Time Domain Integral Equation Method Using FFT-Based Marching-On-in-Degree Method for Analyzing PEC Patches On Substrate

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***Abstract*-One time domain method, based on the TDIE solved by the marching-on-in-degree (MOD) scheme, is presented in this paper for capturing transient electromagnetic responses of dielectric-metallic composite structures. For perfect electrically conducting (PEC) surfaces patched on the dielectric substrate, the time-domain electric field integral equations (TD-EFIE) is used to analyze PEC patch, while PMCHW (Poggio, Miller, Chang, Harrington and Wu) integral equation is utilized for describing the dielectric substrate. The whole set of integral equations is solved simultaneously, and the temporal basis function is chosen to be the Laguerre polynomials (LP), which is called marching-on-in-degree scheme. For the spatial basis function, the classical RWG basis function is usually the suitable choice. Since in the right-hand-side of the iteration equation, during the matrix-vector multiplication, the impedance matrix has the form of Toeplitz matrix, the fast Fourier transform (FFT) can be used to accelerate this multiplication. Numerical results are given to demonstrate our proposed method used for analyzing characteristics of structures containing PEC patch on the substrate.**

I. INTRODUCTION

Time-domain integral equation can be widely used for analyzing transient scattering responses of some classic electromagnetic structures [1-2]. Time domain electric field integral equations (TD-EFIE) are used for analyzing perfect electrically conducting surface, while PMCHW (Poggio, Miller, Chang, Harrington and Wu) integral equation is very suitable for dealing with homogeneous dielectric structures.

As for TDIE formulation, there are usually two schemes to solve it, including marching-on-in-time (MOT) method and marching-on-in-degree (MOD) method [3-5]. Both schemes have their strengths and weaknesses. MOT methods are prone to late time instabilities while MOD schemes appear highly stable due to the high-order limit. Both MOT and MOD methods suffer from a high computational cost. Since MOD is stable even in very late time, it is adopted to solve the integral equation in this paper. In the MOD scheme, the weighted Laguerre polynomials form a set of the temporal basis function, by which the temporal currents are represented together with the spatial Rao-Wilton-Glisson (RWG) basis function.

What is more, the computational cost of MOD scales as $O(N_O^2 N_S^2)$, where N_S is the number of spatial unknowns, and N_O is the highest order of the LP. A fast Fourier transform (FFT)-based blocking scheme [4] can be used for accelerating the temporal convolutions, and the computational complexity is reduced to $O(N_S^2 N_O \log^2(N_O))$.

As we discuss about dielectric-metallic composite structures, the patch structure is one classical part of them [6-7]. The PEC patch is directly attached to the dielectric substrate, so in the analysis using time domain integral equations, the dielectric-metallic junctions must be treated carefully in order to obtain high accuracy. One of the methods is the contact-region modelling (CRM) technique [8], and it supposes the dielectric-metallic surface to be two surfaces infinitesimally close to each other. However, this increases the number of unknowns because one more surface needs to be considered. Another effective way is to use triangular half-basis functions based on the full RWG basis to treat the current on the junction [9].

The organization of this paper is arranged as follows. In Section II, the basic knowledge of the electromagnetic problem and mathematical treatment is presented. The TDEFIE-PMCHW equation is solved using MOD scheme, while the FFT is used to accelerate the order iteration of MOD. In Section III, numerical results of some patch structures are given to demonstrate the capabilities of our proposed method. Then the conclusion is made in Section IV.

II. BASIC THEORY

A. Description of The Geometry

Figure 1 shows a generalized composite structure in free space, including homogeneous dielectric objects and PEC patches, both of arbitrary shape. The structure is illuminated by an electromagnetic pulse (EMP) with an arbitrary magnitude, polarization and incident direction. From Fig.1, it is shown that one PEC patch is directly attached on the dielectric substrate, and there are two metallic surfaces need to be considered, including the one toward the free space and the other one toward the inner region of the dielectric [5]. The dielectric surface is denoted by S^D, while the PEC surfaces toward the free space and the dielectric are denoted by S^P and S^I, respectively.

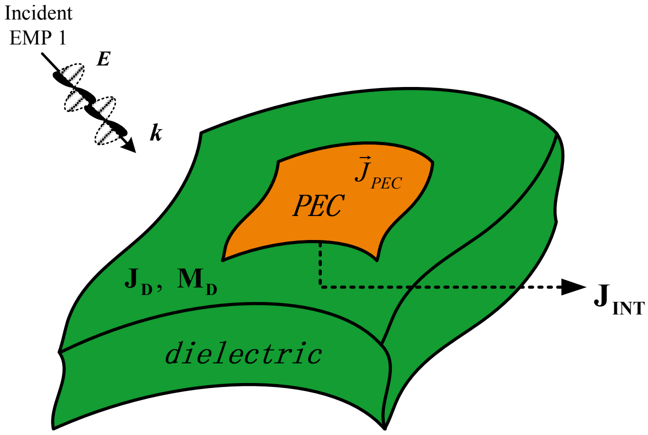

Fig. 1. Description of the simulated dielectric-metallic patch structure.

