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A Hyperchaotic Circuit with Impulsive Switching Controlled by Refractory Threshold and Spike-Train Input

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Abstract—This paper studies a continuous-time non-autonomous spiking oscillators that can exhibit hyperchaos. The circuit has a firing switch depending on the threshold of a state variable and periodic clock signal. As parameters vary, the circuit can exhibit a variety of chaotic/periodic spike-trains. We analyze the spike-trains by three tools: the return map, the histogram and the recurrence plot. Using these tools, we investigate characteristics of typical spike-trains.

1. Introduction

This paper studies a hyperchaotic circuit (HCC) with impulsive switching controlled by refractory threshold and spike-train input. HCSC closely relates to spiking neurons model [1]-[3] and it can output various spike-trains based on integrate-and-fire behavior. They can be a building block of various engineering systems including image processing and ultra wide band communications [4]-[6]. A 4D system constructed by three capacitors, three voltage controlled current sources (VCCSs), a firing switch and a voltage source. A firing switch is constructed by two switches: state-dependent switch S_S and time-dependent switch S_T. When firing switches are opened, capacitor voltage vibrates divergently. We introduce three analysis tools of the spike-trains of the HCC. The first tool is the return map (Rmap) of the state variable at every firing moment. This circuit has 2D piecewise-linear Rap. Rmap is useful to analyze HCC's stability and bifurcation of spike-train generators, provided the state variables are observable. HCC can output various phenomenon, we remark a hyperchaotic phenomenon. We derive Lyapunov exponents from Rmap. The second tool is the histogram of inter-spike intervals (ISI). It corresponds to the Fourier spectrum for smooth signals and is convenient to extract basic information from spike-trains. The third tool is the recurrence plot (RP [7] [8]) for ISI sequences. The RP has been used to characterize chaotic/periodic attractors and has potential to extract information hidden in the ISI sequence. We apply these tools to the HCCs and demonstrate typical date. The results provide basic information to develop systematic analysis of spike-trains in nonlinear circuits and systems. We studied various circuits related the HCC [9]-[17], this is the first paper that analyze 4D systems using firing switch constructed by S_S and S_T. And this is the first paper that analyze hyperchaos by piecewise linear 2D Return map.

2. Hyperchaotic Circuit

The HCC is constructed by three capacitors C_1, C_2, C_3, three voltage controlled current sources VCCS$_1$,VCCS$_2$,VCCS$_3$, the state-dependent switch S_S, the time-dependent switch S_T and the voltage source E. Fig.1(a) shows circuit model of the HCC. These VCCS's output currents are described by the following equation.

$$(i_1, i_2, i_3) = (g_1 v_3, g_2(v_2 - v_3), g_3(v_2 - v_1)) \quad (1)$$

Therefore dynamics of the HCC and spike-trains $z(t)$ described by the following circuit equation.

$$\frac{d}{dt}\begin{bmatrix} C_1 v_1 \\ C_2 v_2 \\ C_3 v_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & g_1 \\ 0 & g_2 & -g_2 \\ -g_3 & g_3 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \text{ for } z = 0 \quad (2)$$

$$(v_1(t+), v_2(t+), v_3(t+)) = (E, v_2(t), v_3(t)) \text{ for } z = 1 \quad (3)$$

$$z(t) = \begin{cases} 1 & \text{if } v_1 \geq V_T \text{ and } d = nT \\ 0 & \text{otherwise} \end{cases}$$

Fig.1(b) shows the dynamics of the HCC. If $z = 0$ then capacitor voltage vibrates divergently. If the capacitor voltage v_1 exceeds the threshold voltage V_T, the switch S_S is closed. And a clock signal arrives, the switch S_T is closed. If the switch S_S and the switch S_T are closed, z becomes 1 and the capacitor voltage v_1 jumps to the base level E. Repeating this vibrate-and-fire behavior, the HCC can output various spike-trains $z(\tau)$.

