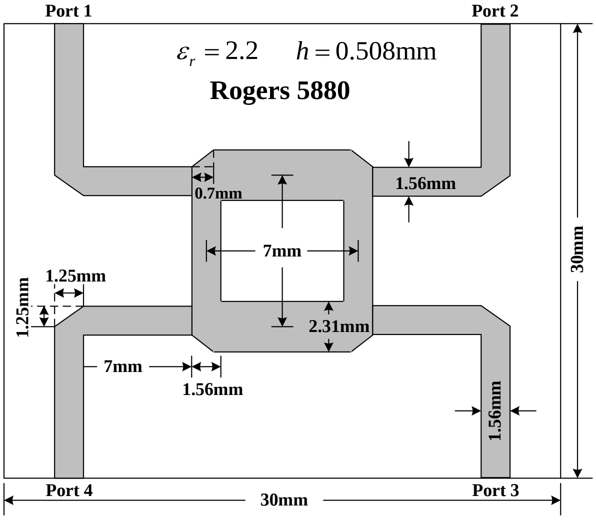
As is well-known, the calculation of matrix element will encounter singularities when the field and source points are very close to each other. In this paper, the singular parts of the matrix element are treated with the method derived in [15]. To observe the recognizable standing-wave feature on the feed line, the reference planes should be selected away from not only the discontinuities but also the exciting ports [12], [14], [17]. The matrix pencil method (MPM) is adopted in this paper. After the fitting operation, the current distribution on feed lines can be written as [16]:

$$\begin{aligned} I(z) &\approx \sum_{i=1}^N p_i\exp(\gamma_i z) \\ &= \sum_{i=1}^N p_i\exp\left[(\alpha_i + j\beta_i)z\right]\ , \qquad z>0 \end{aligned} \tag{12}$$

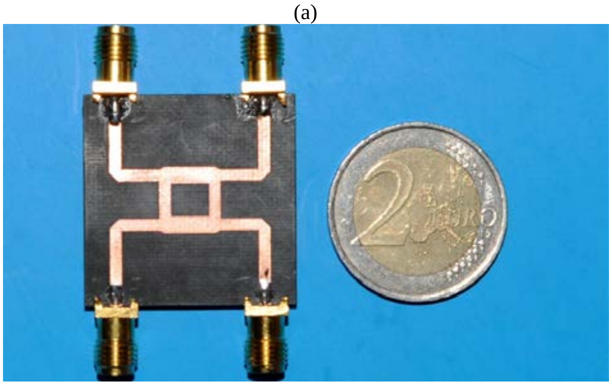
where p_i is the amplitude of the ith mode. α_i and β_i stand for the propagation constant of the ith mode. The reference plane is selected at z = 0. From the physical point of view, the first two terms, ($p_1\ \alpha_1\ \beta_1$) and ($p_2\ \alpha_2\ \beta_2$), are just the incident and reflected wave of the dominant mode [14]. The S_{11} can be easily obtained from them. By applying this fitting algorithm to each port, the scattering matrix can be finally obtained.

IV. Numerical Examples

To verify the method involved in this paper, a microstrip branch-line hybrid is simulated, fabricated and measured, as is shown in Fig. 1. The relative permittivity of the substrate is 2.2 and the thickness is 0.508mm (Rogers 5880). The detailed configuration is provided in Fig.1(a), while the photos of the hybrid is shown in Fig.1(b). In order to facilitate the meshing procedure with triangular patches, the oblique cutting is used on the feed lines.



(a)



(b)

Fig 1 Microstrip branch-line hybrid.

(a) configuration and material. (b) photo of the filter

As is mentioned above, all the others should be perfectly matched except the exciting port. Both the simulation and measurement results are plotted in Fig. 2. From this figure, very good agreement can be found in the waveform of the S-parameters, however, a frequency offset can also be observed. This can be explained by the unstable constitutive parameters of the substrate in this high frequency band.

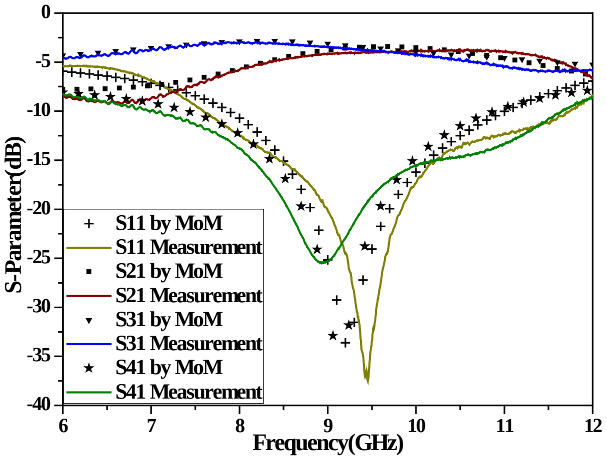


Fig 2 Simulation and Measurement results of the branch-line hybrid

V. Conclusion

In this paper, a full wave analysis of microstrip multiport circuits is presented, which is based on the efficient computation of Green's functions and the accurate modeling of method of moments. With extraction of all the surface wave modes and adequate leaky wave modes, the spatial domain Green's functions of layered medium structure can be obtained by combining the DCIM and the all modes method in near and non-near regions, respectively. The Delta-Gap model is adopted to describe the excitation ports, while all the other non-exciting ports are perfectly matched by artificially enforcing the current distribution exponentially decaying on the feed line. To calculate the S-parameters, the MPM is used to extract the incident and reflected wave of the dominant mode. A 4-port branch-line hybrid is simulated, fabricated and measured to verify the method involved in this paper. Very good agreement between simulation and measurement results have been observed from the example.

VI. Acknowledgement

This work was supported in part by the National Basic Research Program of China (No. 2009CB320203, and No. 2010CB327400) and in part by the National Nonprofit Industry Specific Research Program of China (No. 200910041-2).

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