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Noise Effects on Generalized Chaos Synchronization in Semiconductor Lasers

Kenichi Arai[1], Susumu Shinohara[1], Satoshi Sunada[2], Kazuyuki Yoshimura[1], Takahisa Harayama[1,3], and Atsushi Uchida[4]

[1] Communication Science Laboratories, Nippon Telegraph and Telephone Corporation,
2-4, Hikaridai, Seika-cho, Soraku-gun, Kyoto 619-0237, Japan
[2] Institute of Science and Engineering, Kanazawa University,
Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan
[3] Graduate School of Engineering, Toyo University,
2100 Kujirai, Kawagoe, Saitama 350-8585, Japan
[4] Department of Information and Computer Sciences, Saitama University,
255 Shimo-Okubo, Sakura-ku, Saitama City, Saitama 338-8570, Japan

Abstract—We investigate the effects of noise in drive and/or response systems on generalized synchronization. We show numerically that noise in a drive system can induce synchronization in an unidirectionally coupled Lorenz system, while noise in a response system disrupts the synchronization. In addition, through experiments using coupled semiconductor lasers with optical feedback, we observe that noise in a drive system enhances the synchronization.

1. Introduction

There has been a lot of interest in the synchronization of chaotic systems since the early work on synchronized motion in two coupled chaotic systems [1, 2, 3]. One reason is that synchronization itself is an interesting phenomenon induced by the nonlinear nature of systems. Moreover, it has a variety of applications, especially in the field of secure communications. Some of these applications are implemented utilizing the generalized synchronization of circuit and/or laser systems in a drive-response configuration. [4, 5, 6, 7].

With synchronization realized by using physical devices, noise is inevitable and, therefore, it is important to take account of the effects of the noise on the synchronization. For example, although synchronization occurs in a noiseless system, intermittent desynchronization is induced even with a small amount of noise or parameter mismatches in a bubbling regime [8, 9]. On the other hand, noise is also known to have counter effects, that is, common noise induced synchronization [10, 11, 12, 13, 14, 15, 16]. In this paper, we investigate the effects of noise on generalized synchronization in a drive-response configuration when noise is added to drive systems and/or to response systems [17].

This paper is organized as follows. We focus on the effect of noise on the synchronization of unidirectionally coupled Lorenz systems in Sec. 2 and chaotic lasers injected by another chaotic laser in Sec. 3. We provide a summary in Sec. 4.

2. Lorenz systems

In this section, we consider the generalized synchronization of a coupled Lorenz system in a drive-response configuration as shown in Fig. 1. An auxiliary system, which

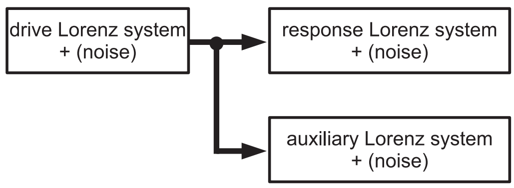

Figure 1: Unidirectionally coupled Lorenz systems in a drive-response configuration. The auxiliary system is used to detect synchronization.

is equivalent to the response system, is used to detect generalized synchronization [18, 19]. This system is described by the following stochastic differential equations.

$$\text{(drive)}\begin{cases} \dot{x}_d &= \sigma(y_d - x_d),\\ \dot{y}_d &= \rho_d x_d - y_d - x_d z_d + s_d \xi_d,\\ \dot{z}_d &= -b z_d + x_d y_d,\end{cases}$$

$$\text{(response)}\begin{cases} \dot{x}_r &= \sigma(y_r - x_r),\\ \dot{y}_r &= \rho_r x_r - y_r - x_r z_r + k y_d + s_r \xi_r,\\ \dot{z}_r &= -b z_r + x_r y_r,\end{cases}$$

$$\text{(auxiliary)}\begin{cases} \dot{x}_a &= \sigma(y_a - x_a),\\ \dot{y}_a &= \rho_r x_a - y_a - x_a z_a + k y_d + s_r \xi_a,\\ \dot{z}_a &= -b z_a + x_a y_a,\end{cases}$$

