Light Scattering by a Chain of Electrically Small Plasmonic Particles

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Abstract

The plasmons represent a well established collective excitation of metals in the visible and near UV regions. Because of some specific characteristics (such as the permittivity) of the specified particles, electromagnetic wave can be focused and enhanced in certain cases. The different energy distribution depends on the size parameter $q = 2\pi\, a/\lambda$ *(where a stands for the radius of the particle and* λ *denotes the wavelength of incident wave). We have obtained the energy distribution of a periodic structure of electrically small sized plasmonic particles.*

1. Introduction

In the past few years, the noble metallic nanoparticles (*e.g.* Ag and Au) attract lots of attention, because of the so-called surface plasmon resonance (SPR), a collective oscillation of conductive electron in the overall metal. Due to the free electrons of metals, the permittivity exhibits frequency dependent (dispersive), and its real part usually is negative in certain range of visible and near UV regions. When an illuminating light interacts with the metallic nanoparticles, SPR can be induced at a specific frequency. The unique feature leads to a near field enhancement in electric field and a strong scattering. The plasma frequency, ω_p, can be expressed as:

$$\omega_p^2 = \frac{n\, e^2}{\varepsilon_0\, m_{eff}}.$$

The plasmons [1,2] have a profound impact on properties of metals, not least on their interaction with electromagnetic radiation where the plasmon produces a dielectric function of the form:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\tau)},$$

which is approximately independent of wave vector, and the parameter τ is a damping term representing dissipation of the plasmon's energy into the system. When a particle is illuminated by light, the problem has an exact solution presented by the classical Mie theory[3]. The fields around small particles are generally investigated in terms of specific far-field quantities such as absorption, scattering and extinction cross-sections or its far-field scattering diagram. More detailed information follows from the analysis of near-field effects and the investigation of the energy distribution. Although this problem also refers to the classical problem of electromagnetism, it continues to attract attention until now. Due to the development of nanotechnologies, this problem became especially important lately due to the study of light scattering by small particles. It is important for many modern applications, *e.g.*, for field concentration for nanopatterning, [4-6] near-field optical microscopy and other studies, [7-9] laser cleaning, [10,11] high-Q cavity devices.[12-15] When $\varepsilon(\omega_p) = -2 + i\varepsilon''$, according to Reference[16], the absorption cross-section increases inversely proportional to ε''.

In the present work, we investigate the influence of different permittivities on the energy distribution and the peak value of the plasmon. Also we have developed a chain of plasmons of a fixed permittivity. The energy distribution is shown. We analyze the situation depending on the Mie theory.

2. Light Scattering by a Chain of Electrically Small Particles

Geometrical optics [17] yields the simplest approach to understand the energy flux within the weakly absorbing media. This approach is applicable for particles (radius *a*) with a size significantly larger than the radiation wavelength. But in this paper what we are interested is a particle of small size compared with the wavelength.

For an electrically small plasmon (ka = 0.01) illuminated by plane wave propagating in the *z* direction, the incident wave:

$$E_i = E_0 e^{ikz}\, \hat{x}, \text{and } H_i = H_0 e^{ikz}\, \hat{y}$$

where the wave number is denoted by $k = 2\,\pi\, c/\lambda$. Time dependence e^{-iwt} is assumed but omitted herein and subsequently. For simplicity, we assumed a unit amplitude $E_0 = 1$. For fields scattered by a sphere of radius *a*, the fields can be expressed through electric ${}^{e}\Pi^{(s)}$ and magnetic ${}^{m}\Pi^{(s)}$ Debye potentials:

$$r\,{}^{e}\Pi^{(s)} = -\frac{\cos\phi}{k^2}\sum_{l=1}^{\infty} {}^{e}B_l \sqrt{\pi\rho/2}\; H_{l+0.5}^{(1)}(kr)\; P_l^{(1)}(\cos\theta)$$

$$r\,{}^{m}\Pi^{(s)} = -\frac{\sin\phi}{k^2}\sum_{l=1}^{\infty} {}^{m}B_l \sqrt{\pi\rho/2}\; H^{(1)}_{l+0.5}(kr)\; P^{(1)}_l(\cos\theta)$$

