Improvement of Accuracy of Fixed Point Arithmetic FDTD Method

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Abstract

From the viewpoint of application to antenna analysis, this paper examines the calculation error of a fixed point arithmetic that causes a problem in the implementation of the finite difference time domain (FDTD) method in field programmable gate arrays (FPGAs). We evaluate the relative error while focusing on the difference between the fixed point arithmetic and the floating point arithmetic rather than the calculation accuracy of the FDTD method itself. We achieve improvement in the relative error in the analyzed field by introducing a technique that scales the intrinsic impedance in order to cancel significant digits of the magnetic field component that are generated based on the ratio of the electric field to the magnetic field. As a result, we achieve operation with the relative error improvement of more than 50 dB using a 32-bit fixed point arithmetic in free space field analysis.

1. INTRODUCTION

The Finite Difference Time Domain (FDTD) method is widely used as an electromagnetic field analysis technique, and it is implemented using various general-purpose computers such as personal computers and work stations. The FDTD method faces a problem in that a huge amount of memory and a long calculation time are required when the cell size becomes small such as when the analysis model is complicated or for a high dielectric constant. The FDTD calculations related to a large-scale analysis model for the whole human body model or for indoor propagation have the similar problem. There are also demands for calculations related to a large-scale analysis model for the whole human body model or for indoor propagation. In such situations, a super computer [1] or a PC cluster is used to perform the calculations to achieve large-capacity high-speed operation. As for the super computer, high-speed operation using a vector operation machine is possible, but it is extremely expensive. Throughput will be influenced by the conditions presented by the other calculation loads. A PC cluster has a merit in that it can be formed relatively easily in comparison to the super computer. However, the performance of a large-scale PC cluster becomes saturated when the amount of data communications becomes large. Due to this, an enhancement to the performance that is proportional to the number of PCs may not always be achieved.

As an alternative method, a specialized computer is occasionally used. This technique involves implementing the FDTD algorithm into large-scale integration chips (LSIs) such as field programmable gate arrays (FPGAs) and application-specific integrated circuits (ASICs). Since high-speed operation of FDTD is achieved through hardware parallel processing, this technique is very promising in the future. Previous conventional studies examined the hardware architecture related to incorporating the FDTD method into FPGAs and high-speed algorithms [2][3], a simple two-dimensional FDTD model that used Mur for the absorption boundary condition [4], and special uses [5] such as the underground scattering problem. We also have developed a FPGA calculator for three-dimensional antenna analysis [6]. However, the detail examination on the calculation accuracy in order to apply to antenna analysis has not been reported. This paper examines the calculation error of a fixed point arithmetic from the viewpoint of three-dimensional antenna analysis where the perfectly matched layer (PML) absorption boundary condition [7] is used, when the FDTD method is implemented into FPGAs.

2. SIMULATION METHOD AND MODEL

The purpose of this study is high-speed processing based on FPGA circuit implementation. Therefore, it is desirable for the fixed point arithmetic to be applied to update the calculation of the electromagnetic field and PML as the main operation. On the other hand, it is effective that pre-processing such as coefficient calculations or post-processing such as far field calculations are carried out by the floating point arithmetic. As an example, the FDTD formulas of x component of the electric field and magnetic field are expressed as equation (1), and (2), respectively. The calculation accuracy of the FDTD method itself is not evaluated in this paper. We evaluate the relative error of the fixed point arithmetic for a floating point arithmetic. A 32-bit (4-byte) integer type variable is used for the fixed point arithmetic. The range of the variable is approximately 180 dB.

Fig. 1 shows the simulation model. A dipole antenna that is located in the center of the analysis region is used in the evaluation model as the basis of the antenna analysis. It is easy to analyze the dependence of the bit width on the relative error since the electromagnetic near field distribution and input impedance frequency characteristics of this antenna are well known. Table 1 gives the simulation specifications. The analysis region including the absorption boundary consists of uniform cubic cells, and PML is used as the absorption boundary condition.

