Extension of Total-Field/Scattered-Field Formulation to PML Media

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Introduction

Finite Difference Time Domain (FDTD) method has been extensively used to calculated electromagnetic scattering problems. For the linear media, based on the linearity of Maxwell equations, the total field can be expressed as the superposition of incident field and scattered field. When the sources of the incident fields are located far away from the scatterers, the total fields at the outer boundary of the FDTD computing space domain consist of both incoming and outgoing waves. Since the ABCs can only absorb the outgoing waves and the scattered fields are always outgoing, it is necessary to extract the incident fields from the total fields so that only the scattered field are present at the boundary. Usually two standard approaches [1-3] are employed to do this. One is to calculate only the scattered field throughout the entire solution space, which is called "pure-scattered field formulation" and has been extended to Perfect Matching Layer (PML) media in one of our previous papers [4]. The other one is to surround the scatterers with a closed surface. Inside the closed surface the total field is calculated and outside the closed surface only the scattered field is calculated. This is called the "total-field/scattered-field formulation". So far, only the unsplit form of the field components have been used for this formulation, which can not be incorporated into PML equations, thus making it unsuitable for a PML medium. In this paper we extend this method to PML media, with a view to take the advantage of PML equations for computation on parallel computers [5]. The results on electric fields inside a dielectric sphere illuminated by a sinusoidal uniform plane wave have been obtained using a CM-5 parallel computer. The results compare well with the exact results as well as the results predicated using a pure scattered-field formulation.

Theory

Total-field/scattered-field formulation to solve a scattering problem is based on the linearity of Maxwell equations, and the total field, scattered field and incident field all satisfy the Maxwell's equations, and hence the FDTD algorithm can be applied equally to those three fields. The basic idea of this formulation is to surround scatterers with a closed surface (called connecting surface), so that the calculation space is divided into two regions, total field region and scattered field region. In the total field region FDTD algorithm is applied to the total field, while in the scattered field region FDTD algorithm is applied only to the scattered field. On the interface between these two regions, the incident field is taken into account. A very detailed study of this formulation can be found in [1].

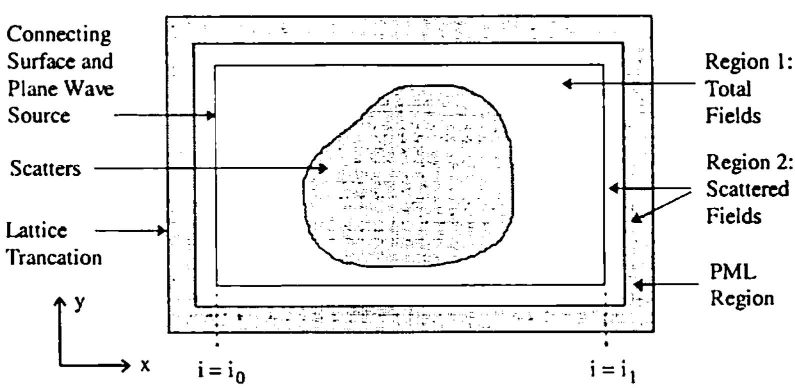

Figure 1 Zoning of the solution space with PML ABC on a xy plane cut, PML region is part of the scattered field region.

The extension of this formulation to PML media is based on the linearity of PML equations. Fig. 1 shows the zoning of the solution space on a xy plane cut. PMLs are in the scattered field region. Only the scattered field propagates into PML region and is absorbed by PMLs. Following the principle in [1], we can derive the 12 discretized PML equations for the total-field/scattered-field formulation as,

$$H_{xy}^{n+1} = R_{aHxy} H_{xy}^{n} - R_{bHxy}\left[E_z^n(i,j+1,k) - E_z^n + C_{HxEz}E_{z,inc}^n\right] \tag{1a}$$

$$H_{xz}^{n+1} = R_{aHxz} H_{xz}^{n} + R_{bHxz}\left[E_y^n(i,j,k+1) - E_y^n + C_{HxEy}E_{y,inc}^n\right] \tag{1b}$$

$$H_{yz}^{n+1} = R_{aHyz} H_{yz}^{n} - R_{bEHz}\left[E_x^n(i,j,k+1) - E_x^n + C_{HyEx}E_{x,inc}^n\right] \tag{1c}$$

$$H_{yx}^{n+1} = R_{aHyx} H_{yx}^{n} + R_{bHyx}\left[E_z^n(i+1,j,k) - E_z^n + C_{HyEz}E_{z,inc}^n\right] \tag{1d}$$

$$H_{zx}^{n+1} = R_{aHzx} H_{zx}^{n} - R_{bHzx}\left[E_y^n(i+1,j,k) - E_y^n + C_{HzEy}E_{y,inc}^n\right] \tag{1e}$$

$$H_{zy}^{n+1} = R_{aHzy} H_{zy}^{n} + R_{bHzy}\left[E_x^n(i,j+1,k) - E_x^n + C_{HzEx}E_{x,inc}^n\right] \tag{1f}$$