We assume that the characteristic root of Equation (2) has complex conjugate eigenvalues described Equation(4).

$$s^3 - \frac{g_2}{C_2}s^2 + (\frac{g_1 g_3}{C_1 C_3} + \frac{g_2 g_3}{C_2 C_3})s - \frac{g_1 g_2 g_3}{C_1 C_2 C_3} = (s - \lambda\omega)(s^2 - \delta\omega s - (\delta^2\omega^2 + \omega^2)) \quad (4)$$

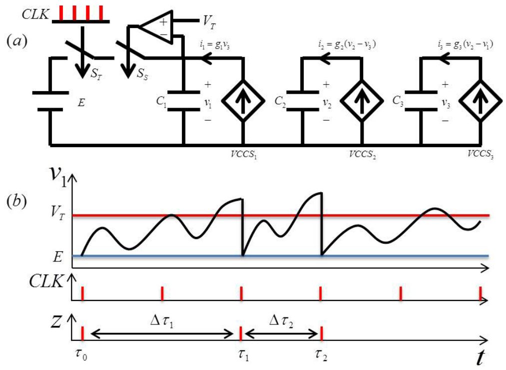

Figure 1: (a)Circuit model of the HCC, (b)Dynamics of the HCC

For convenience of discussions on analysis of spike-trains, using the following dimensionless parameters and variables:

$\tau = \omega t, d = \omega T, q = \frac{E}{V_T}, \boldsymbol{v} = [v_1, v_2, v_3]^T,$
$\boldsymbol{u} = [u_1, u_2, u_3]^T, \boldsymbol{e}_1 = [1, 0, 0]^T, \boldsymbol{p} = [1 - p_3, p_2, p_3],$
$\boldsymbol{v} = \boldsymbol{T}\boldsymbol{u}, \boldsymbol{p} = \boldsymbol{T}^{-1}\boldsymbol{e}_1{}^T$

$$\boldsymbol{T} = V_T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{g_3 C_1}{g_1 C_3} & \frac{g_3 C_1}{g_1 C_3} & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} 1 & 0 & 1 \\ \frac{\delta \omega C_1}{g_1} & \frac{\omega C_1}{g_1} & \frac{\lambda \omega C_1}{g_1} \\ (\delta^2 - 1)(\frac{\omega C_1}{g_1})^2 & 2\delta(\frac{\omega C_1}{g_1})^2 & (\frac{\lambda \omega C_1}{g_1})^2 \end{bmatrix} \quad (5)$$

Equation(3)(4) is transformed into Equation(6).

$$\frac{d}{d\tau}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \delta & 1 & 0 \\ -1 & \delta & 0 \\ 0 & 0 & \lambda \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \text{ for } z = 0$$
$$\begin{bmatrix} u_1(\tau_+) \\ u_2(\tau_+) \\ u_3(\tau_+) \end{bmatrix} = \begin{bmatrix} u_1 - (1 - p_3)(u_1 + u_3 - q) \\ u_3 - p_2(u_1 + u_3 - q) \\ u_3 - p_3(u_1 + u_3 - q) \end{bmatrix} \text{ for } z = 1 \quad (6)$$

$$z(t) = \begin{cases} 1 & \text{if } u_1 + u_3 \geq 1 \text{ and } \tau = nd \\ 0 & \text{otherwise} \end{cases}$$

This is the normal form equation having six parameters: $(\delta, \lambda, p_2, p_3, q, d)$. For simplicity, we define parameters as $(\delta, \lambda, p_2, p_3, q) = (0.02, 0.04, -0.05, 0.5, 0.7)$.

