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Abstract—We developed the algorithm for obtaining saddle quasi-periodic solutions and demonstrated them in a ring of coupled bistable oscillators [1]. This algorithm worked when a saddle quasi-periodic solution existed in the basin boundary of two attractors. The reason why we derived the saddle quasi-periodic solution was that it was indispensable to elucidate the bifurcation mechanism of quasi-periodic solutions such as saddle-node, Neimark-Sackker, and pitchfork, etc.

In this paper, we develop the algorithm for calculating Lyapunov exponents (LEs) of the saddle quasi-periodic solution. Usually, LEs are calculated for "attractors," therefore the algorithm using variational equation along a stable flow works well. However, when the flow is unstable as is the case of saddle quasi-periodic solution, we have to calculate the variational equation by correcting the unstable flow for every short time span. We demonstrate the calculation of LEs of pitchfork and saddle-node bifurcations of saddle quasi-periodic solutions in a ring of several number of coupled hard-type oscillators.

As an example, we will introduce LEs for the saddle quasi-periodic solution obtained for the two coupled hard-type oscillator system shown in Eq. (1).

$$\begin{cases} \dot{x}_0 &= x_1, \\ \dot{x}_1 &= -\epsilon(1-\beta x_0^2 + x_0^4)x_1 - (1-\alpha)x_0 \\ & \quad + \alpha(x_2 - 2x_0 + x_2), \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -\epsilon(1-\beta x_2^2 + x_2^4)x_3 - (1-\alpha)x_2 \\ & \quad + \alpha(x_0 - 2x_2 + x_0). \end{cases} \quad (1)$$

From Eq. (1) we can obtain SICC (a stable (nodal) quasi-periodic solution) and UICC1 (an unstable (saddle) quasi-periodic solution) as shown in Fig. 1 [2]. To obtain UICC1 we use the algorithm to calculate a saddle quasi-periodic orbit shown in NOLTA2012 [1]. Fig. 2 shows the variation of Lyapunov exponents for UICC1 and SICC in terms of β. The upper trace of Fig. 2 shows the variation of three LEs of UICC1 in descending order. They show one positive, two almost zero LEs which are one of the evidences of saddle quasi-periodic solution. The lower trace of Fig. 2 shows that of three LEs in the same order. They show two almost zero and one negative LEs which are one of the evidences of stable (nodal) quasi-periodic solution. From this diagram we notice that a saddle-node bifurcation occurs clearly at $\beta = \beta$*=3.909.

For calculating LEs of SICC, we can use ordinary algorithm. In contrast, for calculating LEs of UICC, we must use our developed new algorithm. The algorithm is presented by flowchart shown in Fig. 3 [3]. In Fig. 3 an operator a' =Solve($t_a, t_b; a$) is defined such that numerically solving Eq. (1) with initial condition a at $t = t_a$ gives a new value a' at $t = t_b$. Therefore, the operator Solve($t, t+T; a_t$) gives the numerically calculated value a_{t+T} at $t = t + T$ where T is chosen as $T = 0.1$ sec. Then, by using the obtained saddle solution as a core orbit, we can calculate LEs for the saddle quasi-periodic solution.

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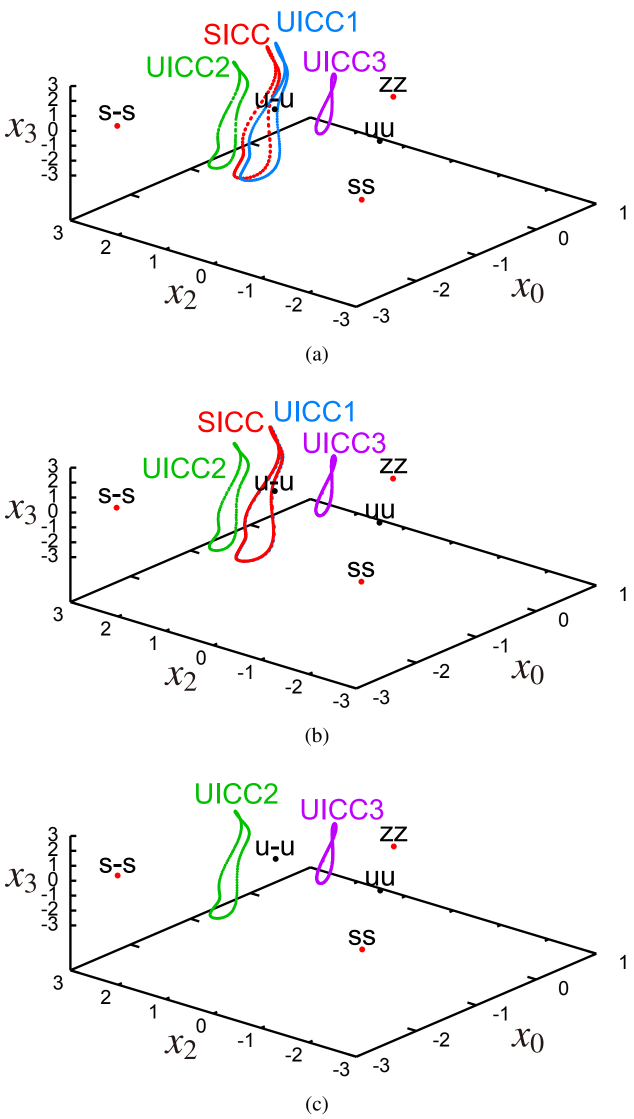