$$E_{xy}^{n+1} = C_{aExy} E_{xy}^{n} + C_{bExy}\left[H_z^{n+1} - H_z^{n+1}(i,j-1,k) + C_{ExHz}H_{z,inc}^{n+1}\right] \tag{1g}$$

$$E_{xz}^{n+1} = C_{aExz} E_{xz}^{n} - C_{bExz}\left[H_y^{n+1} - H_y^{n+1}(i,j,k-1) + C_{ExHy}H_{y,inc}^{n+1}\right] \tag{1h}$$

$$E_{yz}^{n+1} = C_{aEyz} E_{yz}^{n} + C_{bEyz}\left[H_x^{n+1} - H_x^{n+1}(i,j,k-1) + C_{EyHx}H_{x,inc}^{n+1}\right] \tag{1i}$$

$$E_{yx}^{n+1} = C_{aEyx} E_{yx}^{n} - C_{bEyx}\left[H_z^{n+1} - H_z^{n+1}(i-1,j,k) + C_{EyHz}H_{z,inc}^{n+1}\right] \tag{1j}$$

$$E_{zx}^{n+1} = C_{aEzx} E_{zx}^{n} + C_{bEzx}\left[H_y^{n+1} - H_y^{n+1}(i-1,j,k) + C_{EzHy}H_{y,inc}^{n+1}\right] \tag{1k}$$

$$E_{zy}^{n+1} = C_{aEzy} E_{zy}^{n} - C_{bEzy}\left[H_x^{n+1} - H_x^{n+1}(i,j-1,k) + C_{EzHx}H_{x,inc}^{n+1}\right] \tag{1l}$$

where R_{aHxy}, R_{bHxy}, C_{aExy}, and C_{bExy} et al. are coefficient arrays which can be obtained easily by following [5]. As in [5], all of the information for media including PECs can be included in the coefficient arrays to make the code parallel computer efficient. The coefficient array C_{EzHy} for the incident field in Eq. (1k) is constructed as

$$C_{EzHy} = \begin{cases} 0 & \textit{Total field region} \\ 0 & \textit{Scattered field region} \\ -1 & \textit{Connecting surface } i = i_0 \\ 1 & \textit{Connecting surface } i = i_1 \end{cases} \tag{2}$$

Other coefficient arrays for incident fields can be constructed similarly. With such arrays, field components in Eqs. (1) become the total fields when they are located in the total field region and become scattered fields when they are located in the scattered field region (including the PML region), while on the connecting surface incident fields are applied. Eqs. (1) are the general PML equations for the total-field/scattered-field formulation which are suitable for any type of incident wave for both magnetic and non-magnetic problems. They are parallel computer efficient, because calculations in total-field region, scattered-field region as well as on connecting surface are performed simultaneously.

Results

To test our methods, the electric fields inside a dielectric sphere illuminated by a sinusoidal uniform plane wave has been computed. The incident field travels to z direction and has only $E_{x,inc}$ and $H_{y,inc}$ components. All other components are zero. Note the incident E field and H field have half cell distance in space and half time step difference in time. In the calculations, 1) the amplitude of the incident E field is considered to be unity. 2) Yee time stepping, $\Delta t = \delta/(2c)$, rather than exponential time stepping, is used, where c is free space velocity, and $\delta = dx = dy = dz$ as uniform cubic cell is used, 3) eight layer PMLs with parabolic variation of conductivity and normal reflection coefficient 0.0001 are used.

Figure 2 shows the zoning of the solution space. The number of cells along the diameter is 30. The frequency of the incident sinusoidal plane wave is 1 GHz. 4500 time steps are chosen. This is equivalent to 6.25 cycles of sinusoidal wave which is found to be sufficient to reach steady state. The results are shown in Fig. 3. The results for the pure scattered formulation, which are obtained with the method in [4], are also shown in Fig. 3 for the purposes of comparison .

As can be seen from Fig. 3 that both total-field/scattered-field formulation results and pure scattered field formulation results compare well with the exact results. For Ez on x axis (Fig. 3(a)), the accuracies of both formulation are the same, while for Ex on y axis (Fig. 3(b)), Ex ox x axis and Ex on z axis (which are not shown here because of limitation of space), the total-field/scattered-field formulation results are slightly more accurate than the pure scattered-field formulation results.