$${}^{e}B_l = i^{l-1}\frac{2l+1}{l(l+1)}a_l \text{ and } {}^{m}B_l = i^{l-1}\frac{2l+1}{l(l+1)}b_l$$

where a_l and b_l denote the scattering coefficients. For a small particle with $ka << 1$, one can see from the expansion of the Bessel and Hankel functions that the electric amplitude ${}^{e}B_l \sim (ka)^{2l+1}$ is much larger than the magnetic amplitude ${}^{m}B_l \sim (ka)^{2l+3}$. Expanding separately the numerator and denominator in ${}^{e}B_l$, with an accuracy to $(ka)^{2l+1}$ terms, one can find the formula:

$${}^{e}B_l = i^l (ka)^{2l+1}\frac{\varepsilon-1}{[(2l-1)!!]^2}\left\{l^2\left(\varepsilon+1+\frac{1}{l}\right)\right.$$
$$\left. -i(ka)^{2l+1}\frac{\varepsilon-1}{[(2l-1)!!]^2}\frac{l(l+1)}{2l+1}\right\}^{-1}.$$

When Re(ε)= -2, $l=1$, for Im(ε)>> $2\,(ka)^3$ one can use the approximation:

$${}^{e}B_l = i\frac{\varepsilon-1}{\varepsilon+2}(ka)^3 .$$

For a small particle, the scattered electric field components are written as:

$$E_r^{(s)} = 2\frac{\varepsilon-1}{\varepsilon+2}e^{ikr}\left(\frac{a}{r}\right)^3(1-ikr)\sin\theta\cos\phi,$$

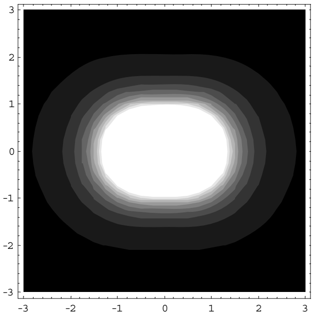
$$E_\theta^{(s)} = \frac{\varepsilon-1}{\varepsilon+2}e^{ikr}\left(\frac{a}{r}\right)^3\left[-1+ikr+(kr)^2\right]\cos\theta\cos\phi,$$

$$E_\phi^{(s)} = -\frac{\varepsilon-1}{\varepsilon+2}e^{ikr}\left(\frac{a}{r}\right)^3\left[-1+ikr+(kr)^2\right]\sin\phi.$$

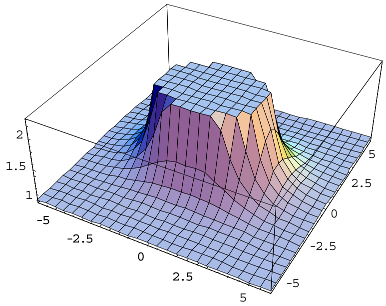
We can see from the formulas that when the real part of the permittivity goes near to -2. The electric field will be very big. The distribution of outgoing energy ($\boldsymbol{E}\cdot\boldsymbol{E}^*$) is shown in Figure 1 where

$$\boldsymbol{E} = \boldsymbol{E}^s + \boldsymbol{E}^{inc} .$$

The permittivity of the particle is $\varepsilon = -2+0.2i$, while its radius is *a*. We assume that the particle is located at the coordinate origin point. Here we have used the normalization on the dimension of the particle. The peak value of the energy is over 2. This is because the real part of the denominator of $\frac{\varepsilon-1}{\varepsilon+2}$ is 0. So the scattered field increases greatly. When we change the permittivity of the particle into $\varepsilon = -2+0.1i$, the peak value increases significantly. This is because the denominator of $(\varepsilon-1)/(\varepsilon+2)$ decreases and the absolute value of the scattered field increases. In Figure 2, we plot the energy distribution, from which the changes of the peak value are apparently made from 2.5 to 6. Also we have obtained the energy distributions of three particles located respectively at *x* = 0, +3, -3 (where we have again used the normalization of the dimensions) in Figure. 3. The permittivity we use is $\varepsilon = -2+0.2i$. It can be seen that the peak value of the energy increases greatly to more than 15. That is because of the great value of the scattered wave.

