

Calculation of the Phase Center of an Ultra-wideband Feed for Reflector Antennas

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Abstract—Next generation ultra-wideband (UWB) radio telescopes require UWB feeds for reflector antennas. Different from narrow band feeds, how to determine the optimal phase center location for a UWB feed over a wide operating frequency band has not been investigated much and therefore analysis and discussions on this issue are needed. In this paper, a method for calculating the optimal phase center of a UWB feed is presented. Examples of the Eleven feeds, a decade bandwidth feed, are used to demonstrate the applications of the new method.

Index Terms—Phase center, ultra-wideband antenna, feed, reflector antenna, Eleven antenna

I. INTRODUCTION

The ongoing developments of the next generation ultra-wideband (UWB) radio telescopes, such as the Square Kilometer Array (SKA) [1] and VLBI2010 [2], have pushed technologies forward on a broad front. For example, UWB feed technologies for reflector antennas have made substantial progress [3], such as the Eleven feed [4]–[6], the quadruple-ridged flared horn [7], the sinuous feed [8] and the quasi self-complementary antenna [9].

The issue of the phase center of a feed for reflectors has been discussed in [10]–[14], mainly for narrow band, and the results have been applied in designs of different feeds, for example in hat feeds [15]–[18].

In this paper, we discuss how to determine the optimal phase center for a wideband feed.

II. DEFINITION

The aperture efficiency e_{ap} of a feed for parabolic reflectors can be expressed by its sub-efficiencies as [19]–[21]

$$e_{ap} = e_{sp}e_{BOR1}e_{ill}e_{pol}e_{\phi}, \quad (1)$$

where e_{sp} , e_{BOR1} , e_{ill} , e_{pol} and e_{ϕ} are the spillover efficiency, the BOR₁ efficiency, the illumination efficiency, the polarization efficiency and the phase efficiency, respectively. The phase efficiency can be expressed by

$$e_{\phi}(f, z) = \frac{\left| \int_0^{\theta_0} G_{co45_BOR1}(\theta, f) \tan(\theta/2) d\theta \right|^2}{\left[\int_0^{\theta_0} |G_{co45_BOR1}(\theta, f)| \tan(\theta/2) d\theta \right]^2}, \quad (2)$$

where $G_{co45_BOR1}(\theta, f) = |G_{co45_BOR1}(\theta, f)| e^{j\phi_z(\theta, f)}$ is the co-polar radiation function of the BOR₁ component in $\varphi = 45^\circ$ plane of the feed. The phase function is $\phi_z(\theta, f) = \phi(\theta, f) - kz \cos(\theta)$ when the phase reference point is moved

from the origin to a point z in the coordinate system of the feed, where $k = 2\pi/\lambda$ is the wave number.

In order to find an optimal phase center over an ultra-wide frequency band, we define first a new characterization - the optimal frequency-weighted phase efficiency as

$$e_{\phi, opt}(z) = \frac{\int_{f_1}^{f_2} w(f) e_{\phi}(f, z) df}{\int_{f_1}^{f_2} w(f) df} = \int_{f_1}^{f_2} \bar{w}(f) e_{\phi}(f, z) df, \quad (3)$$

where $w(f)$ is an optimal frequency weighting function according to the application, and $\bar{w}(f)$ the normalized optimal weighting function defined by

$$\bar{w}(f) = \frac{w(f)}{\int_{f_1}^{f_2} w(f) df}. \quad (4)$$

The choice of $w(f)$ is very much depending on applications. For example, in radio astronomy, the phase efficiency may not be important at low frequency where the sky noise is very high but it is very critical to have high phase efficiency at high frequencies. So the weighting function can be chosen to weight on high frequencies.

Then, the optimal phase center over an ultra-wide band is defined as the phase reference point which maximizes the optimal frequency-weighted phase efficiency, i.e.

$$Z_{pc, opt} = \arg \max_z e_{\phi, opt}(z). \quad (5)$$

III. FORMULATION

Then, the optimal phase center can be determined efficiently by the formulas and procedure as follows.

