A Least Bit Error Adaptive Array for Multi-Level Modulations

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1 Introduction

Co-channel interference (CCI) is one of the most horrible obstacles for radio communications systems to increase system throughput. Adaptive arrays are well known to mitigate the performance degradation caused by CCI [1]. Especially, adaptive arrays based on minimum mean square (MMSE) criteria have been investigated intentionally because they can suppress the CCI sufficiently even if direction of arrival (DOA) of CCI varies dynamically. Actually, when the number of arrays is $N_{\rm R}$, $N_{\rm R}-1$ signals of CCI can be only nulled by the MMSE adaptive arrays. Hence, if more than N_R signals of CCI are received, the transmission performance of the MMSE adaptive arrays degrades seriously. On the other hand, adaptive arrays based on minimum bit error rate (BER) criteria are known to be able to mitigate more than N_R interference signals [2, 3, 4, 5]. However, these adaptive arrays have been applied for only the binary phase shift keying (BPSK) modulation, even though multi-level modulations are indispensable for increasing the system throughput, nowadays.

This paper proposes a novel algorithm for multilevel modulation, especially, amplitude phase shift keying (APSK) based on the minimum BER criteria. The proposed algorithm is defined to minimize the BER function of the APSK. Therefore, the proposed algorithm is named as the least bit error rate (LBER) algorithm for multi-level modulation. Moreover, we derive an equation that is satisfied by the weight vector to which the proposed algorithm converges. Furthermore, the performance of the adaptive array based on the proposed algorithm is evaluated by computer simulation that the proposed adaptive arrays achieves superior performance even when the number of interference signals is equal to or more than the number of the arrays.

2 System Model

In this paper, $N_{\rm R}$ antennas are assumed to be put at the receiver, and one desired signal and $K-1$ interference signals are received at the receiver, simultaneously. In addition, all of the transmission signals are APSK modulation signals. Especially, the desired signal is defined as $x_0(l)=\sum_{i=1}^{M} b_{0,i}(l)2^{-i}d \quad l=1,\dots 2^M$ where M is the number of bits conveyed by one APSK modulation signal. In addition, $b_{0,i}(l)$ is an information bit defined as $b_{0,i}(l)=2l-1-\lfloor \frac{l}{2^i}\rfloor 2^{i+1}$, where $\lfloor\alpha\rfloor$ is the floor function that rounds the input signal α down into an integer. Obviously, $2^{-M+1}d$ is the minimum Euclidean distance of the APSK signal. Moreover, the transmission signal vector is defined as $\boldsymbol{X}_l=[x_0(l),x_1,\cdots,x_{K-1}]^{\rm T}$ where $x_i \ i=1,K-1$ represent the interference signals and suffix T means transpose of a matrix or a vector. Let $\boldsymbol{Y}_l$ denote the received signal vector when transmission signal vector $\boldsymbol{X}_l$ is transmitted, the received signal can be expressed as follows.

$$\boldsymbol{Y}_l=\boldsymbol{H}(k)\boldsymbol{X}_l+\boldsymbol{N}. \tag{1}$$

In (1), $\boldsymbol{H}(k)$ and $\boldsymbol{N}$ denote a channel matrix and an additive white Gaussian noise (AWGN) vector, respectively. The received signal vector is provided to an adaptive array in order to regenerate the desired signal $x_0(l)$. Since received signal vector $\boldsymbol{Y}_l$ is expressed in a complex number, basically, the output of the adaptive array has both real and imaginary parts. However, only the real part is used for the following signal processing and performance evaluation, since the desired signals are only mapped on a real axis when the APSK modulation is applied.

$$z_l = \Re\left[\boldsymbol{W}^{\mathrm{H}}\boldsymbol{Y}_l\right] \tag{2}$$

In (2), z_l, $\Re[a]$ $\boldsymbol{W}$ and suffix H represent the real output signal, a real part of a, the weight vector, and Hermitian transpose of a matrix or vector, respectively.

