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Synchronization of semiconductor lasers by injection of common broadband random light

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Abstract—We numerically study the synchronization of two semiconductor lasers, which have optical self-feedback loops and are subject to injection of common driving light with fast and randomly fluctuating phase and amplitude. We show that the synchronization is possible for broadband random light injection, and clarify the properties of this type of synchronization in detail.

1. Introduction

A variety of physical systems exhibit oscillatory dynamics. Such systems are as diverse as electrical circuits, chemical reaction systems, and neuronal networks. It is well known that these systems can exhibit various types of synchronization phenomena [1, 2]. Lasers are typical such oscillatory systems and exhibit various synchronization phenomena via electrical or optical signals [3, 4].

Recently, it has been revealed that a common random input could give rise to synchronization between two independent limit-cycle or chaotic systems [5, 6, 7, 8, 9, 10, 11, 12]. This type of synchronization has been experimentally observed in semiconductor lasers driven by common light, in which both the amplitude and phase fluctuate randomly [13, 14] or only the phase fluctuates randomly with constant amplitude [15].

The synchronization of lasers has potential applications to secure communications, and many studies have been made for this issue (e.g., [3, 4]). Recently, we have proposed a secure key distribution scheme using correlated randomness in lasers synchronized by injection of common random light with broad bandwidth, which has a fast randomly fluctuating phase or amplitude [16]. The security of this scheme relies on the difficulty of completely observing the broadband common random light with current technology. Such approach using the limits of observation technology is called *bounded observability* approach [17]. In order to achieve higher security in the above scheme, it is necessary to use a common random light with broader bandwidth, which is more difficult to completely observe. In the experiments in Refs. [13, 14, 15], the bandwidth of common random light was of the order of a few GHz, which is not broad enough. It is an important issue to clarify the nature of synchronization phenomenon in broader bandwidth regime beyond the regime of a few GHz.

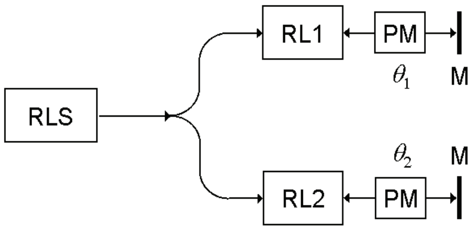

Figure 1: Illustration of the laser system configuration.

In this paper, we consider two semiconductor lasers with optical feedback loops and subject to injection of common random light with much broader bandwidth up to the order of THz. We numerically investigate the condition for their synchronization in detail, focusing on its dependence on the parameters which characterize the lasers or the random light.

2. Model and simulation method

Figure 1 illustrates the configuration of laser system of our study. A portion of light from a random light source (RLS) is injected into two semiconductor lasers, which we call response lasers (RL1, 2). The light has randomly fluctuating phase and amplitude. In experiments, the RLS can be realized by using a super luminescent diode. This optical coupling is unidirectional from the RLS to the response lasers. Each response laser has an external mirror (M) to form an optical self-feedback loop. The loop includes a phase modulator (PM) to vary the phase $\theta_{1,2}$ of the feedback light.

To model the system in Fig. 1, we use the Lang-Kobayashi equation with optical injection [18]:

$$\begin{aligned}\frac{dE_j}{dt} &= \left\{-i\Delta\omega_j + \frac{1+i\alpha}{2}G_N\left(N_j - N_{\mathrm{th}}\right)\right\}E_j \\ &+ \frac{\kappa_{\mathrm{r}}}{\tau_{\mathrm{in}}}E_j(t-\tau)\exp[i\theta_j] + \frac{\kappa_{\mathrm{inj}}}{\tau_{\mathrm{in}}}E_{\mathrm{inj}}(t), \qquad (1)\\ \frac{dN_j}{dt} &= J - \frac{1}{\tau_{\mathrm{s}}}N_j - G_N\left(N_j - N_0\right)|E_j|^2, \qquad (2)\end{aligned}$$

where $j = 1, 2$ indicate the response laser 1 and 2, respectively, E_j represents the complex electric field, N_j the carrier number density, κ_r the optical feedback strength, τ the external-cavity delay time, E_{inj} the complex electric field of the injected common random light, and κ_{inj} the injection strength. The detuning parameter $\Delta\omega_j$ is defined by $\Delta\omega_j = \omega_0 - \omega_j$, where ω_0 is the center optical angular frequency of the injected light and ω_j is that of the jth response laser. For later use, we define the detuning frequency $\Delta f = \Delta\omega_1/2\pi$.