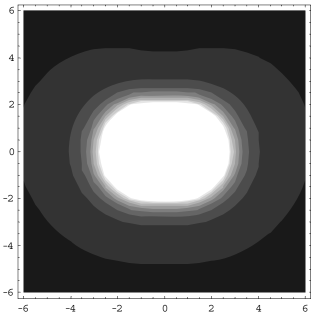


(a) Contour plot

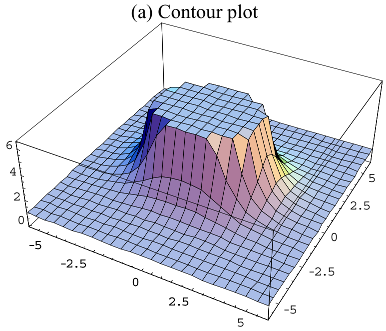


(b) 3-D plot

Fig.1: Outgoing energy distribution of a plasmon with the permittivity of -2+0.2i

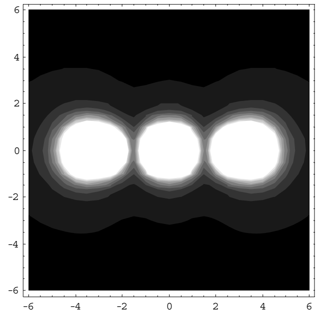


(a) Contour plot



(b) 3-D plot

Fig.2: Outgoing energy distribution of a plasmon with the permittivity of -2+0.1*i*



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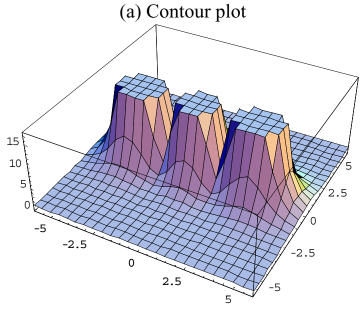
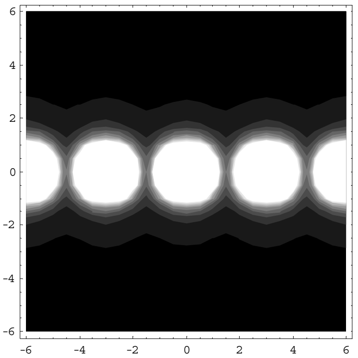
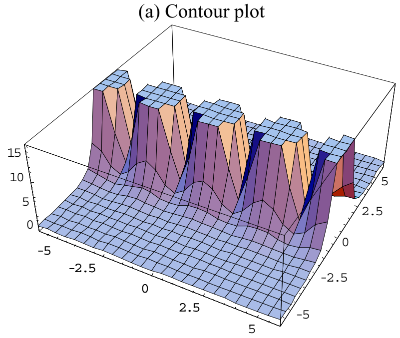


(b) 3-D plot

Fig.3: Outgoing energy distribution of three plasmons with the permittivity of -2+0.2*i*



(a) Contour plot



(b) 3-D plot

Fig.4: Outgoing energy distribution of an infinite chain of plasmon with the permittivity of -2+0.2*i*

Finally, we have considered an infinite chain of plasmons. The particles are placed periodically at $x = 0, \pm 3, \pm 6...$ (where the dimensions have also been normalized). The distribution of outgoing energy ($\boldsymbol{E} \cdot \boldsymbol{E}^*$) is shown in Figure 4. For an infinite chain of particles, the energy distribution is the same regardless of which particle we choose when the incident wave is a plane wave.

3. Conclusion

In the last section of this work, we have shown the energy distribution of different collections of plasmons. Also we have discussed with reasons on the increase of the peak value in the figures. In this present work, we investigated particles of small dimensions only, for micro- and nano-scaled particles. Certainly, we are also interested in, and will discuss in our future work on, some bigger sized particles.

Acknowledgement

The first author wishes to extend his thanks to Dr Anyong Qing for his detailed data and also some formulas provided; and also to Ting Fei and Chengwei Qiu for their efforts in helping with understanding and repeating the Mie theory.

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