Synthesis of Cosecant Array Factor Pattern Using Particle Swarm Optimization

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Abstract-A 24-element symmetrically, equally spaced linear array was synthesize by particle swarm optimization to obtain the cosecant beam pattern in this study. Detailed settings of the array and PSO were presented. Obtained results show that the cosecant squared beam is successfully achieved. Compared the results obtained in this study with those of the published literature, the comparison shows that few numbers of array elements with less number of required iterations by proposed method can achieve the same desired goal.

Figure 1. The geometry of the 24-element symmetrically, equally spaced linear array.

I. INTRODUCTION

Pattern synthesis of antenna array is a vital issue in electromagnetics and antenna engineering. It has become popular for many years. Many techniques have been presented and used [1-6] for synthesizing array patterns. Some array patterns, such as sector beam patterns [7-8], cosecant beam patterns [9-11], and pencil beam patterns with very low sidelobe levels [12], are complicated and difficult, which are not easy to use traditional methods [13] such as Woodward-Lawson method, Taylor method, and Fourier transform method to synthesize them. Hence, using a global optimization algorithm to synthesize a complicated array factor pattern is an alternative way to obtain satisfactory results.

The particle swarm optimization (PSO) is a global optimized technique that can handle a problem with discontinuous and nondifferentiable, and multidimensional characteristics without depending on initial conditions. PSO was proposed in 1995 by Kenney and Eberhar [14], and it has been successfully applied to solve electromagnetic problems [15]. The basic idea of PSO is similar to that of animals to find the food cooperatively. The examples of this concept are the swarm of bees or birds. Bees or birds (particles) are allowed to fly in a finite area (solution space), looking for foods. Assuming there is only one location (the optimal solution) of the food. Once a bee (or bird) finds the location which is better than before, the bee informs other bees to change their location and velocity toward to the food. After certain time (iterations), all bees or birds will gather around the location which close the food (global optimum). This process continues until the location of most of the food is found. PSO algorithm is inspired from this model.

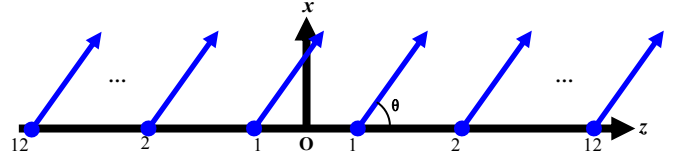In this study, the beam shape synthesis of antenna array is designed and optimized by PSO. The proposed optimization procedure determines the excitation magnitude and phase of each element to synthesis the 24-element linear array to obtain its array factor with cosecant beam shape. Optimized results show that the satisfactory cosecant beam shape of the antenna array has been successfully achieved.

II. COSECANT BEAM ARRAY FACTOR SYNTHESIS USING PSO TECHNIQUES

A 24-element symmetrically, equally spaced linear array aligns along the z-axis is shown in Figure 1. The elements are symmetric with respect to the x-axis. The excitation magnitudes of each symmetric element are the same. However, the phases of each symmetric element are reversed. The spacing between elements is a half-wavelength. For the linear array, there are 12 excitation magnitudes and 12 phases should be determined to synthesis the cosecant beam array factor. Hence, the array factor $AF(\theta)$ can be written as

$$AF(\theta) = I_1 e^{j\beta_1} e^{jkd_1\cos\theta} + I_2 e^{j\beta_2} e^{jkd_2\cos\theta} + \ldots + I_{12} e^{j\beta_{12}} e^{jkd_{12}\cos\theta}$$
$$+ I_1 e^{-j\beta_1} e^{-jkd_1\cos\theta} + I_2 e^{-j\beta_2} e^{-jkd_2\cos\theta} + \ldots + I_{12} e^{-j\beta_{12}} e^{-jkd_{12}\cos\theta}$$
$$= 2\sum_{n=1}^{12} I_n \cos(kd_n\cos\theta + \beta_n). \quad (1)$$

Where, k is the wave number. I_n, β_n, and d_n are the excitation magnitude, phase, and location of the n-th element, respectively. Since the spacing between elements is half-wavelength, the array factor can be simplified as