The trajectory rotates divergently around the u_3 axis. If the trajectory exceeds the threshold plane $u_1 + u_3 = 1$ and clock period reaches, it jumps instantaneously on to the base plane $u_1 + u_3 = q$ along direction vector $(1 - p_3, p_2, p_3)^T$. Equation(6) has the following exact piecewise solution.

$$\begin{bmatrix} u_1(\tau) \\ u_2(\tau) \\ u_3(\tau) \end{bmatrix} = e^{\delta\tau}\begin{bmatrix} \cos\tau & \sin\tau & 0 \\ -\sin\tau & \cos\tau & 0 \\ 0 & 0 & e^{(\lambda-\delta)\tau} \end{bmatrix}\begin{bmatrix} u_1(0) \\ u_2(0) \\ u_3(0) \end{bmatrix} \quad (7)$$

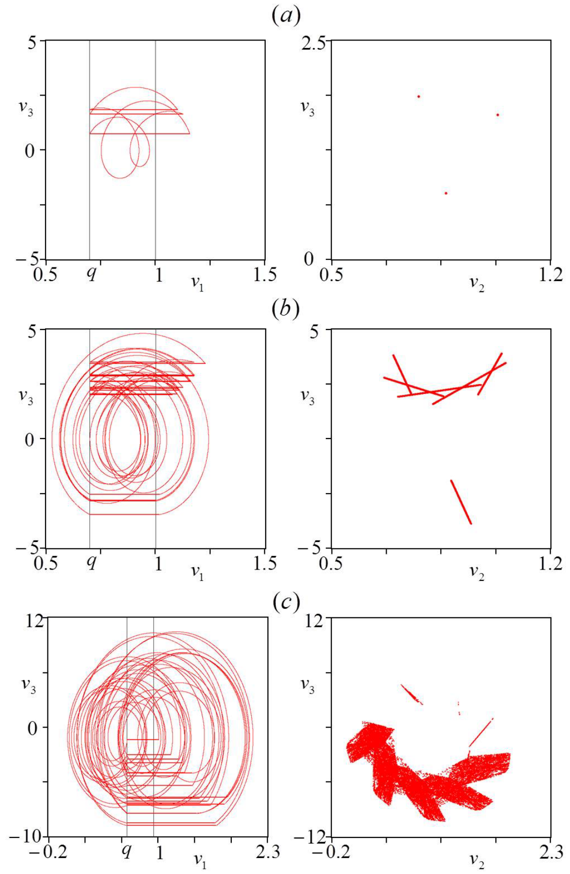

Figure 2: Left: Typical trajectory, Right: 2D Rmap (a) periodic for $d = 2$, (b) chaotic for $d = 3.6$, (c) hyperchaotic for $d = 5.3$

$(u_1(0), u_2(0), u_3(0))$ denotes an initial state vector. Fig.2 show typical trajectories and 2D Rmap calculated and converted back using equation(6)(7).

As $d = 2$, trajectory is periodic and Rmap has 3 points and Lyapunov exponent $\lambda_{11} = -0.34, \lambda_{12} = -0.34$. As $d = 3.6$, trajectory is chaotic and Rmap has some lines and Lyapunov exponent $\lambda_{11} = 0.06, \lambda_{12} = -1.82$. As $d = 5.3$, trajectory is hyperchaotic and Rmap has some lines and planes and Lyapunov exponent $\lambda_{11} = 0.10, \lambda_{12} = 0.06$.

3. Analysis

3.1. Histogram

The histogram is most basic method to consider the ISI characteristic. For using the histogram of ISI, we define two quantities τ_n and $\delta\tau_n$. Let τ_n denote the

n-th spike position and let $\delta\tau_n = \tau_n - \tau_{n-1}$ denote n-th ISI. Using these quantity, we can make histogram of the ISI. Fig.3 shows the histogram corresponding to Fig.2. Since we use clock period d in switching rule, the histogram shows some line spectrum at several multiples of d as shown in Fig.3. The color of the histogram and the RP is depend on own ISI. Fig.3(d) shows the coloring of the histogram and the RP.

