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Abstract—We consider dissipative solitons arising in the Swift-Hohenberg equation and study the effects of adding a pinning defect and drift. The competition between the pinning of dissipative solitons to the defect and the pulling of the drift term give rise to complex dynamical behaviour. In particular we show that dissipative solitons display excitability by a number of different mechanisms.

1. Introduction

Dissipative solitons (also known as localized structures) are states in extended media that consist of one region in one state surrounded by a stable stationary state. These structures were first suggested in Refs. [1, 2] and then described in a variety of systems, such as chemical reactions [3], semiconductors [4], granular media [5], binary-fluid convection [6, 7], vegetation patterns [8], and also in nonlinear optical cavities where they are usually referred as Cavity Solitons (CS) [9, 10, 11, 12, 13]. Their potential in optical storage and processing of information has been stressed [14].

In general, dissipative solitons (DS) may develop a number of instabilities like start moving, breathing, or oscillating. In the latter case, they would oscillate in time while remaining stationary in space, like the oscillons (oscillating localized structures) found in a vibrated layer of sand [5]. The occurrence of these oscillons in autonomous systems has been reported both in optical [15] and chemical systems [16]. It has been shown that they can become unstable leading to excitable solitons in systems for which the local dynamics is not excitable [17]. In this case excitability appears as an emergent property arising from the spatial dependence, which allows for the formation of these structures. The inclusion of a defect in the systems enhance this behaviour and can be used for potential information processing applications [18].

Excitability is a concept arising originally from biology (e.g. in neuroscience), and found in a large variety of nonlinear systems [19]. A system is said to be excitable if perturbations below a certain threshold decay exponentially while perturbations above induce a large response before going back to a resting state. Roughly speaking, excitability needs two ingredients: a barrier in phase space that defines the excitable threshold, and a re-entry mechanism that sets the system back to the original state after a refractory time. In excitability mediated by DS, the excitable threshold is automatically set by the stable manifold of the unstable (middle-branch) DS, which is the barrier one has to overcome to create an DS. The re-entry mechanism is, in some cases, intrinsic to the dynamics of DS, as in [17]. The need for the re-entry mechanism limits the variety of systems showing excitable DS.

In this work we show that a re-entry mechanism leading to excitability can be implemented by adding a defect and drift in a finite system. In this case, when a superthreshold perturbation creates an DS on the defect, the drift pulls it out and drives it to the limits of the system, where the DS disappears and the system goes back to the original state. This makes excitability commonplace in systems displaying DS when a defect and drift are introduced. Excitability appears through a number of different mechanism depending on the size of the defect and the intensity of the drift.

2. Modified Swift-Hohenberg equation

We analyze this scenario in the general Swift-Hohenberg equation close to the degenerate Hamiltonian-Hopf bifurcation where DS appear [20],

$$\frac{\partial u}{\partial t} = -\left(\frac{\partial^2}{\partial x^2} + k_0^2\right)^2 u + c\frac{\partial u}{\partial x} + r(x)u + au^2 - gu^3 + b(x) \quad (1)$$

where u is a real field,

$$b(x) = h \exp\left[-\left(\frac{x - x_0}{\sigma}\right)^2\right]$$

is a defect modeled by a Gaussian function with height h, and c the strength of the drift.

$$r(x) = r_0 - 1 + \exp\left[-\left(\frac{x - x_0}{\epsilon}\right)^{18}\right]$$

is a supergaussian profile to model a finite system where the spatial structures advected away by the drift will die at the boundary. DS exist for $a > \sqrt{27/38}g$, so, throughout this work, we take $a = 1$ and $g = 1$, and $r_0 = -0.09$ roughly in the middle of the subcritical region where DS exist. We also fix $k_0^2 = 0.5$, $\sigma = 2.045$, $\epsilon = 98.17$, and $x_0 = L/2$ with $L = 418.88$. h and c will be the main control parameters in this study.

For numerical simulations, we integrate Eq (1) using a pseudospectral method where the linear terms in Fourier space are integrated exactly while the nonlinear ones are integrated using a second-order in time aproximation [22].

