Model-Based Inversion of 2-D Scattering Problems

Mansor Nakhkash, Reza Saadat and Mehdi Fallah
Department of Electrical Eng., Yazd University
P.O.Box 89195-741,Yazd, Iran, nakhkash@yazduni.ac.ir

Abstract

Model-based inversion (MBI) is robust and stable when the inverse scattering problem is ill-posed. However, it poses large computational burden, making the inversion inefficient. This paper introduces a 2-D electromagnetic MBI, in which the computation time is improved using an efficient implementation of the source-type integral equation. Employing simulated annealing as the optimization algorithm in the proposed MBI method, several examples demonstrate accuracy of the method in the reconstructions of different object profiles.

1. INTRODUCTION

Inverse scattering problem is to reconstruct properties of an object from the scattering data. On the contrary, the goal in a forward problem is to generate the data scattered from a known medium. In electromagnetic theory, the inversion methods aim at reconstructing the complex permittivity profile of objects using the scattering field. They may be classified into two categories [1]. The first category includes all direct inversion methods, such as diffraction tomography [2], which extract the profile directly by applying a mathematical operator to the measured scattering field.

The second category comprises the so-called model-based inversion (MBI) methods. They rely on an optimization procedure in which the profile of a medium is reconstructed by optimizing a suitable cost function [3]-[8]. The value of this function indicates the closeness between the observed (measured) data and the model data that is obtained from the solution of the forward scattering problem. The MBI provides superior performance over the direct inversion when the inverse problem is ill-posed [1]. The nonlinearity of the forward problem, inadequacy of the scattered data and presence of noise in the data make an inversion ill-posed. The MBI, however, suffers from lack of efficiency due to two reasons: the need to apply global algorithms for optimizing the cost function and the intensive computation required for solving the forward problem. This paper tries to alleviate the second reason.

The solution to the forward problem in MBI, mostly, involves the use of numerical methods that are computationally intensive, such as the method of moment (MOM) [4]-[7]. We have, recently, proposed an efficient method, which evaluates the source-type integral equation (STIE) for 2-D forward problems [9]. The present paper reports the implementation of this method in the 2-D MBI problems and the merits of its application. The cost function is defined as the mean square error (MSE) between the observed scattering field and those calculated from the forward model (i.e. STIE). Two global optimization methods, used prominently in the MBI, are genetic algorithms (GA) [5], [6] and simulated Annealing (SA) [7], [8]. GA and SA have the advantages that they require only the value of the cost function and they are simple to implement. In this paper, the SA algorithm is employed to minimize the MSE function.

Several reconstruction examples are presented to demonstrate the performance of the proposed 2-D MBI method. The observed data is synthetic, in a sense that the FDTD method is used to generate the observed scattering field at different points around objects. The results show our efficient MBI technique provides enough accuracy in the reconstructions of various object profiles.

2. MODEL-BASED INVERSION METHOD

The 2-D MBI method reconstructs the object profile by optimizing a cost function. To formulate the problem, consider the geometry of a 2-D electromagnetic problem shown in Fig. 1, where an inhomogeneous object of finite-size is located in a background medium. Under TM

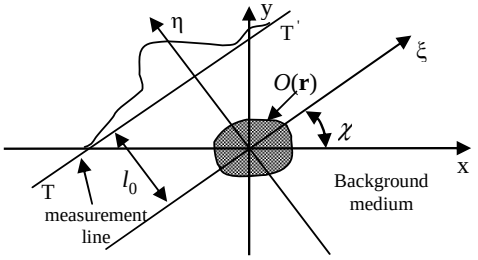

Figure 1. The geometry of a 2-D electromagnetic problem

excitation, the source-type integral equation for a 2-D problem gives the scattered field $\ldots^{sca}(\mathbf{r})$ at any point specified by vector $\mathbf{r} = (x, y)$ as

$$\ldots^{sca}(\mathbf{r}) = \ldots(\mathbf{r}) - \ldots^{inc}(\mathbf{r}) = \nu_0^2 \int_{\vee} G(\mathbf{r} - \mathbf{r}') O(\mathbf{r}') \ldots(\mathbf{r}')\, d^2 r' \quad (1)$$