$$
\begin{aligned}
E_x^{n}(i,j,k) = {} & C_{ex}(i,j,k)E_x^{n-1}(i,j,k) \\
& - C_{exly}(i,j,k)\left[H_z^{n-\frac{1}{2}}(i,j,k) - H_z^{n-\frac{1}{2}}(i,j-1,k)\right] \\
& + C_{exlz}(i,j,k)\left[H_y^{n-\frac{1}{2}}(i,j,k) - H_y^{n-\frac{1}{2}}(i,j,k-1)\right]
\end{aligned} \quad (1)
$$

$$
\begin{aligned}
H_x^{n+\frac{1}{2}}(i,j,k) = {} & C_{hx}(i,j,k)H_x^{n-\frac{1}{2}}(i,j,k) \\
& - C_{hxly}(i,j,k)\left[E_z^{n}(i,j+1,k) - E_z^{n}(i,j,k)\right] \\
& + C_{hxlz}(i,j,k)\left[E_y^{n}(i,j,k+1) - E_y^{n}(i,j,k)\right]
\end{aligned} \quad (2)
$$

3. RELATIVE ERROR CHARACTERISTICS

The characteristics of the near field distribution are examined based on calculations of the dipole antenna in free space as an example. First of all, we must know the magnitude of coefficient of equation in order to determine the incident electric field value at the feeding point of the dipole antenna. Table 2 gives the coefficient values in the electromagnetic field equations in free space. From this table we find that the coefficient of C_{exly} and that of C_{exlz} in the electric field equations, (1) and (2), respectively, are large. On the other hand, the coefficients for C_{hxly} and C_{hxlz} in the magnetic field equation are very small. This fact is greatly caused by the magnitude of electric permittivity and magnetic permeability.

When the incident electric field at the feeding point is 256 V/m, in order to use the bit width effectively, the electric field and input impedance are evaluated. Here, the bit width is varied in the range of 16-32 to examine the dependence of the bit width on the relative error. Fig. 2 shows the maximum (value at the cumulative probability of 100%) of the relative error of the electric field amplitude in the observation plane (*xy* plane) as indicated in Fig. 1. Fig. 2 shows that the relative error of the electric field is –69 dB in the case of 32 bits. The relative error increases with a decrease in the bit width, and the slope is approximately –5 dB/bit. The dependence of the bit width on the relative error of the input impedance is also shown in this figure. The figure also shows that the tendency of the input impedance is similar to that for the electric field.

Fig. 3(a) shows the amplitudes of the electric field and magnetic field on the observation plane in order to analyze the deterioration in the relative error caused by the decrease in the bit width. Variable *ix* is the *i*-th cell in the *x* direction. The range of 8-92 on the horizontal axis represents the field region. The electric field is greater than the magnetic field and the difference is approximately 50 dB. In addition, the probability density distribution of the amplitude ratio in the observation plane is shown in Fig. 3(b). The curve of this ratio has a peak at approximately 360 based on Fig. 3(b). It is thought that the lower amplitude of the magnetic field causes the reduction of effective bit width and the deterioration of the relative error seriously.

4. IMPROVEMENT OF RELATIVE ERROR

A. Normalization of Intrinsic Impedance

From the above results, it is important that the value of the magnetic field be equal to that of the electric field and that the effective bit width be increased to achieve a higher level of calculation accuracy. By the way, the relationship between the electric field and magnetic field of a plane wave in free space is given as

$$E = Z_0 H \quad (3)$$

with intrinsic impedance Z_0 that is 377. When the fixed point arithmetic is carried out, a significant difference in amplitude between the electric field and magnetic field does not occur if Z_0 is equal to 1. Therefore, we define magnetic field H' as the fixed point arithmetic in the following expression with coefficient a.

$$H' = a\, H \quad (4)$$

The dependence of the bit width on the relative error is indicated in Fig. 4. The cumulative probability on the relative error of the electric field for a = 377 using this method is shown in Fig. 4(a). Based on this figure, we find that the relative error of –126 dB is achieved with the cumulative probability of 100% in the case of 32 bits. The relative error of –46 dB is obtained even in the case of 18 bits for a = 377. The shape of the characteristic curve of each bit width is approximately equal. The cumulative probability of 100%, i.e., the maximum of the electric field, is shown in Fig. 4(b). We find that the deterioration in the relative error logarithmically occurs with the decrease in the bit width. The slope is approximately –6 dB/ bit. In addition, the relative error of the input impedance is also shown in Fig. 4(b). From this figure, the relative error of the input impedance is found to be approximately 40 dB lower than that of the electric field when the bit width is less than 24. Because the amplitude of the electromagnetic field in the vicinity of the feeding point is greater than that of the other points, the relative error of the input impedance is lower. Sufficient improvement of 57 dB in the relative error in the case of 32 bits in comparison with that shown in Fig. 2 is achieved.