B. TDIE Formulation

Under the EMP illumination, there are induced electric and magnetic currents on the dielectric surface, which are denoted by $\mathbf{J_D}$ and $\mathbf{M_D}$, respectively. For the PEC object, only induced electric currents $\mathbf{J_{PEC}}$ and $\mathbf{J_{INT}}$ on the surfaces S^P and S^I, respectively.

In the presence of electric and magnetic current sources, the scattered fields of the dielectric object can be described by

$$\mathbf{E}_\mathbf{v}^{\mathbf{sca}}(\mathbf{r},t) = L_v(\mathbf{J_v}) - K_v(\mathbf{M_v}) \tag{1}$$

$$\mathbf{H}_\mathbf{v}^{\mathbf{sca}}(\mathbf{r},t) = L_v(\mathbf{M_v}) + K_v(\mathbf{J_v}) \tag{2}$$

$$L_v(\mathbf{I}) = \frac{-\alpha_v}{4\pi}\int_S \frac{1}{R}\frac{\partial}{\partial t}\mathbf{I}(\mathbf{r},\tau_v)\,dS' + \frac{1}{4\pi\beta_v}\nabla\int_S\int_0^\tau \frac{1}{R}\nabla'\cdot\mathbf{I}(\mathbf{r},t_v)\,dt\,dS' \tag{3}$$

$$K_v(\mathbf{I}) = \frac{1}{4\pi}\nabla\times\int_S \frac{1}{R}\mathbf{I}(\mathbf{r},\tau)\,dS' \tag{4}$$

where $\mathbf{I}$ represents either electric current or magnetic current. v=‘*1*’ and ‘*2*’ represent exterior and interior domains of the dielectric, respectively. So the PEC currents $\mathbf{J_{PEC}}$ and $\mathbf{J_{INT}}$ belong to domain ‘*1*’ and domain ‘*2*’, respectively. $R = |\mathbf{r}-\mathbf{r}'|$ is the distance between an arbitrary observation point $\vec{r}$ and the source one $\vec{r}'$. $\tau_v = t - R/c_v$ is the retarded time. When $\mathbf{I}$ is electric current, $\alpha_v = \mu_v$, $\beta_v = \varepsilon_v$, and for the magnetic current $\mathbf{I}$, they are inverse.

As for dielectric, the current satisfies the property that $\mathbf{J_{D,1}} = -\mathbf{J_{D,2}}$ and $\mathbf{M_{D,1}} = -\mathbf{M_{D,2}}$, so we define $\mathbf{J_{D,1}} = \mathbf{J}_D$ and $\mathbf{M_{D,1}} = \mathbf{M_D}$ for simplicity. Therefore, by applying boundary conditions on the dielectric surface and the two PEC surfaces, we obtain a set of time-domain integral equations for the problem as shown in Fig. 1:

$$-L_2(\mathbf{J_D}) + K_2(\mathbf{M_D}) - L_1(\mathbf{J_D}) + K_1(\mathbf{M_D}) - L_1(\mathbf{J_{PEC}}) + L_2(\mathbf{J_{INT}}) = \mathbf{E^{inc}}(\mathbf{r},t)|_{\tan}, \quad \forall\mathbf{r}\in S^D \tag{5}$$

$$-L_2(\mathbf{M_D}) - K_2(\mathbf{J_D}) - L_1(\mathbf{M_D}) - K_1(\mathbf{J_D}) - K_1(\mathbf{J_{PEC}}) + K_2(\mathbf{J_{INT}}) = \mathbf{H^{inc}}(\mathbf{r},t)|_{\tan}, \quad \forall\mathbf{r}\in S^D \tag{6}$$

While for the metallic surface S_{PEC}, we have

$$-L_1(\mathbf{J_D}) + K_1(\mathbf{M_D}) - L_1(\mathbf{J_{PEC}}) = \mathbf{E^{inc}}(\mathbf{r},t)|_{\tan}, \forall\mathbf{r}\in S^P \tag{7}$$

And for the interface S_{INT}, we have

$$L_2(\mathbf{J_D}) - K_2(\mathbf{M_D}) - L_2(\mathbf{J_{INT}}) = 0, \quad \forall\mathbf{r}\in S^I \tag{8}$$