where $\ldots^{inc}(\mathbf{r})$ is the incident field produced by sources in the absence of the object, $\ldots(\mathbf{r})$ is the total field, $G(\mathbf{r} - \mathbf{r}')$ is known as Green function and $\vee$ defines the object region (the scattering region). The object profile $O(\mathbf{r})$ and the wave

number γ_0 for the background medium are given by

$$O(\mathbf{r}) = \frac{\kappa_r}{\hat{\kappa}_{Br}} - 1 - 2\frac{\omega}{j\varpi\hat{\kappa}_B}, \quad \nu_0 = \varpi\sqrt{\sigma_0\,\hat{\kappa}_B} \qquad (2)$$

where κ_r and ω denote the relative permittivity and conductivity of the object, $\hat{\kappa}_B = \kappa_0\,\hat{\kappa}_{Br} = \kappa_B - 2\,\omega_B / j\varpi$ denotes the complex permittivity of the background medium and ϖ is the angular frequency. It is assumed that the object and background medium are nonmagnetic, i.e. their permeability is σ_0.

The cost function is the MSE between the measured data and those calculated from Eq. (1). Let $\psi^{sca}_{obs}(\mathbf{r}_1), \psi^{sca}_{obs}(\mathbf{r}_2), \ldots, \psi^{sca}_{obs}(\mathbf{r}_N)$ be the scattering field observed (measured) at N locations $\mathbf{r}_n = (x_n, y_n)$ around the object. The observed data can be at different frequencies as well; but, single frequency data is viewed in this paper. Furthermore, let ν be discretized into M rectangular cells, whose size is chosen small enough so that κ_r and ω can be considered constant inside the cells. Introducing the parameter point $\mathbf{x} = (\kappa_{r1}, \omega_1, \kappa_{r2}, \omega_2, \ldots, \kappa_{rM}, \omega_M)$, the MSE function is defined as

$$f(\mathbf{x}) = \frac{1}{2N}\sum_{n=1}^{N}\left|\psi^{sca}_{\mathbf{x}}(\mathbf{r}_n) - \psi^{sca}_{obs}(\mathbf{r}_n)\right|^2 \qquad (3)$$

The field $\psi^{sca}_{\mathbf{x}}(\mathbf{r}_n)$ is obtained from the forward model (Eq. (1)) for the profile specified by the parameters $\mathbf{x}$. The minimization of $f(\mathbf{x})$ converges to the actual constitutive parameters of the object. This is performed using the SA algorithm.

A. *Forward Model*

The STIE equation (1) forms the forward model for the MBI. Because the field inside the object is a function of itself, this equation results in a nonlinear mapping from the object profile to the scattered field. Therefore, the solution to the STIE is not trivial and, mostly involves numerical solutions, such as the MOM, which need intensive computation. An efficient way to solve (1) is to use the Born approximation, in which the incident field $\psi^{inc}(\mathbf{r})$ approximates the total field $\psi(\mathbf{r})$ inside the object. Such an approximation and its variant, Born series, has limited range of validity when the contrast and/or size of the scatterer is small [2].

We have devised an efficient STIE solver [9] that alleviates low contrast/small size requirements of Born series. Starting from a low contrast profile for the complex permittivity of the scatterer, the method obtains iteratively the field by incremental change of the object profile towards the real one. Let $O_k(\mathbf{r})$ denote the profile at the kth iteration. The object profile is modified to $O_{k+1}(\mathbf{r})$ at the next iteration and the field change $\Delta\psi^{sca}_{k21}(\mathbf{r}) = \psi^{sca}_{k21}(\mathbf{r}) - \psi^{sca}_{k}(\mathbf{r})$ that is due to $\Delta O_{k21}(\mathbf{r}) = O_{k21}(\mathbf{r}) - O_k(\mathbf{r})$ can be given by

$$\begin{aligned}\Delta\psi^{sca}_{k21}(\mathbf{r}) &= \nu_0^2\int G(\mathbf{r}-\mathbf{r}')\Delta O_{k21}(\mathbf{r}')\psi^{inc}(\mathbf{r}')\,d^2r' \\ &2\,\nu_0^2\int G(\mathbf{r}-\mathbf{r}')\Delta O_{k21}(\mathbf{r}')\psi^{sca}_{k}(\mathbf{r}')\,d^2r' \\ &2\,\nu_0^2\int G(\mathbf{r}-\mathbf{r}')O_{k21}(\mathbf{r}')\Delta\psi^{sca}_{k}(\mathbf{r}')\,d^2r'\end{aligned} \qquad (4)$$