3 Proposed algorithm

Let $\boldsymbol{P}_{\mathbf{r}}(\alpha > \beta)$ denote the probability that the variable α is greater than β, BER of the adaptive array, P, can be defined as follows.

$$\begin{aligned} P &= \sum_{l=1}^{2^M-1} \boldsymbol{P}_1(l)\,\boldsymbol{P}_{\mathbf{r}}\left(|s(l) - b_{0,1}(l)z_l| > 2^{-M}d\right) + \boldsymbol{P}_1\left(2^M\right)\boldsymbol{P}_{\mathbf{r}}\left(s\left(2^M\right) - b_{0,1}(2^M)z_{2^M} > 2^{-M}d\right) \\ &= \sum_{l=1}^{2^M-1} \boldsymbol{P}_1(l)\left\{\boldsymbol{Q}\left(\frac{2^{-M}d - s(l) + b_{0,1}(l)z_l}{\sigma|W|}\right) + \boldsymbol{Q}\left(\frac{2^{-M}d + s(l) - b_{0,1}(l)z_l}{\sigma|W|}\right)\right\} \\ &\quad + \boldsymbol{P}_1\left(2^M\right)\boldsymbol{Q}\left(\frac{2^{-M}d - s\left(2^M\right) + b_{0,1}(2^M)z_{2^M}}{\sigma|W|}\right) \end{aligned} \tag{3}$$

In (3), $\boldsymbol{P}_1(l)$, $b_{0,1}(l)$, σ^2 and $s(l)$ denote the probability that signal $x_0(l)$ is transmitted, the most significant bit of $x_0(l)$, half of the variance of the AWGN, and amplitude of $x_1(l)$, *i.e.*, $b_{0.1}(l)x(l)$, respectively. In addition, $\boldsymbol{Q(u)}$ represents the Gaussian Q-function defined in the following.

$$\boldsymbol{Q}(u) = \frac{1}{\sqrt{2\pi}}\int_u^{\infty} e^{-\frac{t^2}{2}}dt \tag{4}$$

As is described above, our aim is to minimize the BER of the adaptive array output. Therefore, the optimum weight vector of the proposed algorithm is defined to minimize the BER function defined in (3). To find the optimum weight vector, the gradient descent method is applied in our proposed algorithm as is done in the LBER algorithm for BPSK.

$$\boldsymbol{W}_k = \boldsymbol{W}_{k-1} - \mu\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{W}^{\mathrm{H}}}, \tag{5}$$

where μ is a stepsize parameter. Basically, it can be approximated that transmission signals $x_0(l)$ $l = 1, \ldots 2^M$ are transmitted with equal probability. As a result, the weight vector update of the proposed algorithm can be derived as follows.

$$\boldsymbol{W}_k = \boldsymbol{W}_{k-1} + \frac{\mu_0}{\sqrt{2\pi}\sigma}\begin{cases} e^{-\frac{\left|2^{-M}d - s(l) + b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{k-1}|^2}}\left(\frac{b_{0,1}(l)\boldsymbol{Y}_l}{|\boldsymbol{W}_{k-1}|} - \frac{2^{-M}d - s(l) + b_{0,1}(l)z_l}{|\boldsymbol{W}_{k-1}|^3}\boldsymbol{W}_{k-1}\right) & \\ +e^{-\frac{\left|2^{-M}d + s(l) - b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{k-1}|^2}}\left(-\frac{b_{0,1}(l)\boldsymbol{Y}_l}{|\boldsymbol{W}_{k-1}|} - \frac{2^{-M}d + s(l) - b_{0,1}(l)z_l}{|\boldsymbol{W}_{k-1}|^3}\boldsymbol{W}_{k-1}\right) & l \neq 2^M \\ e^{-\frac{\left|2^{-M}d - s(l) + b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{k-1}|^2}}\left(\frac{b_{0,1}(l)\boldsymbol{Y}_l}{|\boldsymbol{W}_{k-1}|} - \frac{2^{-M}d - s(l) + b_{0,1}(l)z_l}{|\boldsymbol{W}_{k-1}|^3}\boldsymbol{W}_{k-1}\right) & l = 2^M \end{cases} \tag{6}$$

In (6), μ_0 is an actual stepsize defined as $\mu_0 = \frac{\mu}{2^M}$. As is seen from (6), the weight vector is updated in the same manner for the transmission signal vectors except for those with $x_0(2^M)$.