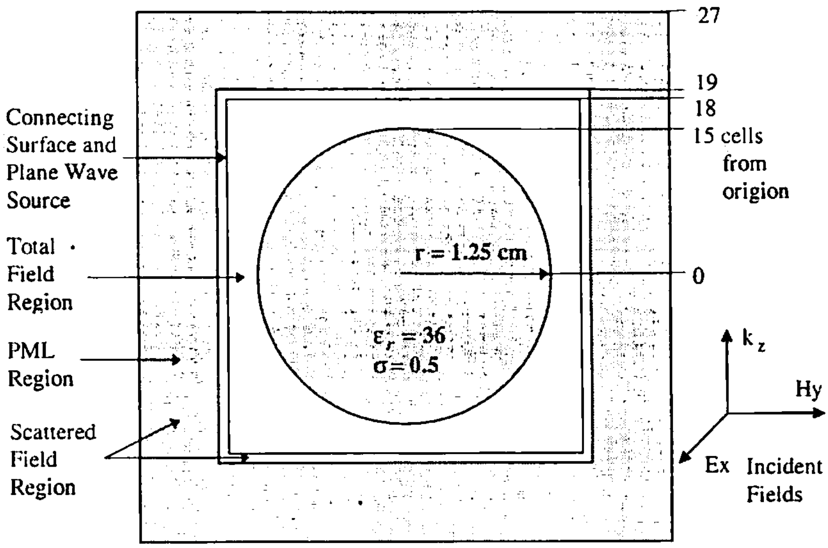

Figure 2. Zoning of the calculation space for the dielectric sphere illuminated by a uniform plane wave. Cubic uniform grids with eight PMLs is used. Cell size is 0.8333 mm. The gap between the PML inner surface and connecting surface is only one cell.

Conclusions

In this paper, the total-field/scattered-field formulation to solve scattering problems has been extended to PML media with the purpose of taking advantages of PML equations on parallel computers. Computer simulation results agree very well with the analytical results.

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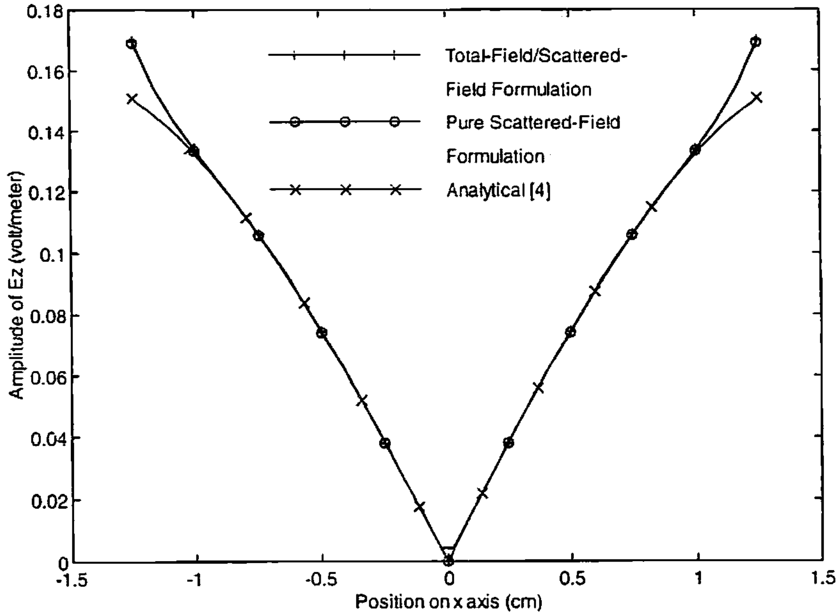
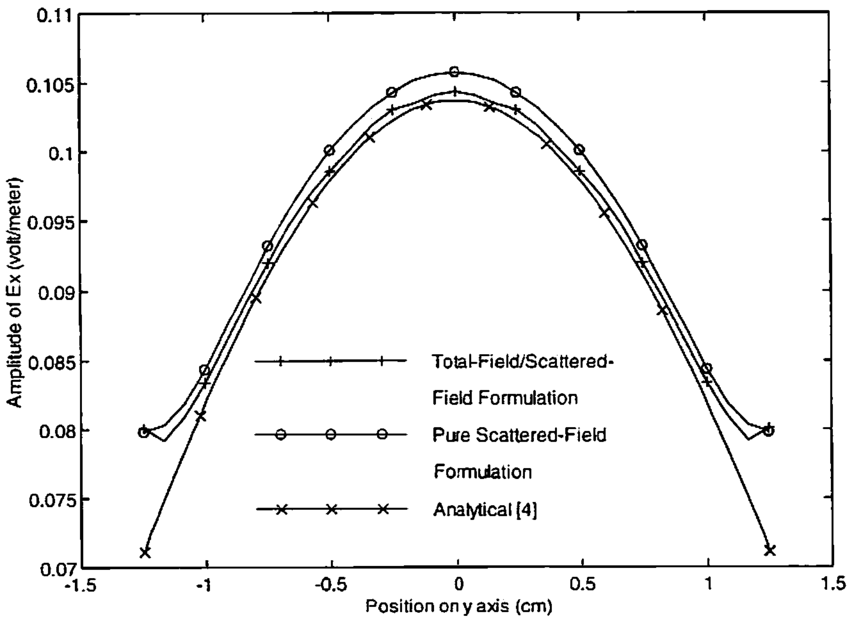


(a) Ez on x axis.



(b) Ex on y axis.

Figure 3 Total fields inside the single layer sphere.