Let $\rho(t)$ and $\phi(t)$ be fluctuations in the amplitude and phase of the injected light E_{inj} defined by $E_{inj}(t) = E_0|1+\varepsilon\rho(t)|\exp[i\phi(t)]$, respectively, where E_0 and ε are positive constants. We assume $\rho(t)$ and $\phi(t)$ are described by the stochastic differential equations

$$\frac{d\rho}{dt} = -\rho/\tau_m + \sqrt{2/\tau_m}\,\xi(t) \tag{3}$$

and

$$\frac{d\phi}{dt} = \sqrt{2/\tau_m}\,\eta(t), \tag{4}$$

where τ_m is a positive constant. In Eqs. (3) and (4), $\xi(t)$ and $\eta(t)$ are the normalized white Gaussian noise with the properties $\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0$, $\langle\xi(t)\eta(s)\rangle = 0$, and $\langle\xi(t)\xi(s)\rangle = \langle\eta(t)\eta(s)\rangle = \delta(t-s)$, where $\langle\cdot\rangle$ denotes the ensemble average and δ is Dirac's delta function. The amplitude $\rho(t)$ is the Ornstein-Uhlenbeck process, and it has the properties $\langle\rho(t)\rangle = 0$ and $\langle\rho(t)\rho(s)\rangle = \exp[-|t-s|/\tau_m]$. This indicates that the correlation time of $\rho(t)$ is given by τ_m. On the other hand, $\phi(t)$ has the property $\langle[\phi(t)-\phi(s)]^2\rangle = 2\tau_m^{-1}|t-s|$. Since $\phi(t)$ has the diffusion constant τ_m^{-1}, its characteristic time for correlation decay can be defined by τ_m. Therefore, τ_m gives the time scale of fluctuation of E_{inj}. This implies that the bandwidth of E_{inj} is of the order of τ_m^{-1}.

In our numerical simulations, the following parameter values were used: $\alpha = 3$, $G_N = 8.4\times10^{-13}\,\mathrm{m^3s^{-1}}$, $N_0 = 1.4\times10^{24}\,\mathrm{m^{-3}}$, $N_{th} = 2.018\times10^{24}\,\mathrm{m^{-3}}$, $\tau_{in} = 8.0\,\mathrm{ps}$, $\tau_s = 2.04\,\mathrm{ns}$, $\tau = 4.0\,\mathrm{ns}$, and $J = 1.19\,J_{th}$, where $J_{th} = N_{th}/\tau_s$ is the lasing threshold of injection current. For this value of J, the response lasers have the relaxation oscillation frequency 2.0 GHz. We assumed a slight detuning between the two response lasers as $\omega_1 - \omega_2 = 0.2$ GHz. As for the injected light, we set $\varepsilon = 0.3$ and $E_0 = [0.19 J_{th}/G_N(N_{th}-N_0)]^{1/2}$. The other parameters κ_{inj}, κ_r, Δf, and τ_m were varied in the simulations.

We are specifically interested in the condition for synchronization of the response lasers, especially its dependence on the parameters which characterize the lasers or the injected light. In order to measure the synchronization, we use the correlation between the output intensities of the two response lasers, $I_j(t) = |E_j(t)|^2$. The correlation between $I_1(t)$ and $I_2(t)$ is defined as

$$C = \frac{\langle(I_1-\mu_1)(I_2-\mu_2)\rangle_T}{\sigma_1\sigma_2}, \tag{5}$$

where μ_j and σ_j are the average and the standard deviation of I_j, respectively, and $\langle\cdot\rangle_T$ denotes the time average. By definition, C is in the range $-1 \le C \le 1$, and it takes the maximum $C = 1$ when the identical synchronization, i.e., $I_1(t) = I_2(t)$ is achieved. We numerically integrate Eqs. (1) and (2) to evaluate C.

