# Maximum Transmission Through Slits in Adjacent Parallel Conducting Plates 

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#### Abstract

In this study, the problem of electromagnetic coupling through slits in two conducting parallel-plates is studied for the case that the structure is illuminated by a TE (to the slit axis) polarized plane wave with main interest on the understanding about maximum transmission mechanism. It is observed that the maximum of the normalized transmitted power with respect to the incident power density becomes $1 / \pi$ wavelengths, irrespective of the slit widths and the incident angle of the TE wave, when the distance between the slitted conducting parallel-plates is much smaller than the wavelength. For the case that the slit is fed by a parallel-plate waveguide (PPW), the transmitted power beyond the slitperforated conducting plates is maximized for the case that the cavity formed between the edges of the slits is resonated. In addition, the equivalent conductance of the coupling slit for the case of maximum transmission is close to that for the narrow slit, $1 / 120 \lambda_{0}$.


## 1. INTORDUCTION

The problem of electromagnetic coupling between two half-space regions through apertures in slitted parallel conducting screens has been considered by Leviatan [1]. In the study [1] where the structure has been treated mainly as a simplified Near-field Scanning Microscope (NSM), it has been observed that the transmitted power through the slits becomes maximized when the transverse shift between the slits approaches multiples of half wavelength.

In this study, the problem of electromagnetic coupling through slits in two conducting parallel-plates which separate two half spaces is studied for the case that the structure is excited by a TE (to the slit axis) polarized plane wave with
main interest on the maximum transmission mechanism (see Fig. 1) [1]. It is observed that the transmitted power beyond the slit-perforated conducting plates is maximized when the lateral displacement(shift) between the slits becomes approximately multiples of the half wavelength, as found in [1].

For the case that the distance between the slitted conducting plates is much smaller than the wavelength, the maximum of the normalized transmitted power with respect to the incident power density amounts up to $1 / \pi$ wavelengths, independent of the incident angle of the TE wave and the slit widths. Some discussions are given to explain this interesting phenomenon from the viewpoint of cavity resonance and to connect it with the maximum transmission through narrow slit in a thick conducting screen [2], which are thought to be, in working principle, identical with each other.

In addition, for the case that the slit is excited by the TEM mode wave in a parallel-plate waveguide (PPW) (see Fig. 2), instead of the TE polarized plane wave as in Fig. 1, it is observed that the power coupled(transmitted) to the half space region is maximized when the cavity resonator, formed between the edges of the coupling slits in the parallel conducting planes, is resonated.

## 2. THEORY

Fig. 1 shows a cross-sectional view of a pair of slitted conducting parallel-plates ( $z=0$ and $z=d$ ) which separates two half spaces regions 1 and 3. In Fig. 1, $a(A)$ is the width of slit $S_{a}\left(S_{A}\right), d$ is distance between the plates, and $L$ is the distance, in the lateral $(x)$ direction, between the edges of two slits.

The structure is invariant to the $y$-axis and excited by a

TE (to the slit axis, i.e., $E_{y}=0$ ) polarized plane which is incident on the slit $S_{a}$ with an incident angle $\theta_{0}$. The $y$ component incident plane wave can be given by

$$
\begin{equation*}
H_{y}^{i}=H_{0} \exp \left(-j \underline{k_{1}} \cdot \underline{\rho}\right) \tag{1}
\end{equation*}
$$

where $k_{\underline{1}}=k_{1}\left(\sin \theta_{0} \hat{x}+\cos \theta_{0} \hat{z}\right), \underline{\rho}=x \hat{x}+z \hat{z}, \quad k_{n}=k_{0} \sqrt{\varepsilon_{r n}}$, $k_{0}=2 \pi / \lambda_{0}$, and $\lambda_{0}$ is the free space wavelength.

The equivalence principle is employed to divide the original problem into three equivalent situations where the equivalent magnetic current sheets $\pm \underline{M_{a}}\left[= \pm \hat{y} E_{x}^{a}(x)\right.$ $= \pm \hat{y} E_{x}(x, 0)$ over the slit $\left.S_{a}\right]$ and $\pm \underline{M_{A}}\left[= \pm \hat{y} E_{x}^{A}(x)\right.$ $= \pm \hat{y} E_{x}(x, 0)$ over the slit $\left.S_{A}\right]$ on both sides of the shorted slits $S_{a}$ and $S_{A}$, respectively [1].


Fig. 1. Slitted conducting parallel-plates illuminated by a $\operatorname{TE}\left(E_{y}=0\right)$
polarized plane wave.