One of the advantages of the method used here is that the magnetic field, which is the main cause of the cancellation of significant digits, is directly increased. The numerical value range of approximately an 8 bit equivalency is obtained by this method. Another advantage is that the amplitude of the coefficient that is less than or more than 1 becomes approximately 1 as much as possible. Due to these two effects, the numerical equalization of both the coefficient and electromagnetic field is achieved and this contributes to a remarkable improvement in the relative error. These problems originally occurred because the MKSA unit system is used. This method is calculated using a unit system in which the intrinsic impedance of free space is normalized to 1.

B. *Influence of Absorption Boundary Condition*

The relative error in the PML region is evaluated to investigate the influence of the absorption boundary condition. Fig. 5 shows the electric field and the relative error at the observation line. The relative error in the field region is less than –140 dB. On the other hand, the electric field decreases significantly due to the influence of the loss term of the updated equations because PML (ix = 2 to 8) is a virtual electric wave absorber. The electric field decreases slightly in the third to first layer (ix = 8 to 6), but decreases significantly after the fourth layer (ix = 5). The difference between the fixed point arithmetic and floating point arithmetic occurs outside the region in PML. However, in the field region where originally a high accuracy level is necessary, the relative error has only a slight influence in PML.

Table 4 indicates the calculation results obtained using several PML parameters, the number of layers L, degrees M of a decrement curve, and reflection coefficient R. Here, the relative errors for the cumulative probability of 100% and 50% are shown. We find that the variation in the relative error curve is less than 1 dB for each parameter. It is clarified that the PML parameters do not almost affect the relative error of FDTD method from these results.

5. CONCLUSION

In this paper, the calculation error particular to a fixed point arithmetic that causes a problem when the FDTD method is implemented into FPGAs was examined from the viewpoint of applicability to antenna analysis. Not the calculation accuracy of the FDTD method, but the relative error between the fixed point arithmetic and floating point arithmetic were evaluated. We achieved a remarkable improvement in the relative error in the analysis region by introducing a technique that scales the intrinsic impedance for the cancellation of significant digits of the magnetic field component generated by the ratio of the electric field to the magnetic field. As a result, we achieve operation with the relative error improvement of more than 50 dB using a 32-bit fixed point arithmetic in free space field analysis.

REFERENCES

[1] A. Taflove, "Computational Electrodynamics: The Finite-Difference Time-Domain Method,"Chap. 16 Artech House, 1995.

[2] J.P. Durbano, F.E. Ortiz, J.R. Humphrey, M.S. Mirotznik, and D.W. Prather, "Hardware Implementation of a Three-Dimensional Finite-Difference Time-Domain Algorithm,"IEEE Antennas Propagat. Letters, Vol. 2, pp. 54-57, 2003.

[3] R.N. Schneider, L.E. Turner, and M.M. Okoniewski, "Application of FPGA Technology to Accelerate the Finite-Difference Time-Domain (FDTD) Method,"in Proc. ACM Int. Symp. Field Programmable Gate Arrays, pp. 97-105, CA, 2002.

[4] S. Matsuoka, K. Ohmi, and H. Kawaguchi, "Study of a Microwave Simulation Dedicated Computer, FDTD/FIT Data Flow Machine," IEICE Trans. Electron., Vol.E86-C, no.11, pp. 2199-2206, Nov. 2003.

[5] W. Chen, P. Kosmas, M. Leeser, and C. Rappaport, "An FPGA Implementation of the Two-Dimensional Finite-Difference Time-Domain (FDTD) Algorithm," in Proc. ACM Int. Symp. Field Programmable Gate Arrays, pp. 213-222, CA, 2004.

[6] H. Suzuki, Y. Takagi, R. Yamaguchi, and S. Uebayashi, "FDTD Implementation of FDTD Algorithm," in Proc. APMC2005, P5.91, Dec. 2005.

[7] J.P. Berenger, "A perfectly matched layer for the absorption electromagnetic waves," J.Computational Physics, vol. 114, no. 1, pp. 185-200, 1994.