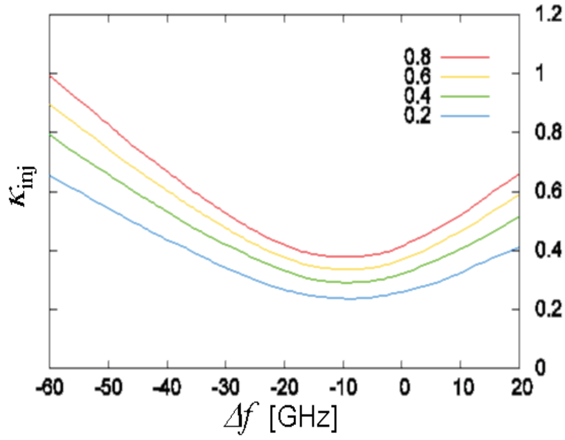

Figure 2: Contour plot of correlation C in $(\Delta f, \kappa_{inj})$ plane for response lasers with $\kappa_r = 0.2$, where $\tau_m = 10\,\mathrm{ps}$.

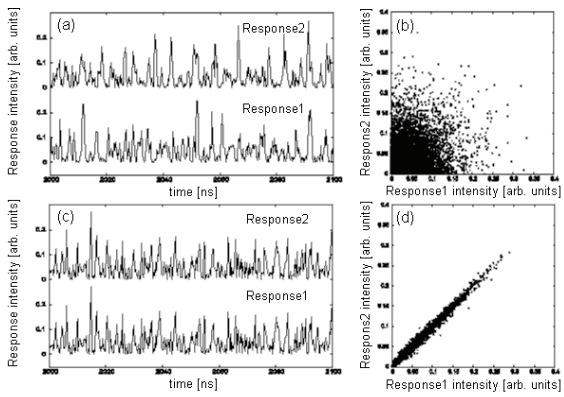

Figure 3: Temporal waveform and correlation plot of outputs of response lasers with $\kappa_r = 0.2$, where $\tau_m = 10\,\mathrm{ps}$. Parameters are $\kappa_{inj} = 0.5$ and (a), (b) $\Delta f = -50\,\mathrm{GHz}$ and (c), (d) $\Delta f = -10\,\mathrm{GHz}$.

3. Numerical results

We show an example of the parameter region for synchronization. It was found that the phase shifts θ_1 and θ_2 of feedback light are important parameters, which significantly affect the degree of synchronization. They were set as $\theta_1 - \theta_2 = 2\pi(\omega_1-\omega_2)\tau$ to maximize C. We will use θ_1 and θ_2 satisfying this relation in what follows. Figure 2 shows contour plot of C as a function of $(\Delta f, \kappa_{inj})$ for $\kappa_r = 0.2$ and $\tau_m = 10\,\mathrm{ps}$. The injected light has a broad bandwidth of the order of 100 GHz. The condition $C > 0.8$

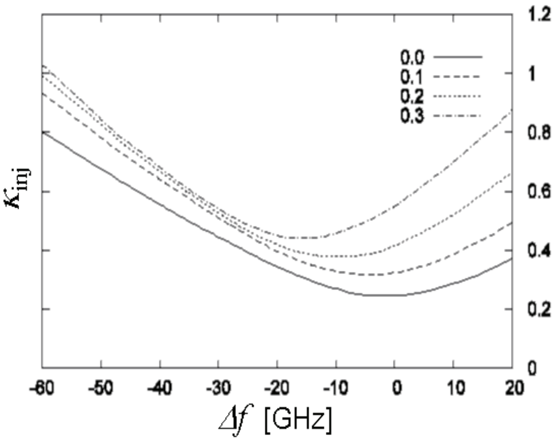

Figure 4: Synchronization region with $C > 0.8$ in $(\Delta f, \kappa_{\rm inj})$ plane for $\kappa_{\rm r} = 0$ (solid line), 0.1 (dashed line), 0.2 (dotted line), and 0.3 (dash-dotted line), where $\tau_{\rm m} = 10\,{\rm ps}$.