Considering $O_0(\mathbf{r}) = 0$ (that corresponds to the profile of the background medium), the second and third integrals are zero for k=0. The algorithm can be, therefore, started knowing the incident field. Also, the modification of $O_k(\mathbf{r})$ with regard to k should be nonlinear. We use exponential increment in accordance with difference equation $\Delta\kappa_{r(k21)} = a\,\kappa_{r(k)}$ for the relative permittivity and $\Delta\omega_{(k21)} = b\,\omega_{(k)}$ for the conductivity.

B. *Simulated Annealing Algorithm*

Metropolis algorithm [10] provides a simple method for simulating the physical annealing process of solids. In an annealing process, the temperature of a solid in a heat bath is initially increased to a maximum value at which the solid melts and becomes disordered. The temperature is then lowered slowly so that the system at any time is approximately in thermal equilibrium. As cooling proceeds, the system becomes more ordered and approaches a frozen ground state (minimum energy state) at temperature zero. It is presumed that the initial temperature of the solid is sufficiently high and the cooling is carried out adequately slow; otherwise the system become quenched forming defects or freezing out in meta-stable states (i.e. trapped in a local energy state).

Making an analogy between the system energy and the value of a cost function, simulated annealing algorithm can be viewed as Metropolis algorithm, applied for minimization of the function [10]. In every iteration (k) of the SA, the current point $\mathbf{x}_{k-1}$ is perturbed slightly to generate a new point $\mathbf{x}_k$. If the resulting change in the cost function $\Delta f = f_k - f_{k-1}$ is negative, i.e. the modified cost is lower, the new point approaches the optimum and is accepted. If the change is positive, the new point will not necessarily be rejected and it will be accepted according to the *Boltzmann* probability distribution

$$P_k = \exp(-\Delta f / T_k) \qquad (5)$$

where T_k plays the role of the temperature in the Metropolis algorithm. Random numbers between zero and one are generated for this purpose.

Considering $U[0, 1]$ as a uniform random number in the interval [0, 1], the SA algorithm, applied for the minimization of the MSE function in (3), can be described in pseudo language as follows:

1. Initialize the temperature (T_0).
2. Generate a sample parameter point $\mathbf{y}$ and its MSE value $f(\mathbf{y})$.
3. Select the new point $\mathbf{x}_k$. Acceptance probability $P_k(\mathbf{y})$ is calculated using (5). If $P_k(\mathbf{y}) > U[0, 1]$ then $\mathbf{x}_k = \mathbf{y}$; otherwise $\mathbf{x}_k = \mathbf{x}_{k-1}$.

4. Whether the thermal equilibrium is achieved to update the temperature? If it is not, go to Step2.
5. Decrease the temperature T_k.
6. If stopping rule is satisfied, terminate the algorithm; otherwise go to Step2.

Some Steps in the above algorithm need more elaboration. The initial temperature should be enough high so that all new points are, almost, accepted at the beginning. There are different strategies to determine T_0. In our work, generating a number of points randomly before the start of SA , the initial temperature is set to the standard deviation of the function values at these points. Constant grains $\div\kappa_r$ and $\div\omega$ are randomly added to or subtracted from the permittivity and conductivity of all cells so as to obtain the sample point **y** in Step 2. The required precision for the reconstruction imposes the value of the grains. At each temperature, enough iterations are performed so that thermal equilibrium is roughly achieved. We interpret the equilibrium when the relative change in mean of the MSE is less than a small value (e.g. 0.01). Different annealing schedules exist for the temperature change [10]. In this context, T_k is decreased with respect to

$$T_k \mid \iota\, T_{k41} \qquad (6)$$

where ι is a constant smaller but close to unity. Typical values of ι lie between 0.8 and 0.99. Finally, the algorithm is stopped when the ratio of the number of accepted points to the total number of generated points in a concluding temperature fells below a value $\bullet$, where $\bullet \subset$ [0, 0.1]. The parameters that give the lowest MSE in the entire algorithm are considered as the global solution to the inverse problem.

3. RECONSTRUCTION EXAMPLES

Because of paper deadline, the examples will be included in the paper later on.

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