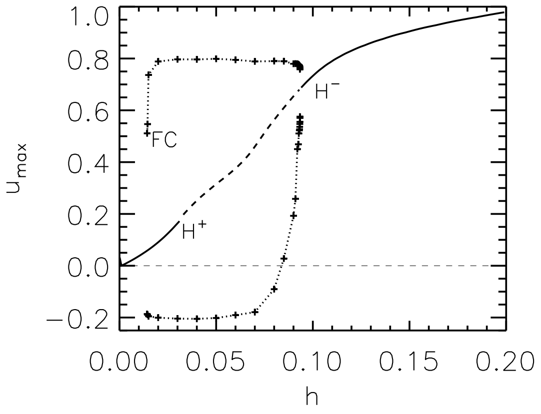

Figure 1: Bifurcation diagram showing the maximum of stationary localized structures as a function of the height of the localized Gaussian defect h for $c = 0.2$. The crosses indicate the maximum and minimum value of the oscillations of a fixed point in space due to the cycle created at the Hopf bifurcations.

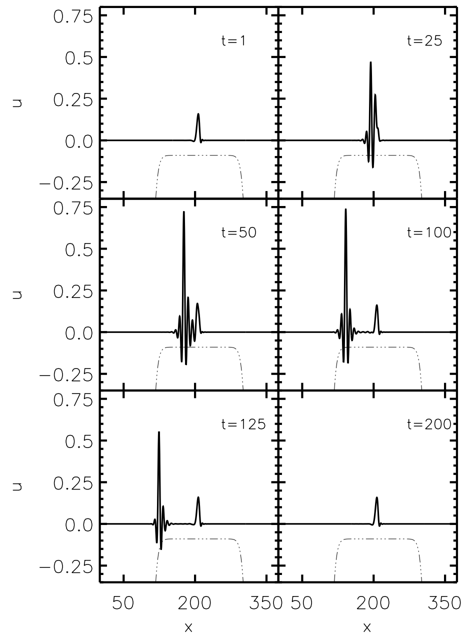

Figure 2: Excitable excursion of the low amplitude fundamental solution close to the Hopf point H^{+}. The dashed-dotted line shows the supergaussian profile $r(x)$ defining the limits of the system.

Periodic boundary conditions have been used. The steady-state DS, stable and unstable, are found by equating to zero the left-hand side of Eq.1. Discretizing the space one obtains a set of coupled nonlinear equations wich can be solved using a Newton-Raphson method [23].

3. Dynamical analysis

One of the consequences of the term $b(x)$ in Eq. (1) is that the translational symmetry is broken. Solutions are now pinned at x_0 where the Gaussian defect is. This also affects the transverse profile of the solutions, in particular the fundamental solution is no longer spatially homogeneous but it exhibits a small bump (see first panel of Fig. 2). The competition between the defect that produce a pinning of the localized state and the advection term that try to drift the solution away leads to different pinning-unpinning transitions. Figure 1 shows the bifurcation diagram of the localized solutions as a function of the size of the defect h for $c = 0.2$.

DS are unpinned from the defect through two Hopf bifurcations. The first one (H^{+}) is subcritical and leads to a limit cycle where DS grow out of the (low amplitude) fundamental solutions and are then drifted away to die at the boundary of the system, creating a train of soliton or soliton tap [21]. Generally, the unstable cycle emerging from a H^{+} bifurcation folds in a Fold of cycles (FC) bifurcation. The region just below the H^{+} bifurcation exhibits excitable behavior, specially if the FC and H^{+} are close (otherwise the system is bistable and excursions may stay in the oscillatory upper state)(see Fig. 4).

If the system is set below H^{+}, a perturbation applied to the (low amplitude) fundamental solution grows to generate a DS that is advected away to die at the boundary, and the system comes back to the resting state. The excitability found in this region is of Type II [24], characterized by the fact that the oscillations created at the bifurcation point H^{+} start with non zero frequency. An excitable excursion in this case is shown in Fig. 2 for $c = 0.7$ and $h = 0.055$.