A. Initial optimal UWB phase center

As it is known from [10], the phase center for a single frequency point can be determined preliminarily (or approximately) by

$$Z_{0pc}(f) = \frac{\phi(\theta_0, f) - \phi(0, f)}{(\cos(\theta_0) - 1)k}, \quad (6)$$

where θ_0 is the half subtended angle of the reflector, which makes $\phi_z(\theta_0, f) = \phi_z(0, f)$.

Then, we define the preliminary optimal phase center location $Z_{0pc, opt}$ as

$$Z_{0pc, opt} = \int_{f_1}^{f_2} \bar{w}(f) \frac{\phi(\theta_0, f) - \phi(0, f)}{(\cos(\theta_0) - 1)k} df. \quad (7)$$

B. Determining the optimal UWB phase center

We assume then that the optimal UWB phase center $Z_{pc,opt}$ is not far from the preliminary optimal UWB phase center $Z_{0pc,opt}$, i.e.,

$$Z_{pc,opt} = Z_{0pc,opt} + \Delta z. \quad (8)$$

and $|k\Delta z| \ll 1$ over the operating frequency band of (f_1, f_2) , which means that $|k_2\Delta z| = |2\pi/\lambda_2\Delta z| \ll 1$, where λ_2 is the wavelength at f_2 . Now the phase function can be expressed as

$$\phi_z(\theta, f) = \phi(\theta, f) - kZ_{0pc,opt} \cos(\theta) - k\Delta z \cos(\theta). \quad (9)$$

Then, by using the Taylor expansion

$$e^{-jx} = 1 - jx - \frac{x^2}{2}, \quad \text{when } |x| \ll 1,$$

the optimal phase efficiency in (3) can be expressed to second order in Δz as

$$e_{\phi,opt}(z) = I_0 + I_1(\Delta z) + I_2(\Delta z)^2, \quad (10)$$

where I_0 , I_1 and I_2 can have analytic expressions in a format similar (but much more complicated) to those in [11], [12], [14]. Actually, there is no advantage to use the analytic but very complicated formulas to find the optimal phase center. Instead, we can use the following simple method to determine the phase center location.

Choose three points of Δz around $\Delta z = 0$ randomly, say Δz_1 , Δz_2 and Δz_3 . Calculate the optimal phase efficiencies $e_{\phi,opt}(\Delta z_1)$, $e_{\phi,opt}(\Delta z_2)$ and $e_{\phi,opt}(\Delta z_3)$ so we can obtain the values of I_1 , I_2 and I_3 from (10)

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & \Delta z_1 & (\Delta z_1)^2 \\ 1 & \Delta z_2 & (\Delta z_2)^2 \\ 1 & \Delta z_3 & (\Delta z_3)^2 \end{bmatrix}^{-1} \begin{bmatrix} e_{\phi,opt}(\Delta z_1) \\ e_{\phi,opt}(\Delta z_2) \\ e_{\phi,opt}(\Delta z_3) \end{bmatrix} \quad (11)$$

Thus, the optimal phase center of a feed for paraboloids over a ultra-wide band can be determined by

$$Z_{pc,opt} = Z_{0pc,opt} - \frac{I_1}{2I_2}, \quad (12)$$

and the maximum phase efficiency is

$$e_{pc,opt} = I_0 - \frac{I_1^2}{4I_2}. \quad (13)$$

IV. EXAMPLE

The Eleven feed is a decade bandwidth feed [22], [23]. Fig. 1 shows a 1.5–14GHz Eleven feed for reflectors [24]. This feed has been measured and we have the measured complex $G_{co45_BOR1}(\theta, f)$. Applying the above formulas, we can obtain the phase center as shown in Fig.2. First, we calculate $Z_{0pc,opt}$ by (7), which is $Z_{0pc,opt} = -62.0$ mm in this case. Then, choose three values of Δz , as shown by the red circles in Fig. 2. Using (11)-(13), the phase center can be calculated ($\Delta z = -10.86$ mm), depicted by the black square in Fig. 2. As a verification, we calculate the phase efficiency $e_{\phi,opt}(z)$ over $Z_{0pc,opt} + \Delta z$ where $k_2\Delta z$ is from -0.6 to 0.6, shown by the blue line. From this curve, we see that the phase center is