4 Performance analysis

Because the nonlinear function is included in the proposed algorithm written in (6), it is quite difficult to analyze the performance of the adaptive arrays based on the proposed algorithm, theoretically. However, we try to get some insight of the performance in this section. In principle, the optimum weight vector $\boldsymbol{W}_{\mathrm{opt}}$ which minimizes the BER is defined as the vector with respect to which the ensemble average of the partial differential of the BER is the zero vector. In a word, $\mathrm{E}\left[\frac{\partial P}{\partial \boldsymbol{W}_{\mathrm{opt}}^{\mathrm{H}}}\right]=0$ where $\boldsymbol{W}_{\mathrm{opt}}$ and $\mathrm{E}[\beta]$ denote the optimum weight and ensemble average of β. Of course, the zero vector multiplied by $\boldsymbol{W}_{\mathrm{opt}}^{\mathrm{H}}$ also becomes zero. So, by using the definition of the BER function in (3), the following equation can be derived.

$$\Re\left[\boldsymbol{W}_{\mathrm{opt}}^{\mathrm{H}}\mathrm{E}\left[\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{W}_{\mathrm{opt}}^{\mathrm{H}}}\right]\right]=\sum_{l=1}^{2^M-1}\mathrm{E}\left[\left(-e^{-\frac{\left|2^{-M}d-s(l+1)+b_{0,1}(l+1)z_{l+1}\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}+e^{-\frac{\left|2^{-M}d+s(l)-b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}\right)\frac{2^{-M}d+s\left(l\right)}{|\boldsymbol{W}_{\mathrm{opt}}|}\right]=0 \tag{7}$$

If a channel matrix is given, the optimum weight vector is expected to stay steady. This means that $\frac{2^{-M}d+s(l)}{|\boldsymbol{W}_{\mathrm{opt}}|}$ $l=1,\ldots,2^M$ are regarded as constants in the above equation. Hence, the solutions of the following equations obviously satisfy the equation in (7).

$$\begin{aligned}\mathrm{E}\left[e^{-\frac{\left|-2^{-M}d+s\left(2^M\right)-b_{0,1}\left(2^M\right)z_{2^M}\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}\right]&=\mathrm{E}\left[e^{-\frac{\left|-2^{-M}d+s\left(2^M\right)-b_{0,1}\left(2^M-1\right)z_{2^M-1}\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}\right]\\&=\mathrm{E}\left[e^{-\frac{\left|2^{-M}d+s\left(2^M-1\right)-b_{0,1}\left(2^M-1\right)z_{2^M-1}\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}\right]=\cdots=\mathrm{E}\left[e^{-\frac{\left|b_{0,1}\left(2^{M-1}\right)z_{2^M-1}\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}\right]\end{aligned} \tag{8}$$

In (8), the property of the APSK modulations, $s(l)+2^{M}d=s(l+1)-2^{-M}d$, is used. Obviously, the equation in (8) is satisfied with z_l $l=1,\ldots,2^M$ If $b_{0,1}\left(l\right)z_l$ has the probability density function that is symmetric with respect to $s(l)$. It is obvious that z_l with this probability density function minimizes the BER. Therefore, the proposed algorithm is expected to converge to the weight vector with which z_l has the probability density function. Therefore, the equation in (8) is satisfied by the proposed algorithm. By using the equation in (8), consequently, the optimum weight vector of the proposed array can be derived from the equation $\mathrm{E}\left[\frac{\partial P}{\partial \boldsymbol{W}_{\mathrm{opt}}^{\mathrm{H}}}\right]=0$.

$$\boldsymbol{W}_{\mathrm{opt}}=\boldsymbol{G}^{-1}\mathrm{E}\left[\sum_{l=1}^{2^M}e^{-\frac{\left|2^{-M}d-s(l)+b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}b_{0,1}\left(l\right)\boldsymbol{Y}_l-\sum_{l=1}^{2^M-1}e^{-\frac{\left|2^{-M}d+s(l)-b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}b_{0,1}\left(l\right)\boldsymbol{Y}_l\right] \tag{9}$$

$$\boldsymbol{G}=\frac{1}{|\boldsymbol{W}_{\mathrm{opt}}|^2}\mathrm{E}\left[\sum_{l=1}^{2^M}e^{-\frac{\left|2^{-M}d-s(l)+b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}b_{0,1}\left(l\right)z_l-\sum_{l=1}^{2^M-1}e^{-\frac{\left|2^{-M}d+s(l)-b_{0,1}(l)z_l\right|^2}{2\sigma^2|\boldsymbol{W}_{\mathrm{opt}}|^2}}b_{0,1}\left(l\right)z_l\right] \tag{10}$$