where ξ_d, ξ_r, and ξ_a are white Gaussian noise and independent of each other ($\langle \xi^2 \rangle = 1$, and $\langle \xi_i \xi_j \rangle = 0$ if $i \neq j$). k is a coupling coefficient and s_d and s_r are the noise amplitudes for the drive and response (auxiliary) systems, respectively. The other parameters are set at the following values : $\sigma = 10$, $b = 8/3$, $\rho_d = 28.5$, and $\rho_r = 28$. An Euler-Maruyama method is employed to simulate noisy Lorenz systems.

In the noiseless case ($s_d = s_r = 0$), the response system can be synchronized with the drive system in the sense of generalized synchronization when the coupling strength exceeds a critical value. Figure 2 shows conditional Lyapunov exponent, and the average $d_{\rm rms}$ and maximum $d_{\rm max}$ values of the distances d $(= |(x_r, y_r, z_r) - (x_a, y_a, z_a)|)$ between the response and auxiliary systems as a function of the coupling strength k. $d_{\rm rms}$ and $d_{\rm max}$ become zero

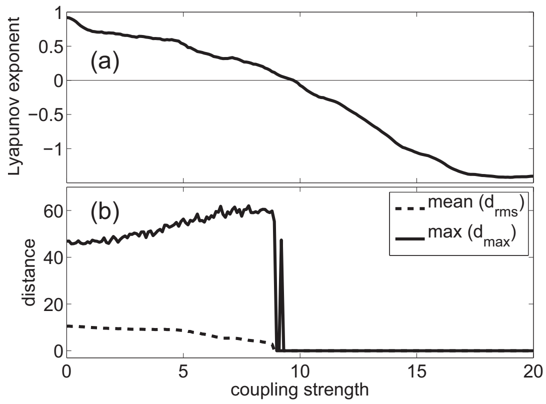

Figure 2: Generalized synchronization for a noiseless system. (a) Conditional Lyapunov exponents and (b) the average $d_{\rm rms}$ and the max $d_{\rm max}$ distances are shown as a function of the coupling strength k.

and the conditional Lyapunov exponent becomes negative at almost the same coupling strength, $k \sim 9.3$. It was observed that generalized synchronization is achieved stably when $k > 9.3$.

Secondly, we consider the effects of noise in the response system on the generalized synchronization. Figure 3 (a) shows the dependence of the distances ($d_{\rm rms}$ and $d_{\rm max}$) on the noise amplitude s_r of the response (auxiliary) system when $k = 12.0$. $d_{\rm rms}$ is an increasing function of the noise amplitude and $d_{\rm max}$ also tends to grow as the noise increases. This indicates that the noise of the response system degrades the synchronization. Figure 3 (b) shows the dependence of the distances ($d_{\rm rms}$ and $d_{\rm max}$) on the coupling strength k for $s_r = 10^{-3}$. The average $d_{\rm rms}$ and maximum $d_{\rm max}$ distances approach 0 at $k \sim 10$ and $k \sim 30$, respectively. This indicates that even with response noise, generalized synchronization can be achieved through intermittent desynchronizations ($10 < k < 30$) while the coupling strength becomes large.

Next, noise is added to the drive system. Examples of the time evolution of distances are shown in Fig. 4 (b) and (c) for small ($s_d = 10^{-2}$) and large ($s_d = 10$) noise, respectively. It is observed that drive noise can induce synchronization. Figure 4 (a) shows the average distance $d_{\rm rms}$ and the conditional Lyapunov exponent as a function of noise