By expressing the fields in the each region in terms of the fields due to the incident TE wave $\left(H_{y 1}^{s c}\right)$ and the magnetic current sheets $\pm \underline{M_{a}}$ and $\pm \underline{M_{A}}$ over the slits $S_{a}$ and $S_{A}$, one finds that the $y$-component magnetic fields in each region can be expressed as

$$
\begin{align*}
& H_{y 1}=H_{y 1}^{s c}+H_{y 1}^{M a}  \tag{2}\\
& H_{y 2}=-H_{y 2}^{M a}+H_{y 2}^{M A}  \tag{3}\\
& H_{y 3}=-H_{y 3}^{M A} \tag{4}
\end{align*}
$$

where the subscripts denote the $y$-component and the region while the superscripts represent the sources of the fields.

The continuity of tangential fields across the slits $S_{a}$ and $S_{A}$ leads to the coupled integral equations for the magnetic current distributions, which are equal to the tangential electric field distributions $E_{x}^{a}$ and $E_{x}^{A}$ over the slits. By employing the pulse (piecewise constant) basis function and the point matching method, the equations are solved numerically by the method of moments. From the knowledge of the electric distributions $E_{x}^{a}$ and $E_{x}^{A}$, one can compute all the quantities of interests such as the coupled power $P_{a}$ through $S_{a}$, the guided powers $P_{u p}$ and $P_{d n}$ along the guiding structure (formed in the region 2 between the conducting parallel-plates at $z=0$ and $z=d$ ), and the coupled power $P_{A}$ through $S_{A}$ into the region 3.

Similarly, for the case that the structure is excited by the TEM wave in a PPW as shown in Fig. 2, the problem can be solved following the same procedure as in the above case of a TE polarized plane wave incidence as in Fig. 1.


Fig. 2. Slitted conducting parallel-plates excited by the TEM wave in a PPW.

A simple equivalent circuit representation for the TEM wave in the PPW is shown in Fig. 3 in which the normalized
equivalent admittance $y_{s}$ of the slit $S_{a}$ can be obtained from the voltage reflection coefficient $\Gamma_{v}$ as [3]

$$
\begin{equation*}
y_{s}=Y_{s} / Y_{0}=g_{s}+j b_{s}=\left(1-\Gamma_{v}\right) /\left(1+\Gamma_{v}\right) \tag{5}
\end{equation*}
$$

with $Y_{0}=1 /\left(\eta_{1} h\right)$ and $\eta_{1}=\sqrt{\mu_{0} / \varepsilon_{0} \varepsilon_{r 1}}$.


Fig. 3. Simple equivalent circuit representation for the TEM wave in the PPW in Fig. 2. (a) unnormalized. (b) normalized.

## 3. RESULTS AND DISCUSSIONS

In the prior work [1], in which the proposed coupling problem was studied mainly from the viewpoint of a Nearfield Scanning Microscope (NSM), it has been observed that the peaks of the transmitted power $P_{A}$ into the region 3 are occurred when the lateral displacement(shift) $L$ between the slits $S_{a}$ and $S_{A}$ approaches multiples of the half wavelength of the TEM mode wave guidable along the $\pm x$ direction in the region 2. In addition, it has been shown that the peak values of $P_{A}$ strongly depend on the distance $d$ between the slitted conducting plates. However, in the study [1], it seems to have paid little attention on the maximum of the peak values and the dependency of the transmitted power $P_{A}$ on the incident angle $\theta_{0}$.

Fig. 4 shows the normalized powers $\left(P_{a}, P_{A}, P_{u p}\right.$, $\left.P_{d n}\right)$ to the incident power density $P_{0}\left(=\eta_{0}\left|H_{0}\right|^{2} / 2\right)$ of the TM wave and normalized angular power density $D(\theta) / D_{\text {max }}$ of the transmitted wave into the region 3. For reference, note
that $P_{a}=P_{A}+P_{u p}+P_{d n}$ and $P_{A}=\int_{-\pi / 2}^{\pi / 2} D(\theta) d \theta$.
In Fig. 4, all the quantities are nearly independent of the incident angle $\theta_{0}$. Most of the coupled power $P_{a}$ through the slit $S_{a}$ becomes transmitted power $P_{A}$ into the region 3, i.e., $P_{A} \approx P_{a}$. Between the guided TEM mode powers ( $P_{u p}$ and $P_{d n}$ ) along the region 2, the coupled power $P_{u p}$ beyond the wide slit $S_{A}$ is very small and negligible [4].


Fig. 4. Normalized powers w.r.t the incident power density.
Case of $\varepsilon_{r 1}=\varepsilon_{r 2}=\varepsilon_{r 3}=1.0, a=0.01 \lambda_{0}, A=0.2 \lambda_{0}, d=0.001 \lambda_{0}$,
and $L=0.4944 \lambda_{0}$ in Fig. 1.