A different kind of behavior is the one shown in Fig. 3. Here for larger values of h, large amplitude DS are again pinned to the defect. In this case, decreasing h the drift is eventually able to detach the DS and a new one is formed creating again a cycle in a Hopf bifurcation H^{-}. In this case the bifurcation is supercritical, showing small oscillations close to threshold, but decreasing h very little this small oscillations become already very large, in a sort of canard, creating a source of DS. This is again a mechanism leading to Type II excitability. In this case the resting state is, though, a high amplitude DS (see first panel). In both cases excitable excursions are induced by increasing transiently the height of the defect, h in the first case and decreasing in the second. The competition between the defect and the drift unfolds, actually, a much richer scenario, shown in Fig. 4

In addition to the two cases explained above, this scenario has a third mechanism leading to excitability through

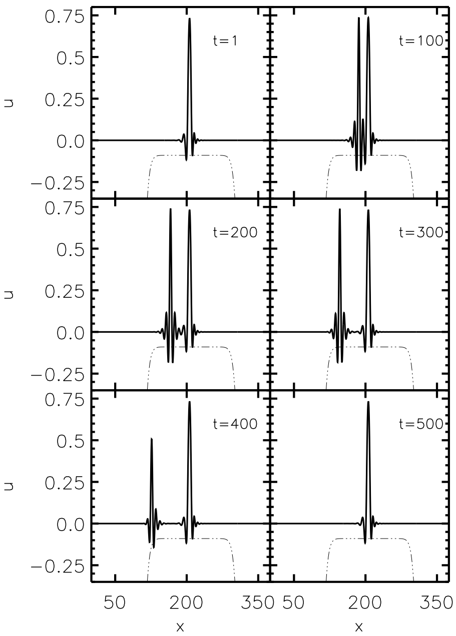

Figure 3: The same as in Fig.2 but close to the second Hopf point H^-, for $c = 0.2$ and $h = 0.12$.

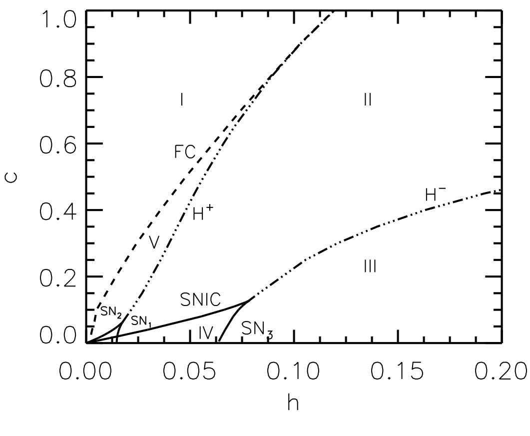

Figure 4: Two-parameter c vs h phase diagram of the soliton solutions in the modified Swift-Hohenberg equation for $a = 1$, $g = 1$, $r = -0.09$ and $k_0^2 = 0.5$. Bifurcation lines are: SN (SN_1, SN_2, SN_3) saddle-nodes, SNIC saddle-node in the invariant circle, H^+ Subcritical-Hopf, H^- Supercritical-Hopf, and FC (Fold of Cycles). Regions delimited by bifurcation lines are as follows: I fundamental solution (excitable close to FC), II oscillations (train of solitons), III DS (excitable close to H^-, IV DS (excitable close to SNIC), and V bistability between fundamental solution and oscillations.

a saddle-node in the invariant circle (SNIC) bifurcation. In this case a (high amplitude) DS is also unpinned leading to the excitable excursion, similar to the case of the supercritical Hopf H^-, but in a Type I fashion, i.e., the oscillatory regime created at the bifurcation exhibits frequencies with arbitrary low values. The appearance of oscillations with a divergence of the period has been experimentally observed in semiconductor lasers [21], making the observation of the scenario described in this work experimentally relevant.

4. Concluding remarks

In summary, we have shown how a defect and drift make excitability a common feature of DS in extended systems. Our results are completely general and show the different mechanisms leading to this behavior. Type I and Type II excitability can be observed through the unpinning of DS from the defect.

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