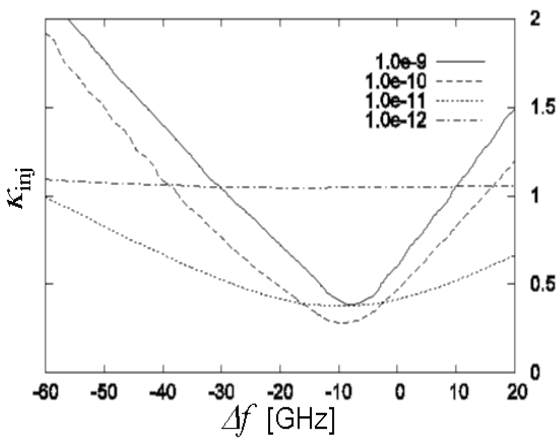

Figure 5: Synchronization region with $C > 0.8$ in $(\Delta f, \kappa_{\rm inj})$ plane for $\tau_{\rm m} = 1\,{\rm ns}$ (red line), $100\,{\rm ps}$ (green line), $10\,{\rm ps}$ (blue line), and $1\,{\rm ps}$ (blue line), where $\kappa_{\rm r} = 0.2$.

is satisfied inside the wedge-shaped region bounded by red line. It was observed that C is very close to unity over most part of this region. This numerical result indicates that synchronization by common random light injection is possible even when the injected light has a broad bandwidth. In what follows, we use $C > 0.8$ as a criterion for the synchronization.

In Figs. 3 (a)-(d), we show temporal waveforms and correlation plots of outputs of the response lasers. Figures 3 (a) and (b) correspond to the case of $\Delta f = -50\,{\rm GHz}$ and $\kappa_{\rm inj} = 0.5$, which is outside the synchronization region in Fig. 2 and has a small correlation. These figures clearly show that the response lasers do not synchronize with each other. Figures 3 (c) and (d) correspond to the case of $\Delta f = -10\,{\rm GHz}$ and $\kappa_{\rm inj} = 0.5$, which is a set of values inside the synchronization region. The correlation is $C \simeq 1$ and a straight line of $I_1 = I_2$ appears in Fig. 3 (d). This clearly indicates that the synchronization of response lasers occurs.

We examine the effects of the feedback strength $\kappa_{\rm r}$. Figure 4 shows the synchronization regions in $(\Delta f, \kappa_{\rm inj})$ plane for different values of $\kappa_{\rm r}$, where $\tau_{\rm m} = 10\,{\rm ps}$. The contour lines of $C = 0.8$ are shown for four different values of $\kappa_{\rm r}$. For each contour line, the synchronization occurs in a region above the line. The synchronization region becomes smaller as the feedback strength $\kappa_{\rm r}$ increases: it is necessary to supply stronger injection light to achieve the synchronization for larger $\kappa_{\rm r}$.

Figure 5 shows how the synchronization region depends on the time scale $\tau_{\rm m}$ of random fluctuations in the injected light. The regions for $C > 0.8$ are shown in $(\Delta f, \kappa_{\rm inj})$ plane for different values of $\tau_{\rm m}$. The synchronization occurs above each boundary curve. This result shows that the synchronization is possible over a wide range of $\tau_{\rm m}$ values, although the minimum value of $\kappa_{\rm inj}$ necessary for synchronization increases as $\tau_{\rm m}$ decreases in the regime $\tau_{\rm m} \leq 100\,{\rm ps}$. The shape of synchronization region changes depending on $\tau_{\rm m}$. A sharp wedge-shaped region appears when $\tau_{\rm m}$ is large and the injected light has a relatively narrow bandwidth. In contrast, the region does not have a wedge shape when $\tau_{\rm m}$ is small and the injected light has a broad bandwidth: the value of $\kappa_{\rm inj}$ on the boundary curve is almost independent of Δf for $\tau_{\rm m} = 1\,{\rm ps}$.