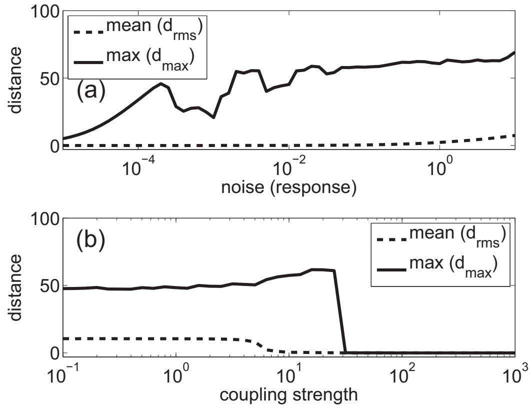

Figure 3: Distances between the coordinates of the response and auxiliary systems when independent noise is added to the response and auxiliary systems. The distance dependence on (a) the noise amplitude s_r when $k = 12.0$ and (b) the coupling strength k when $s_r = 10^{-3}$ are shown.

amplitude s_d when $k = 8.6$, where generalized synchronization does not occur without the drive noise. For large

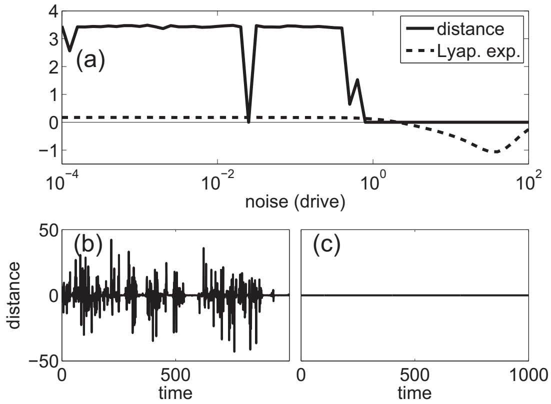

Figure 4: (a) Dependence of the average distance and the conditional Lyapunov exponent on the noise strength of the drive system when $k = 8.6$. Examples of the time evolution of the distance between the coordinates of the response and auxiliary systems are shown (b) when $s_d = 10^{-2}$ and (c) $s_d = 10$.

noise, $d_{\rm rms}$ is almost zero and the conditional Lyapunov exponent is negative. We can say that generalized synchronization is realized stably by adding noise to the drive system.

Finally, we investigate the effects of drive noise when there is also noise in the response and auxiliary systems. Figure 5 (a) shows the dependence of $d_{\rm rms}$ and conditional Lyapunov exponent on drive noise amplitude s_d. For certain range of noise amplitudes, we observe that $d_{\rm rms}$ is almost zero and the conditional Lyapunov exponents are negative, which indicates that generalized synchronization occurs. The example distances shown in Fig. 5 (b) and (c) illustrate desynchronization and synchronization for small ($s_d = 10^{-2}$) and large ($s_d = 10$) drive noise, respectively.

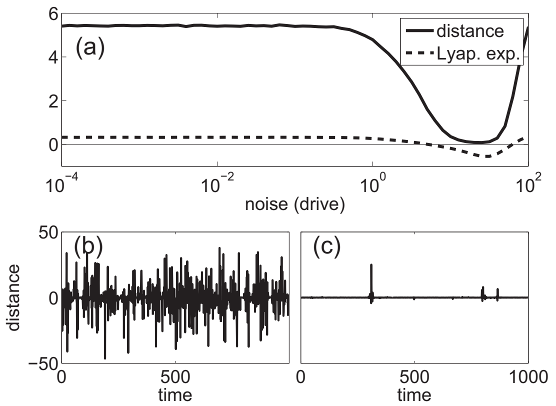

Figure 5: (a) Dependence of the average distance and the conditional Lyapunov exponent on the noise strength of the drive system when $k = 7.0$ and $s_r = 10^{-3}$. Examples of the time evolution of the distance between the coordinates of the response and auxiliary systems are shown (b) when $s_d = 10^{-2}$ and (c) $s_d = 10$.