C. FFT-based MOD Procedure

A set of weighted Laguerre polynomials $L_j(st)$ forms the temporal basis function, as given by $\phi_j(st) = e^{-st/2}L_j(st)$, where j is its order, and s is the scaling factor. Then, the electric and magnetic currents expanded in general as follows

$$\mathbf{I}(\mathbf{r},t) = \sum_{n=1}^{N_S}\frac{\partial}{\partial t}\left[\sum_{j=0}^{\infty}\gamma_{n,j}\phi_j(st)\right]\mathbf{f}_n^S(\mathbf{r}) \tag{9}$$

where $\mathbf{f}_n^S(\mathbf{r})$ is the RWG basis function [10] based on a pair of triangles, *i.e.*

$$\mathbf{f}_n^S(\mathbf{r}) = \mathbf{f}_n^+(\mathbf{r}) + \mathbf{f}_n^-(\mathbf{r}) \tag{10a}$$

$$\mathbf{f}_n^\pm(\mathbf{r}) = \begin{cases} \pm\dfrac{l_n}{2A_n^\pm}\boldsymbol{\rho}_n^\pm, & \mathbf{r}\in T_n^\pm \\ 0, & otherwise \end{cases} \tag{10b}$$

And here N_S denotes the number of edges on the surfaces S^D, S^P and S^I, respectively. $\mathbf{I}(\mathbf{r},t)$ can be $\mathbf{J_D}$, $\mathbf{M_D}$, $\mathbf{J_{PEC}}$ and $\mathbf{J_{INT}}$, corresponding to different current coefficients $\gamma_{n,j}$.

In the final order iteration equation, the temporal convolutions on the right-hand side can be accelerated by the applying FFT, since it satisfies the properties of Toeplitz matrix. Thus the LP order iteration can be accelerated.

III. Numerical Results and Discussion

Here, a temporal modulated Gaussian pulse is used as the incident wave and described by

$$\mathbf{E_{inc}}(\mathbf{r},t) = \mathbf{E_0}(4/\sqrt{\pi}/T)e^{-\gamma^2}\cos(2\pi f_0 t) \tag{30}$$

$$\gamma = 4\left(c_0 t - c_0 t_0 - \mathbf{r}\cdot\hat{\mathbf{k}}\right)/T \tag{31}$$

where E_0 is the amplitude of the incident EMP, $\hat{\mathbf{k}}$ is its wave vector, f_0 is its central frequency, c_0 is its velocity in free space, T is its pulse width, and $c_0 t_0$ is its time delay.

The first structure consists of one dielectric hemisphere and one circular PEC patch just directly attached on the hemisphere as shown in Fig.2. The relative permittivity is ε_r=4.4, and the radius of the composite hemisphere is set to be R=0.2m. It is illuminated by the incident EMP with f_0=800MHz, $\hat{k} = -\hat{z}$ and its polarization chosen along the $+\hat{x}$ axis. The dielectric is meshed by 862 edges of unknowns, and the PEC surface S^P and S^I have both 485 unknowns. Particularly, there are 40 edges at the junction.

Fig.2 shows the calculated bi-static radar cross section (RCS) at f=800 MHz, which is also compared with the results from the commercial software FEKO.

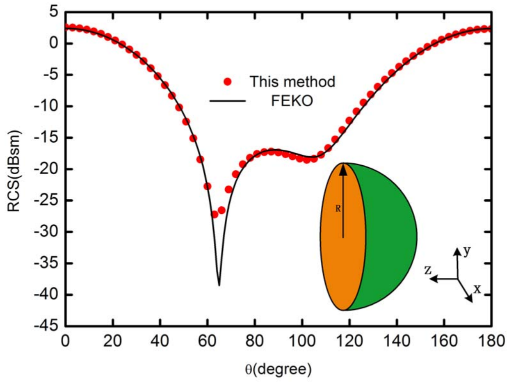

Fig. 2. Bi-static RCS at f=800MHz of the composite hemisphere.

The second example is four square patches mounted on the cylindrical substrate. The radius and the height of the substrate are 0.6m and 0.1m, respectively, with the relative permittivity ε_r=2.0. The four squares are the same, and the dimension of each is 0.3m×0.3m. The centres of these PEC patches are (±0.2, ±0.2) in the x-y plane. The parameters of the incident EMP include f_0=300MHz, $\hat{k}=-\hat{z}$, the polarization chosen along the $+\hat{x}$ axis, and the amplitude E_0=2000V/m.

Fig.3 and Fig.4 show the transient electric current and magnetic current we captured at the point very near the center of the upper surface of the substrate. By our proposed MOO method, we can capture transient electromagnetic characteristics for the composite patch structures.