Hence the wide transverse slit in a PPW with small guide height can be approximately regarded as an open end, as shown in Fig. 5, which behaves like a slightly lossy magnetic wall [4] and the equivalent slit admittance $Y_{s}\left(=G_{s}+j B_{s}\right)$ has small radiation conductance $G_{s}\left(=1 / 120 \lambda_{0}\right)$ which is much smaller than the susceptance $B_{s}$, i.e., $G_{s} \ll B_{s}$.


Fig. 5. Radiation problem from an open slit fed by a PPW with small guide height and its reciprocal (coupling) problem through open slit illuminated by plane wave.

Since most of the fields are confined in the vicinity of the slit, the radiation pattern $D(\theta)$ from the slit $S_{A}$ into the region 3 shows nearly omni-directional characteristics. From this, one can expect that the coupled power into the PPW region would be nearly constant, irrespective of the incident angle of the plane wave, when the slit is illuminated by a plane wave as a reciprocal problem, as shown in Fig. 5.

Though the slit $S_{a}\left(a=0.01 \lambda_{0}\right)$ is small compared to the wavelength $\lambda_{0}$, it should be regarded as a wide slit because the PPW formed between two conducting plates ( $z=0$ and $z=d$ ) has a much smaller guide height $d$ than the slit width $a$, i.e., $a \gg d$. Hence the wide slit $S_{a}$ can be regarded as two open ends $S_{a+}$ and $S_{a-}$ as shown in Fig. 6.

Now one can notice that there are three narrow slits $S_{a+}$, $S_{a-}$, and $S_{A}^{\prime}$ of open end type as shown in Fig. 6 in which two slits $S_{a+}$ and $S_{a-}$ which are illuminated by the plane wave are coupling edges while $S_{A}^{\prime}$ becomes a radiating edge slit. As such the independence of all the power quantities on the incident angle $\theta_{0}$ in Fig. 4 can be understood.


Fig. 6. Enlarged view of the region near the coupling slits to illustrate a cavity formed between the slits.

It is worth while to note that the coupled power $P_{A}$ amounts up to $0.3183 \cong 1 / \pi$, irrespective of the slit width $a$, which means the effective slit width $a_{e f f}$ of the slit $S_{a}$ of width $a$ (actually slit $S_{a+}$ of with $d$ ) can be increased to its maximum value $\left.a_{\text {eff }}\right|_{\max }=\lambda_{0} /\left(\pi \cos \theta_{0}\right)$ when the cavity (of length $L$ ) formed between the narrow open slits $S_{a+}$
and $S_{A}^{\prime}$ which behave like the lossy magnetic walls as mentioned above. Note that this resonance phenomenon is the same as the transmission resonance in the coupling problem through narrow slit in a thick conducting screen [2] which requires the same condition, from the viewpoint of the equivalent circuit representation, as the maximum radiation condition (resonance of an input admittance) in the transmission line model [5] of the rectangular microstrip patch antenna.


Fig. 7. Coupled power $P_{A}$ and equivalent slit admittance $y_{s}=g_{s}+j b_{s}$.

Next, we are to examine the transmission characteristics of the slitted conducting plates for the case that the slit $S_{a}$ is excited by the TEM wave of a PPW as shown in Fig. 2. In order to investigate the structure the normalized equivalent admittance $y_{s}$ of the slit $S_{a}$ is computed, in addition to the coupled powers $\left(P_{a}, P_{A}, P_{u p}\right.$, and $P_{d n}$ ). The plots for the normalized equivalent admittance $y_{s}$ and the transmitted power $P_{A}$ are shown in Fig. 7 in which most of the coupled power $P_{a}$ are transmitted into the region 3, i.e., $P_{a} \approx P_{A}$ while the guided wave powers $P_{u p}$ and $P_{d n}$ are
negligible. As shown in Fig. 7, the conductance $g_{s}=G_{s} / Y_{0}$ linearly increases along with the guide height $h$ and it is very close to the dashed line $\pi h / \lambda_{0}$ which corresponds to the conductance $G_{s}=1 / 120 \lambda_{0}$ of a narrow slit [4]. In order to achieve impedance matching ( $y_{s} \simeq 1+j 0$ ) for obtaining maximum coupling $P_{a}$ through $S_{a}$, there might be lots of possible combinations of the geometrical parameters which would be different from those for the maximization of $P_{A}$.

## 4. CONCLUSION

The problem of electromagnetic coupling through adjacent slitted conducting parallel plates are investigated for the case that a TM polarized plane wave is incident on the slit. From some results, it is observed that the maximum transmitted power normalized to the incident power density is $1 / \pi$ wavelengths independent of the actual slit width and the incident angle of the plane wave. The mechanism of the maximum coupling in the proposed geometry is found to be the same as that through a narrow slit in a thick conducting screen.

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