TABLE 1: SIMULATION SPECIFICATIONS

Number of cells	100×100×100
Cell size	5 mm
Absorbing boundary condition	PML
Number of layers L	7
Number of orders M	2
Reflection coefficient R	10^{-6}
Feed	Delta gap
Incident field	Sine wave
Convergence factor	0.001
Frequency	2.0 GHz

TABLE 2: COEFFICIENT VALUES

	C_{ex}	C_{exly}, C_{exlz}	C_{hx}	C_{hxly}, C_{hxlz}
$\varepsilon_r = 1$, $\sigma = 0$	1.0	215.1	1.0	1.530e-3

TABLE 3: IMPROVED COEFFICIENT VALUES

	C_{ex}	C'_{exly}, C'_{exly}	C_{hx}	C'_{hxly}, C'_{hxly}
$\varepsilon_r = 1$, $\sigma = 0$	1.0	1.02	1.0	0.577

TABLE 4: EFFECT OF PML PARAMETER

Parameters	Relative error (100%) [dB]	Relative error (50%) [dB]
$L = 7, M = 2, R = 10^{-6}$	-126	-141
$L = 4$	-127	-142
$L = 10$	-126	-141
$M = 3$	-126	-142
$M = 4$	-127	-141
$R = 10^{-4}$	-125	-141
$R = 10^{-5}$	-127	-141

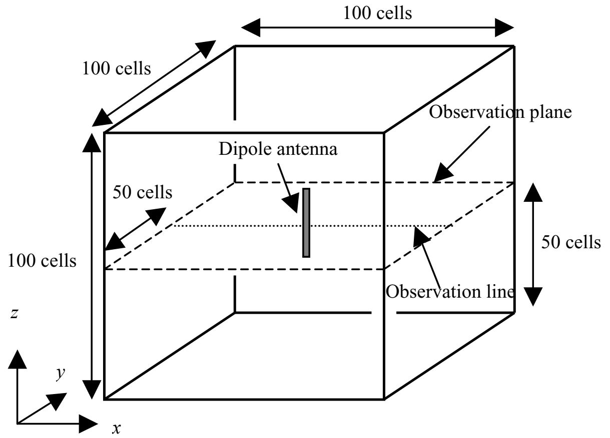


Fig. 1 Simulation model.

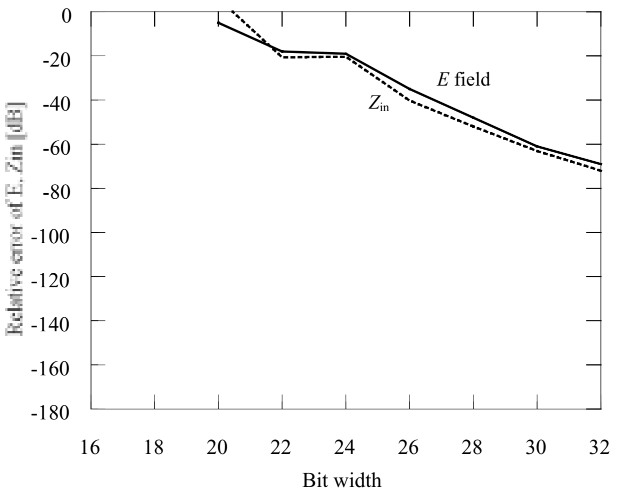
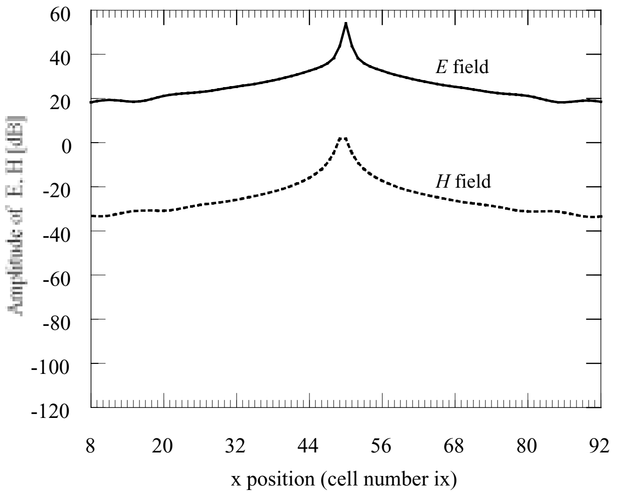
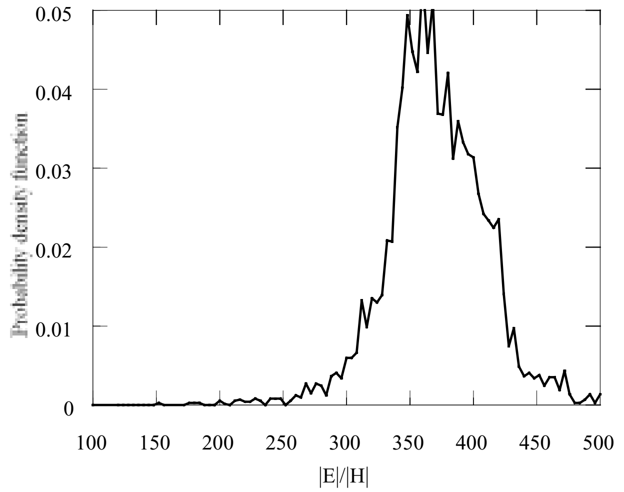