Obviously, (9) and (10) are regarded as simultaneous nonlinear equations. Still, it is not easy to find the solution of the equations directly. However, since G is a scalar, it is obvious that the directivity pattern of the array is determined by (9). Though the performance of the adaptive array is difficult to analyze in the CCI channel, it can be only found that the adaptive array based on proposed algorithm achieves the performance of the maximum ratio combining (MRC) in channels without interference, because the $\boldsymbol{W}_{\mathrm{opt}}$ is given by the weighted sum of the correlations between $b_{0,1}$ and $\boldsymbol{Y}_l$ in (9). In the next section, therefore, the performance in CCI channels is evaluated by computer simulation.

5 Computer simulation

In the performance evaluation, 2 antennas are applied to the receiver, while 3 signals including one desired signal are simultaneously received with equal power. Modulation scheme of these three signals are 4-APSK. Figure 1(a) shows the BER performance with respect to the DOA of the desired signal when DOAs of the two interference signals are 0 degree and 90 degree. In the figure, the performance of the least mean square (LMS) adaptive array is also drawn. Obviously, the proposed array achieves better performance than the LMS array at ±30 degree of DOA. Figure 1(b) shows the BER performance vs. carrier to noise ratio (CNR) when DOA of the desired signal is fixed to 30 degree. In addition to the performance of the LMS adaptive array in the system model, the performance of the LBER adaptive arrays for the quaternary phase shift keying (QPSK) is added as a reference. Apparently, the proposed adaptive array attains the best performance in the three arrays.

6 Conclusion

A least bit error rate algorithm for multi-level modulations is proposed in this paper. In addition, an equation is derived for the optimum weight vector of the proposed algorithm to satisfy. Computer simulation reveals that the adaptive array based on the proposed algorithm achieves much better performance than that based on the LMS. In addition, it is also shown that APSK modulations are more suitable for the least BER algorithm than the QPSK modulation. Anyway, it can be concluded that the adaptive array based on the proposed algorithm achieve superior performance even if the number of interference signals is as many as that of the received antennas.

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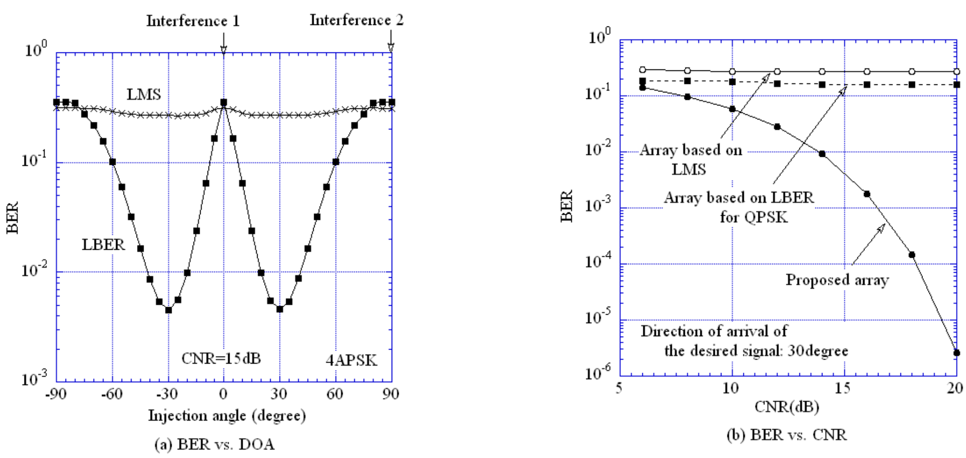

Figure 1: BER performance of the proposed adaptive array