Figure 1: Saddle-node bifurcation of the quasi-periodic attractor for $\alpha = 0.495$ and $\epsilon = 0.1$: the "SICC" (red) incorporates with the "UICC1" (blue) and disappears as β is increased. (a) Before the SN bifurcation for $\beta = 3.898$ (b) At the SN bifurcation for $\beta = \beta^* = 3.909$ (c) After the SN bifurcation for $\beta = 3.910$. The green colored closed curve associates with the "UICC2" and violet colored closed curve associates with the "UICC3," which are out of concern so far.

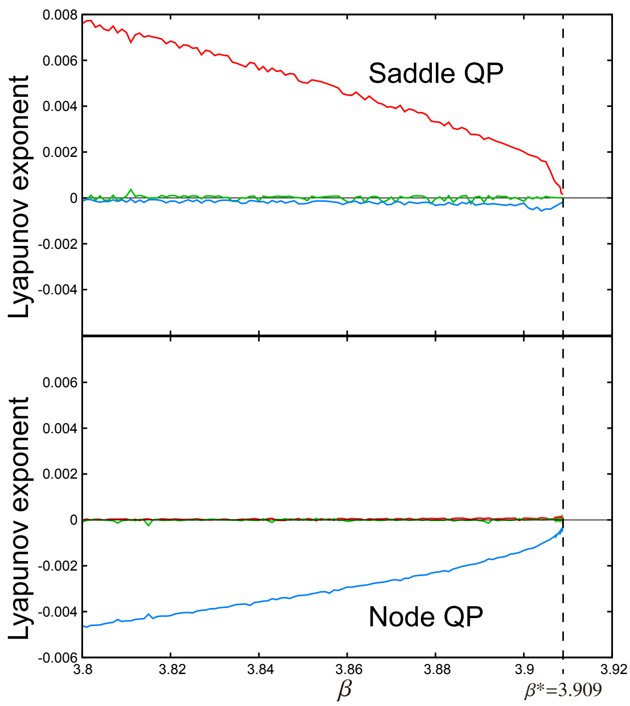

Figure 2: The variation of Lyapunov exponents shown in descending order for $\alpha = 0.495$ and $\epsilon = 0.1$. The smallest LE around -0.35 is omitted. β* presents the saddle-node bifurcation point.

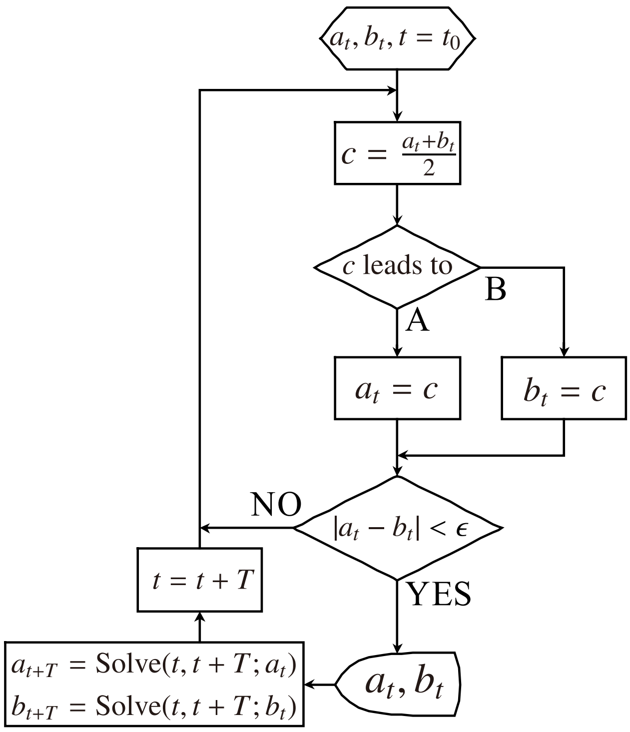

Figure 3: Flowchart for tracing a saddle quasi-periodic solution via the bisection algorithm.