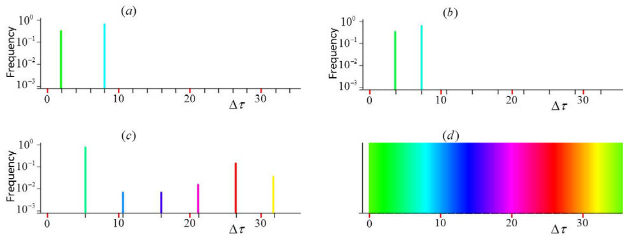

Figure 3: Histogram of the ISI (a) periodic for $d = 2$, (b) chaotic for $d = 3.6$, (c) hyperchaotic for $d = 5.3$, (d) coloring rule

As $d = 2$, histogram has two peaks. As $d = 3.6$, histogram has two peaks. These two histogram of the ISI are similar to each shape. As $d = 5.3$, histogram has many peaks. These many peaks means HCC output complex spike-trains. It is difficult to classify the ISI only by the Histogram of the ISI. Hence, we consider to the ISI by RP.

3.2. Recurrence plot

The RP is known as an analyzing method of chaotic dynamics. Using the RP, we can transform the time series ISI date to graphics.

Let us consider the RP for ISI sequence of the CSO. Let P be a two-dimensional plane. Calculate the distance $D(i,j)$ between the i-th and j-th ISI data: $D(i,j) = |\Delta\tau_i - \Delta\tau_j|$. If $D(i,j) = 0$, then we plot the (i,j) cell of P where $\theta_D = 0$. Color of cell is depend on the width of ISI. Repeating this process for all the $D(i,j)$, we can make the RP. It should be noted that we can set $\theta_D = 0$ because the ISI is restricted in a multiple of d. If spike-train is periodic, the RP is uniform image. If spike-train is chaotic/hyperchaotic, the RP is complex image. As $d = 2$, the RP is repeating uniform image. As $d = 3.6$, many blue box and few green box exhibits randomly because the same ISI is repeated many times and the other ISI is exhibited rarely. As $d = 5.3$, the RP has many green box and some colors box is exhibited. The difference of color means the distance of the ISI.

In order to derive a 2D Return map, we define a plane

$$P_c \equiv \{(u_1, u_2, u_3) | u_1 + u_3 = q\} \tag{8}$$

Let a point on P_c be represented by its u_2 and u_3-coordinates. As a trajectory starts from a point (u_{20}, u_{30}) on P_c at $\tau = 0$. it will exceed the threshold at some positive time $\tau = \tau_0$. At the next clock signal arrives, the trajectory jumps to a point(u_{21}, u_{31}) on P_c. Then we can define the following 2D Return map.

$$\begin{aligned} F : P_c \to P_c, (u_{20}, u_{30}) &\mapsto (u_{21}, u_{31}) \\ (u_{21}, u_{31}) &= F(u_{20}, u_{30}) \\ = (f(u_{20}, u_{30}, nd), g&(u_{20}, u_{30}, nd)) \end{aligned} \tag{9}$$

The function f and g are calculated using Equations(6)(7).

$$\begin{aligned} f(u_{20}, u_{30}, nd) &= e^{\delta nd}(u_{20} + p_2(u_{30} - q)\cos nd \\ &-(q - u_{30} + p_2 u_{20})\sin nd) + p_3(q - e^{\lambda nd}u_{30}) \end{aligned} \tag{10}$$

$$\begin{aligned} g(u_{20}, u_{30}, nd) &= e^{\lambda nd}(1 - p_3)u_{30} \\ -e^{\delta nd}p_3((q - u_{30})&\cos nd + u_{20}\sin nd) + p_3 q \end{aligned} \tag{11}$$

Right of Fig.1 shows 2D Rmaps. Using these maps, Lyapunov exponents can be calculated analytically using following equations.

$$DF(u_{20}, u_{30}) = \frac{\partial(u_{21}, u_{31})}{\partial(u_{20}, u_{30})} \tag{12}$$

$$\lambda_{21} = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N-1} DF(u_{2j}, u_{3j}) \tag{13}$$

$$\lambda_{11} = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N-1} DF(u_{2j}, u_{3j})e_j \tag{14}$$

$$\lambda_{12} = \lambda_{21} - \lambda_{11} \tag{15}$$

$$e_{j+1} = \frac{DF(v_{2j}, v_{3j})e_j}{|DF(v_{2j}, v_{3j})e_j|} \tag{16}$$

λ_{11} and λ_{12} denote first and second 1D Lyapunov exponent. And λ_{21} denote 2D Lyapunov exponent. When $0 > \lambda_{11} > \lambda_{12}$, the HCC generates periodic dynamics and Rmap has some points. When $\lambda_{11} > 0 > \lambda_{12}$, the HCC generates chaotic dynamics and Rmap has some lines. When $\lambda_{11} \geq \lambda_{12} > 0$, the HCC generates hyperchaotic dynamics and Rmap has some lines and planes.