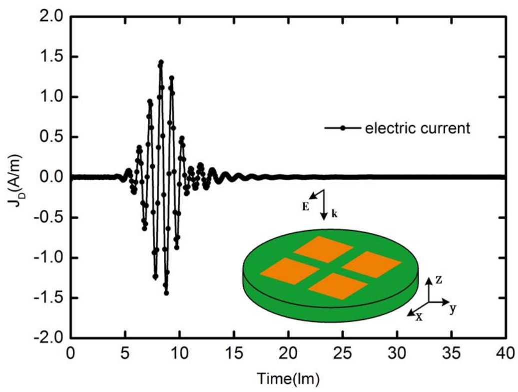

Fig. 3. Transient electric current response on the dielectric.

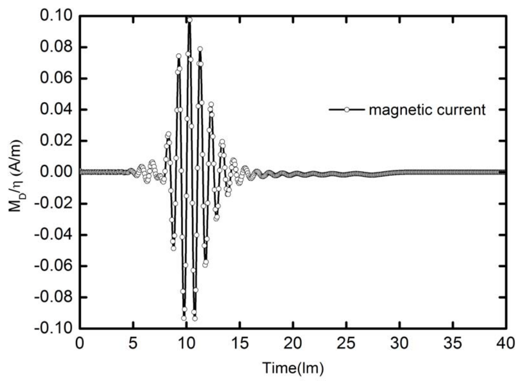

Fig. 4. Transient magnetic current response on the dielectric.

IV. Conclusion

In this paper, one time domain method, based on the TDIE solved by the marching-on-in-degree (MOD) scheme is developed for predicting time- and frequency-domain responses of perfect electrically conducting (PEC) surfaces patched on the dielectric substrate. The TDEFIE-PMCHW formulation is derived for the problem, and FFT-based MOD is utilized to solve the iteration. Numerical results show that our proposed method is accurate for handling composite structures, especially PEC patch mounted on the substrate.

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References

[1] D. S. Weile, G. Pisharody, N. W. Chen, B. Shanker, and E. Michielssen, “A novel scheme for the solution of the time-domain integral equations of electromagnetics,” *IEEE Trans. Antennas Propagat.*, vol. 52, no. 1, pp. 283–295, Jan. 2004.

[2] W. Luo, W. Y. Yin, M. D. Zhu, J. F. Mao, and J. Y. Zhao, “Investigation on time- and frequency-domain responses of some complex composite structures in the presence of high-power electromagnetic pulses,” *IEEE Trans. Electromagn. Compat.*, vol. 54, no. 5, pp. 1006-1016, Oct. 2012.

[3] Z. Ji, T. K. Sarkar, B. H. Jung, M. Yuan, and M. Salazar-Palma, “Solving time domain electric field integral equation without the time variable,” *IEEE Trans. Antennas Propagat.*, vol. 54, no. 1, pp. 258–262, Jan. 2006.

[4] M. D. Zhu, X. L. Zhou, and W. Y. Yin, “An adaptive marching-on-in-order method with FFT-based blocking scheme,” *IEEE Antennas Wireless Propag. Lett.*, vol. 9, pp. 436–439, 2010.

[5] J. Y. Zhao, W. Y. Yin, M. D. Zhu, and Wei Luo, “Time domain EFIE-PMCHW method combined with adaptive marching-on-in-order procedure for studying on time- and frequency- domain responses of some composite structures,” *IEEE Trans. Electromagn. Compat.*, 2013. *(in press)*

[6] B. Shanker, M. Y. Lu, J. Yuan, and E. Michielssen, “Time domain integral equation analysis of scattering from composite bodies via exact evaluation of radiation fields,” *IEEE Trans. Antennas Propagat.*, vol. 57, no. 5, pp.1506–1520, May 2009.

[7] S. N. Makarov, S. D. Kulkarni, A. G. Marut, and L. C. kempel, “Method of moments solution for a printed patch/slot antenna on a thin finite dielectric substrate using the volume integral equation,” *IEEE Trans. Antennas Propagat.*, vol. 54, no. 4, pp. 1174–1184, Apr. 2006.

[8] Y. Chu, W. C. Chew, J. Zhao, and S. Chen, “A surface integral equation formulation for low-frequency scattering from a composite object,” *Trans. Antennas Propagat.*, vol. 51, no. 10, pp. 2837–2844, Oct. 2003

[9] P. Ylä-Oijala, M. Taskinen, and J. Sarvas, “Surface integral equation method for general composite metallic and dielectric structures with junctions,” *Progress in Electromagn. Research, PIER 52*, 81–108, 2005.

[10] S. M. Rao, D. R. Wilton, and A. W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape,” *IEEE Trans. Antennas Propagat.*, vol. 30, no. 3, pp. 409–418, May 1982.