$$AF(\theta) = 2\sum_{n=1}^{12} I_n \cos\left[\frac{(2n-1)}{2}\pi\cos\theta + \beta_n\right]. \quad (2)$$
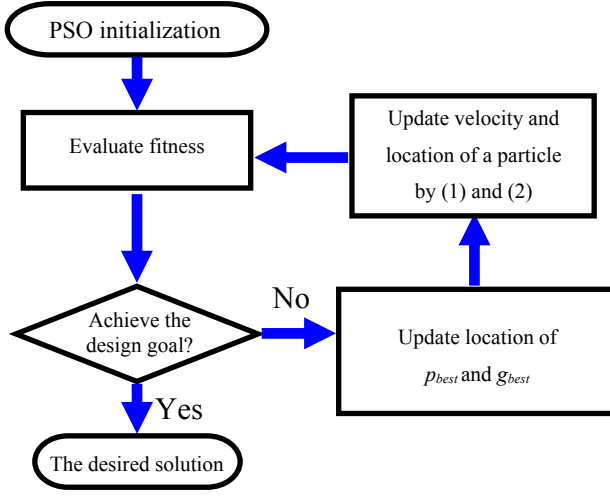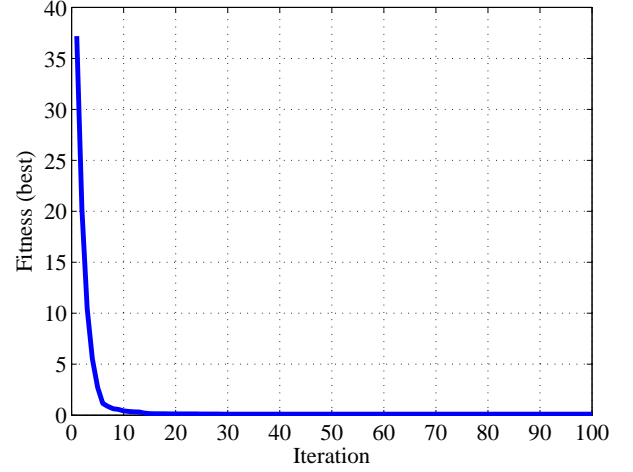
Figure 2. The PSO flow chart.

Figure 3. The fitness curve of the 24-element linear array for the cosecant pattern design by PSO.

The optimized ranges of all magnitudes and phases are set to 0 to 1 and 0 to π, respectively. The desired cosecant pattern is characterized by equations of (3) and (4).

$$f_1(\theta) = \begin{cases} 1 & , \text{ for } 90° \leq \theta < 97° \\ 1.122\,\csc(\cos\theta)\,\csc(\cos 99°) & , \text{ for } 97° \leq \theta < 120° \\ 1.122\,\csc(\cos 135°) - \csc(\cos 99°) & , \text{ for } 120° \leq \theta \leq 127° \\ 10^{-\frac{25}{20}} & , \text{ elsewhere} \end{cases} \quad (3)$$

$$f_2(\theta) = \csc(\cos\theta) - \csc(\cos 99°)/1.122 \quad , \text{ for } 95.8° \leq \theta \leq 120° . \quad (4)$$

As can be seen later in Figure 4, $f_1(\theta)$ is denoted by red dashed lines and $f_2(\theta)$ is denoted by blue dotted lines. The $f_1(\theta)$ allows 1.0 dB (1.122 in linear scale) tolerance between 97^0 and 120^0 and restricts sidelobe levels below -25 dB between 0^0 and 90^0 and between 127^0 to 180^0. The $f_2(\theta)$ allows 1.0 dB tolerance with $f_1(\theta)$ between 95.8^0 and 120^0.

The PSO optimization procedure flow chart is shown in Figure 2. In PSO, p_{best} is the location of the best result of each particle; g_{best} is the location of the best result of the entire particles in history. In each iteration, the velocity and location of each particle are updated by equations of (5) and (6), respectively.

$$v_n = w \cdot v_n + c_1\, rand_1()\,(p_{best} - x_n) \qquad (5)$$
$$+ c_2\, rand_2()\,(g_{best} - x_n),$$

$$x_n = x_n + v_n . \qquad (6)$$

Where, v_n is a particle's velocity. x_n is a particle's location. c_1 and c_2 are scaling factors, which are both set to 1.8. w is an inertia weight, which is decreased linearly from 0.9 to 0.4 over the course of the iteration. $rand_1()$ and $rand_2()$ are uniform random values with the range between 0 and 1. In this study,

the number of particles is set to 81. The maximum number of iteration is set to 100. Once an x_n of a particle is out of its optimization range, the x_n is changed to the minimum or maximum boundary. Then, the sign of the particle's velocity v_n is changed which forces the particle reflected back toward to the solution space.

A fitness function is applied to evaluate the performance of current optimization process by the obtained result. The following fitness function is used in this optimization

$$\text{Fitness} = w_1 \sum_{\theta=0°}^{\theta=180°} (AF(\theta) - f_1(\theta))[\frac{1 + \text{sgn}(AF(\theta) - f_1(\theta))}{2}]\Delta\theta \qquad (7)$$
$$+ w_2 \sum_{\theta=95.8°}^{\theta=120°} (f_2(\theta) - AF(\theta))[\frac{1 + \text{sgn}(f_2(\theta) - AF(\theta))}{2}]\Delta\theta .$$

Where, the $\Delta\theta$ is the angular interval, which is set to 0.1^0. W1 and W2 are the weights of the fitness from the regions shown in (3) and (4), respectively. Both W1 and W2 are set to 1.0 in this study. The fitness shown in (7) accumulates the difference area between the desired pattern and the obtained pattern. The smaller the fitness reflects the better obtained pattern.