3. Laser system

We investigate the effects of noise on the synchronization of a semiconductor laser induced by the injection of chaotic signals from a drive laser. Figure 6 shows our experimental setup for the synchronization. We used three

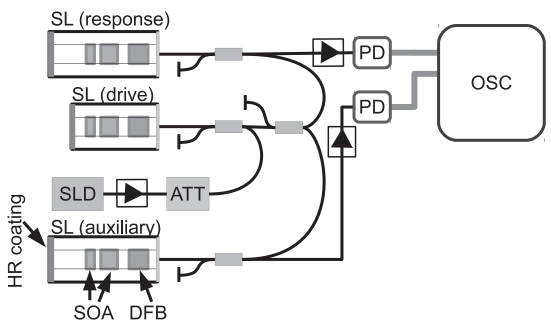

Figure 6: Experimental setup for synchronization. The abbreviations stand for the following: SL, semiconductor laser; SLD super-luminescent diode, ATT, attenuator; PD, photodetector; OSC, digital oscilloscope.

monolithically integrated chaos laser chips, each of which consisted of a distributed-feedback (DFB) laser, two semiconductor optical amplifiers (SOAs) and a passive waveguide. A high-reflective (HR) coating was employed to achieve delayed optical feedback strong enough to obtain chaotic behaviors of laser outputs. One laser chip had a 5 mm external cavity and was used as a drive laser, and the others had 10 mm external cavities and were used as a response laser and an auxiliary laser. The injection currents and the temperatures of the semiconductor lasers were adjusted by a current-temperature controller (ILX, LDC 3902). The relaxation frequencies of the response and the auxiliary lasers were set at 2.6 GHz by adjusting their injection currents. We set the optical wavelength at 1550.030 nm for the response and auxiliary lasers by precisely controlling the temperatures. To investigate the dependence of the noise strength in a drive chaotic laser on the synchronization, the output strength of a super-luminescent diode (SLD), which can be regarded as an optical noise source, was controlled by an attenuator and the outputs were injected into the drive laser. The outputs of the drive laser were injected into the response and auxiliary lasers. The outputs of the response and the auxiliary lasers were converted into electrical signals at photodetectors (New Focus, 1544B) and these signals were sent to a digital oscilloscope (Tektronix, DPO 71694C).

First, without SLD injection, we set parameter values of the drive laser such as the injection current and the temperature so that the response and auxiliary lasers were weakly synchronized. Example wave forms are shown in Fig. 7 (a), where the cross correlation is 0.665. Then, we increased the noise of the drive laser by injecting the SLD outputs. The cross correlations of the observed temporal dynamics of the response and auxiliary lasers are shown for various noise levels in Tab. 1. We can see that noise increases the correlations and the highest correlation was achieved when the attenuator's level was -30 dB. Example wave forms for −30 dB are shown in Fig. 7 (b).

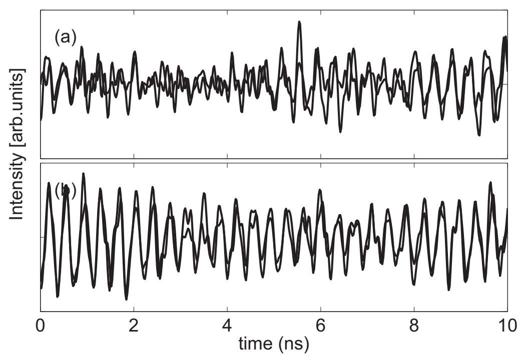

Figure 7: Temporal wave forms of the response and auxiliary lasers (a) without SLD and (b) with SLD (-30dB) injection into the drive laser.

4. Summary

We showed numerically and experimentally that the noise of a drive system can enhance synchronization in a

Table 1: Cross correlations between the response and auxiliary lasers for certain noise strengths.

noise strength (attenuator's scale)	cross correlation
no noise	0.665
-50dB	0.724
-40dB	0.728
-30dB	0.832
-20dB	0.825
-15dB	0.819

coupled chaotic system in a drive-response configuration. Numerical simulations of a coupled Lorenz system showed that the noise of a drive system can induce generalized synchronization even in the presence of response noise, which can disrupt the synchronization. In addition, our experiments with semiconductor lasers showed that the noise of a drive laser can enhance synchronization.

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