It is known that the optical frequencies of two synchronized lasers coincide with that of the injected common light due to the injection locking when the injected light has a relatively narrow bandwidth [13, 14, 15]. We calculated the frequency Ω_j of each laser to examine whether the frequency locking in the synchronized state still occurs for the injection light with a broad bandwidth. We define the frequency Ω_j as

$$\Omega_j = \lim_{T\to\infty} \frac{1}{2\pi T} \left[\arg E_j(T) - \arg E_j(0)\right]. \qquad (6)$$

Figure 6 shows Ω_1 plotted as a function of Δf for different values of $\tau_{\rm m}$, where the injection and feedback strengths are fixed as $\kappa_{\rm r} = 0.2$ and $\kappa_{\rm inj} = 1.2$. The results for Ω_2 are similar to those of Ω_1. It is clear that $\Omega_1 \simeq 0$ holds in a vicinity of $\Delta f = 0$ for large $\tau_{\rm m}$, namely, $\tau_{\rm m} = 1\,{\rm ns}$ and $100\,{\rm ps}$. This indicates the frequency locking between the optical frequency of a laser and that of the injected light. It turns out from Figs. 5 and 6 that $\Omega_1 \simeq 0$ holds roughly over a range in Δf where the synchronization occurs, for these two $\tau_{\rm m}$ values. This observation clearly confirms that the synchronization is accompanied with the frequency locking in the case of large $\tau_{\rm m}$. In contrast, for small $\tau_{\rm m}$ cases, i.e., broad bandwidth cases, there is no plateau where $\Omega_1 \simeq 0$ holds in Fig. 6. Especially for $\tau_{\rm m} = 1\,{\rm ps}$, Ω_1 depends on Δf almost linearly. This fact

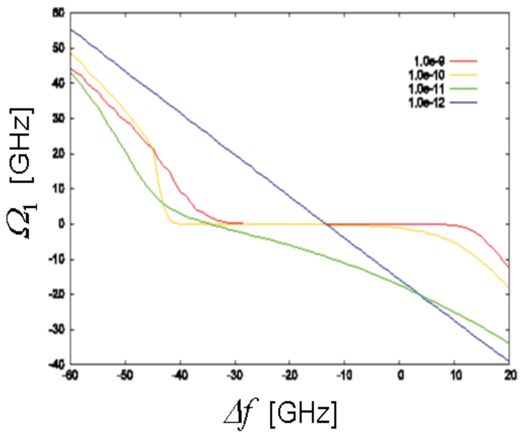

Figure 6: Frequency Ω_1 of electric field E_1 vs. Δf for $\tau_{\mathrm{m}} = 1\,\mathrm{ns}$ (red line), $100\,\mathrm{ps}$ (yellow line), $10\,\mathrm{ps}$ (green line), and $1\,\mathrm{ps}$ (blue line), where $\kappa_{\mathrm{r}} = 0.2$ and $\kappa_{\mathrm{inj}} = 1.2$.

indicates the lack of frequency locking even in the synchronized state in the case of small τ_{m}. The same phenomenon has been theoretically revealed for two detuned limit-cycle oscillators driven by common white Gaussian noise [10]. Thus, synchronization without frequency locking may be regarded as a characteristic of the case of common random signal driving with broad bandwidth.

4. Conclusions

We numerically studied the synchronization of two semiconductor lasers with optical self-feedback loops, which is induced by common injection of random light with broad bandwidth. We have clarified the parameter conditions for the synchronization in detail. The synchronization is possible over a wide range of bandwidth of the injection light, i.e., for $\tau_{\mathrm{m}} = 1\,\mathrm{ns}$ to $1\,\mathrm{ps}$. It has been found that a common random light with fairly broad bandwidth $\tau_{\mathrm{m}} = 1\,\mathrm{ps}$ can induce the synchronization. The minimum value of κ_{inj} for synchronization strongly depends on Δf when the bandwidth is relatively narrow, while it is almost independent of Δf when the bandwidth is broad enough. It was found that the synchronization is accompanied with frequency locking in the narrow bandwidth regime, while it is without frequency locking in the broad bandwidth regime. In addition, we studied the effects of the feedback strength κ_{r} on synchronization and showed that the stronger injection strength is necessary for the larger κ_{r}.

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