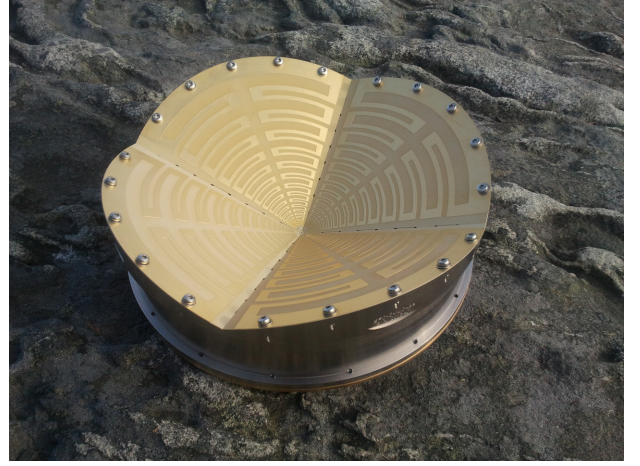


Fig. 1. Photo of the 1.5–14GHz Eleven feed.

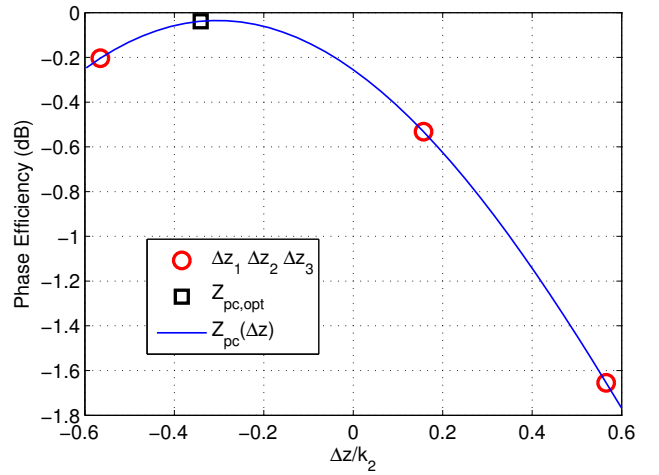


Fig. 2. Phase center calculation.

located at the point marked by the black square. Fig. 3 shows the aperture efficiency and its sub-efficiencies, including the phase efficiency, calculated based on the measured radiation patterns, when the feed illuminates a primary-focused reflector with a subtended angle of $2 \times 60^\circ$. From this figure, the phase efficiency e_ϕ is very high (close to 1) over the whole frequency band of 1–14 GHz.

V. CONCLUSION

In the paper, a simple and fast formula for calculation of the phase center of a wideband feed for reflectors is presented and applied to an example of 1.5–14GHz Eleven feed.

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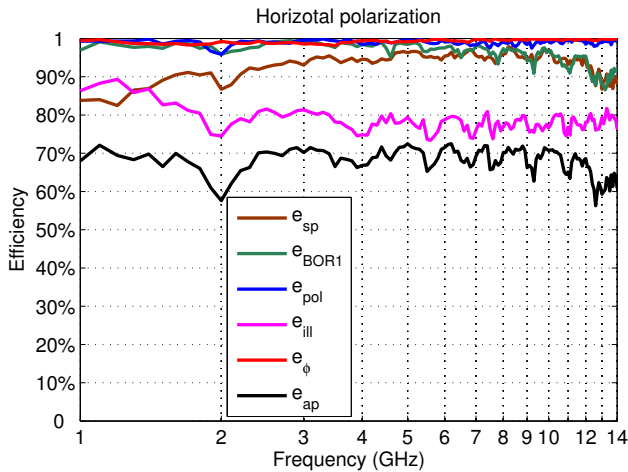


Fig. 3. Efficiency of the 1.5–14GHz feed based on the measured data, with a subtended angle of $2 \times 60^\circ$.

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