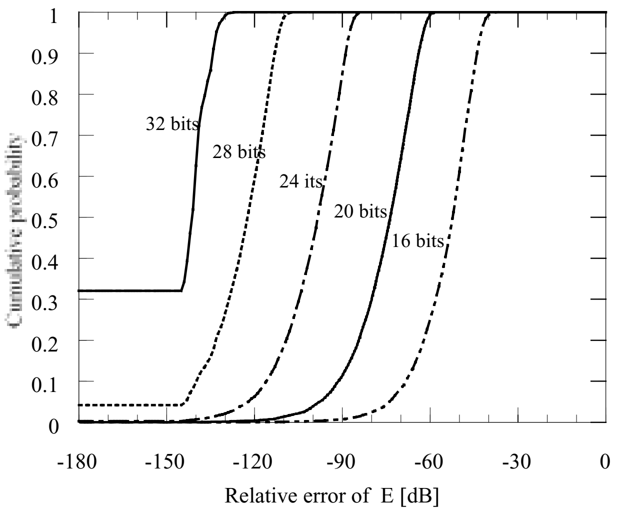
Fig.2 Simulation results.



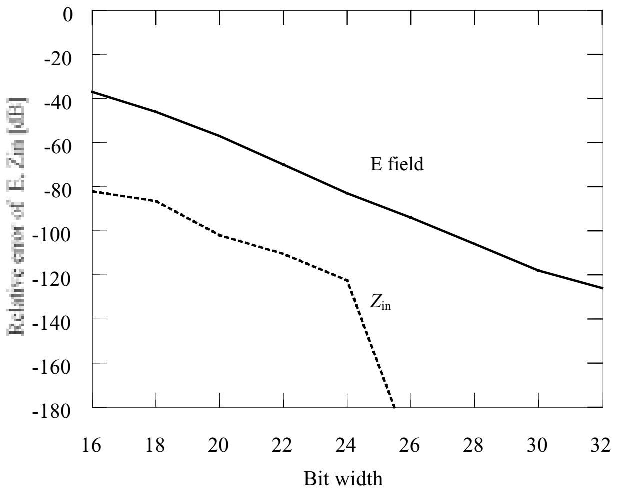
(a) Amplitude of *E* field and *H* field on the observation line.



(b) Ratio between |E| and |H| in the observation plane
Fig.3 Distribution of *E* field and *H* field.



(a) Cumulative probability of relative error



(b) Relative error of *E* field and *Z*in
Fig.4 Relationship between relative error and bit width.

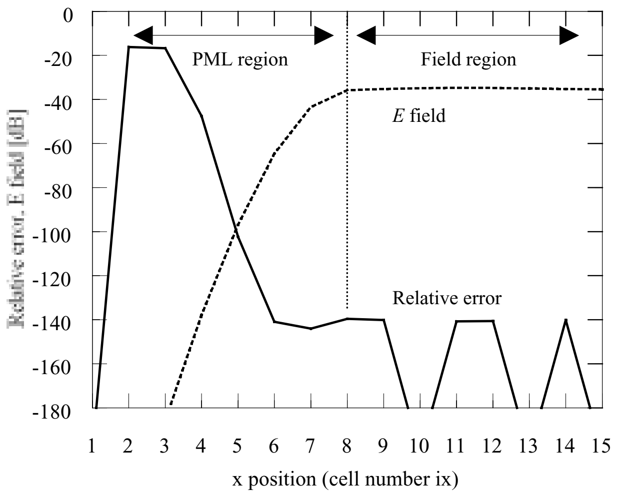


Fig.5 Relative error and *E* field in PML region.