III. OPTIMIZATION RESULTS

Figure 3 shows the fitness curve versus iteration. The fitness drops significantly during the first 10 iterations. After that, it is converged between 10-th to 100-th iterations. It finally reaches the value of 0.095 at the maximum number of iteration 100. The optimized cosecant array factor pattern is shown in Figure 4. The desired cosecant array factor pattern is successfully achieved. Sidelobe levels are below -25 dB. Compared with the results shown in [11], it can be found that there are 30 elements used in the same linear array to achieve the same desired goal by a the tabu search algorithm (TSA). The reduction in the number of array element is six, which shows that the optimization performance of the proposed PSO
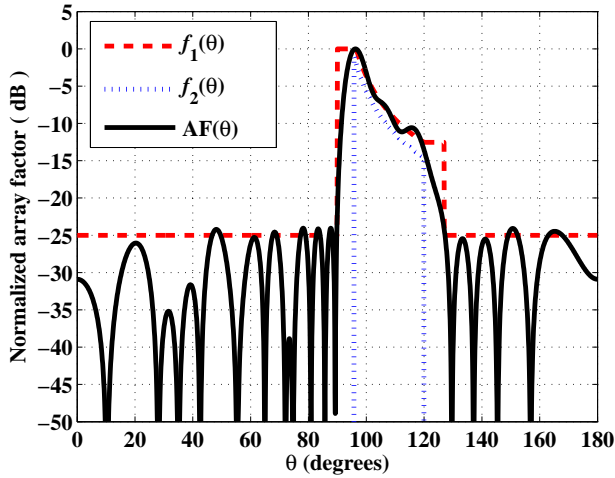
Figure 4. Optimized array factor (normalized) with cosecant array factor pattern obtained by PSO.

TABLE I
OPTIMIZED EXCITATION MAGNITUDES AND PHASES OF THE ELEMENTS

Element numbers	Optimized magnitudes (Normalized)	Optimized phases (Degree)
1	1.0000	20.6908
2	0.8433	57.5668
3	0.5438	86.2708
4	0.3609	91.4545
5	0.3115	87.1325
6	0.3029	124.6747
7	0.2977	138.9065
8	0.1539	143.8578
9	0.1454	148.8518
10	0.2045	179.9873
11	0.1050	151.3374
12	0.1584	179.9196

is better in this problem. The optimized excitation magnitudes and phases of array elements are shown in Figure 5. Detailed values of magnitudes and phases of elements are listed in Table I.

IV. CONCLUSION

The paper describes a cosecant beam pattern of the 24-element symmetrically, equally spaced linear array was successfully achieved by the proposed PSO method. The fitness value converges to the optimal result quickly with few numbers of iterations. The proposed optimization method is easy to implement. Results of the optimization comparison
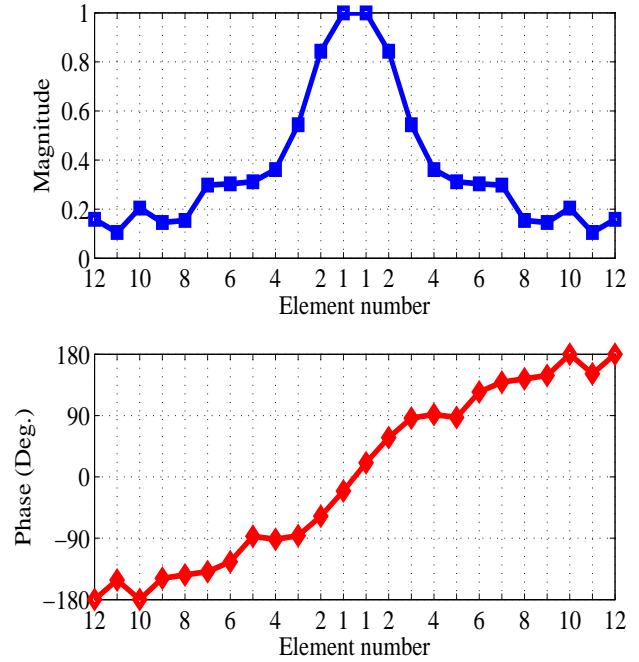

Figure 5. Optimized excitation magnitudes (normalized) and phases of the 24-element linear array for the cosecant array factor pattern.

show that six array elements reduction and less number of required iterations can achieve the same desired goal by the proposed PSO method.

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