4. Conclusions

We have analyzed dynamics and spike-trains from the HCC with impulsive switching controlled by refractory threshold and Spike-train input. Using the Rmap, we have visualized stability. Using the histogram of the ISI and the RP, we have visualized dynamics of spike-trains. We show the HCC can exhibit various dynamics and we can analyze by using these tools.

Future problems include detail analysis of bifurcation and finding more simple analysis tools.

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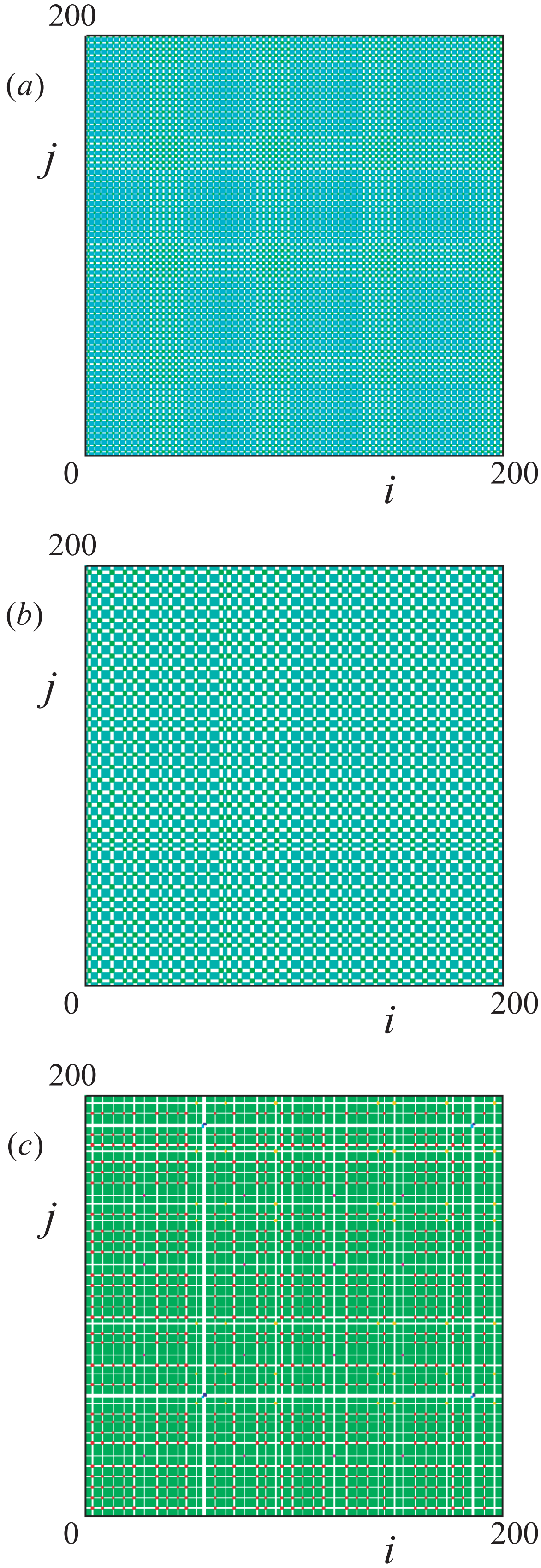

Figure 4: Recurrence plot of the ISI (a) periodic for $d = 2$, (b) chaotic for $d = 3.6$, (c) hyperchaotic for $d = 5.3$